Online Appendix

A Theory: Extensions, Microfoundations, and Discussions

In this appendix, I discuss some extensions and a few additional issues relating to my theory. First, I show how within-sector heterogeneity can be incorporated into the model. Second, I provide other microfoundations for market imperfections, including markups and Marshallian externalities. Third, I discuss the issue of multiple factors. Fourth, I characterize the distinctions between market imperfections and iceberg costs. Fifth, I discuss an extension in which policy instruments have an advantage over market transactions and can directly counteract deadweight losses. Sixth, I discuss the role of social welfare functions. In Section A.7 I solve for expressions in the vertical network example of 2.2; in Section A.8, I provide an additional example to highlight intuitions for how distortionary effects of market imperfections accumulate through backward demand linkages.

A.1 Within-Sector Heterogeneity

The model in the main text features constant-returns-to-scale (CRTS) sectoral production and market imperfections χ_{ij} at the sector-pair level. I now incorporate within-sector heterogeneity into the model. In what follows, I maintain the use "intermediate good" when referring to output aggregated at the sector level, and I use "variety" when referring to differentiated products within each sector. I show that, under a reasonable condition, distortion centrality remains sector-specific (rather than variety-specific), so that ξ_i captures the social value of government subsidies to any firms in sector i. In this case, using sectoral input-output data to compute distortion centrality is without loss of generality.

Consider modifying the model such that each intermediate good i is combined from a continuum of varieties $\nu \in [0,1]$ using a CRTS aggregator

$$Q_i = G_i\left(\{q_i(\nu)\}\right).$$

Producers of variety ν in sector i have a CRTS production function f_i^{ν}:

$$q_i(\nu) = \bar{z}_i z_i(\nu) f_i^{\nu}\left(\ell_i(\nu), \left\{m_{ij}(\nu)\right\}_{j=1}^{S}\right),$$

where $\bar{z}_i$ represents a sector-specific productivity shifter that affects all varieties in sector i, $z_i(\nu)$ is a variety-specific productivity shifter, and $\ell(\nu)$ and $m_{ij}(\nu)$ represent variety-specific input quantities. The fact that cross-sector demand must go through an aggregator $G_i(\cdot)$ implies that different buyers of each intermediate good purchase the same bundle of underlying varieties; this is an important assumption.

Suppose market imperfections and subsidies are variety-specific, i.e. a producer of variety ν in sector i faces imperfection wedge $\chi_{ij}(\nu)$ and subsidy $\tau_{ij}(\nu)$. Given that each variety features CRTS in production, one can relabel each variety as a sector and compute variety-specific distortion centrality $\xi_i(\nu)$. My theoretical results in Section 2 trivially extend to this case, and $\xi_i(\nu)$ captures the social value of policy expenditures on subsidizing variety ν in sector i. More interestingly, distortion centrality remains sector-specific in this environment: $\xi_i(\nu) = \xi_i$ for all ν, where ξ_i is the distortion centrality computed by treating good i as a homogeneous sectoral good, with price index

$$P_i \equiv \min_{\{q_i(\nu)\}} \int p_i(\nu)\, q_i(\nu)\, d\nu \quad \text{s.t. } G_i(\{q_i(\nu)\}) \geq 1.$$

The equivalence $\xi_i(\nu) = \xi_i$ implies that ξ_i captures the social value of policy expenditure in sector i, regardless of which variety ν is being subsidized.

To prove this, note that it is cumbersome to write out the notations for sectoral influence based on nonparametric production elasticities with respect to individual variety of inputs. That being said, Lemma 1 implies that influence can be re-defined using factor price elasticity with respect to productivity shocks. Hence, distortion centrality of variety ν must equal

$$\xi_i(\nu) = \frac{d\ln W/d\ln z_i(\nu)}{p_i(\nu)\, q_i(\nu)}.$$

On the other hand, sector-specific distortion centrality equals

$$\xi_i \equiv \frac{d\ln W/d\ln \bar{z}_i}{\int_0^1 p_i(\nu)\, q_i(\nu)\, d\nu}.$$

The existence of a sectoral aggregator $G_i(\cdot)$ implies

$$\frac{d\ln W/d\ln z_i(\nu)}{d\ln W/d\ln \bar{z}_i} = \frac{\partial \ln P_i}{\partial \ln p_i(\nu)} = \frac{p_i(\nu)\, q_i(\nu)}{\int p_i(\nu)\, q_i(\nu)\, d\nu}. \tag{A.1}$$

The first equality states that the impact of a variety-level productivity shock $z_i(\nu)$ on the factor price, relative to a sectoral-level common shock z_i, can be summarized by the partial derivative of the sectoral price index P_i with respect to the price of variety ν. This is a direct consequence of the aggregator $G_i(\cdot)$, through which cross-sector transactions take place. The second inequality follows by applying the envelope theorem to the definition of sectoral price index. Equation (A.1) implies $\xi_i(\nu) = \xi_i$, proving the claim.

Intuitively, the restriction that cross-sector transactions must take place through sectoral aggregators, implies that all buyers of good j must purchase the same bundle of varieties within sector j, which in turn implies each buyer must be subject to a common market imperfection wedge when purchasing varieties produced by a common sector (i.e. buyer ν in sector i faces the same market imperfection $\chi_{ij}(\nu)$ when buying different varieties in sector j). Without this restriction, imperfections can be variety-pair-specific, in which case the notion of *variety* indeed coincides with the notion of *sector* in the baseline model, and distortion centrality would become variety-specific.

Note that the formulation normalizes the measure of varieties to one but does not impose any restriction on the measure of firms within each sector. Indeed, the notion of *variety* is the level of differentiation at which *heterogeneity* is defined and could differ from the notion of *firm*. For instance, the framework can nest microfoundations with endogenous entry, in which case output per firm of variety v, $\tilde{q}_i(\nu)$, might differ from variety-level total output $q(\nu)$; the two objects relate by the density $\lambda(\nu)$ of firms for variety ν, with $q(\nu) = \lambda(\nu)\tilde{q}(\nu)$. This distinction is conceptually important because my theoretical results rely on CRTS at the level of differentiation (variety), yet models with endogenous entry can feature firm-level production functions without constant returns (e.g., in an earlier version of this paper, I adopted convex-concave production functions as in Buera et al. (2011)).

Lastly, the fact that social value of policy expenditures is independent of within-sector heterogeneity is a feature of the first-order effects that my theory concerns; the selection of firms within each sector does matter for higher-order effects.

A.2 Microfoundations

I provide additional microfoundations to market imperfections. All microfoundations below emit a "wedge" representation with two features: wedges raise production costs, and payments associated with imperfection wedges are quasi-rents that are competed away as deadweight losses in terms of the consumption good.

A.2.1 Financial Frictions

The first microfoundation expands on the running narrative in the main text, i.e. financial frictions. This microfoundation takes the form of working capital requirements on intermediate transactions. Specifically, orders for intermediate inputs must be placed before production and, to prevent hold-up, seller j requires buyer i to pay $\delta_{ij} \geq 0$ fraction of transactional value up front in the form of working capital. Producer i borrows working capital loans Γ_i from a lender at interest rate λ, solving the following profit maximization problem:

$$\max_{\Gamma_i, L_i, \{M_{ij}\}_{j=1}^S} P_i z_i F_i\left(L_i, \{M_{ij}\}_{j=1}^S\right) - \left(\sum_{j=1}^S (1-\tau_{ij}) P_j M_{ij} + (1-\tau_{iL}) W L_i + \lambda \Gamma_i\right) \text{ s.t. } \textstyle\sum_{j=1}^S \delta_{ij} P_j M_{ij} \leq \Gamma_i,$$

where $\delta_{ij} P_j M_{ij}$ is the working capital required for input j, and $\lambda \Gamma_i$ is the total financial costs. Because the producer flexibly chooses how much to borrow, the working capital constraint always binds, and financial costs can be represented by reduced-form proportional wedges $\chi_{ij} \equiv \lambda \delta_{ij}$. For every unit of input j, producer i makes interest payments of $\chi_{ij} P_j$ to the lender; equilibrium prices solve the cost-minimization in (3).

Issuing working capital is costly to the lender because producers cannot commit to repay and instead have an incentive to default that increases with loan size. In order to monitor and enforce repayment, the lender incurs a proportional disutility cost of $\lambda \geq 0$ for every dollar of loans issued. The lender charges a competitive interest rate on loans, earning total income $\Pi \equiv \lambda \sum_{i=1}^{S} \Gamma_i$ as compensation. He spends the income on consumption but earns zero utility net of monitoring costs. The interest payments can therefore be seen as total consumption lost due to the imperfection.

A.2.2 Other Transaction Costs

Note that the financial friction formulation can also be interpreted more broadly as other transaction costs. For instance, if market imperfections arise due to contracting frictions, one can interpret λ as the disutility cost of contract enforcement, incurred by an arbitrator, and δ_{ij} as the fraction of transaction value to be arbitrated.

A.2.3 Markups

Consider sectoral production as a two-stage entry game. In the first stage, a large measure of potential entrepreneurs decide whether to enter each sector i. Entry requires each entrepreneur x to pay a disutility cost κ_i in exchange for a constant returns to scale production function:

$$q_i(x) = z_i F_i\left(\ell_i(x), \left\{m_{ij}(x)\right\}\right).$$

For notational simplicity, I assume firms are identical within each sector. The extension that allows for within-sector heterogeneity, as outlined in Appendix A.1, can be easily integrated into this microfoundation.

Suppose buyers' demand for firm-level output by sector j can be represented by the aggregator

$$Q_i = N_i^{\frac{1}{1-\sigma_i}} \left(\int_0^{N_i} q_i(x)^{\frac{\sigma_i-1}{\sigma_i}} dx\right)^{\frac{\sigma_i}{\sigma_i-1}}, \tag{A.2}$$

where N_i is the measure of firms that entered. The multiplicative term $N_i^{\frac{1}{1-\sigma_i}}$ in the aggregator neutralizes the taste-for-variety effect, so that sectoral production features constant-returns-to-scale in sectoral inputs. Firms behave identically and monopolistically, charging a constant markup $\frac{\sigma_i}{\sigma_i-1}$ over marginal costs. Let $M_{ij} \equiv N_i m_{ij}$ and $L_i \equiv N_i \ell_i$ denote the total inputs used in the sector. Simple substitution shows that sectoral production features CRTS, with total output equal to

$$Q_i = z_i F_i\left(L_i, \left\{M_{ij}\right\}\right).$$

Entrepreneurs receive accounting profits from markups and spend the income on the consumption good, compensating for their disutility entry cost. In equilibrium, free-entry condition pins down the measure of firms in each sector:

$$\underbrace{\kappa_i N_i}_{\text{entry cost}} = \underbrace{\frac{1}{\sigma_i} P_i Q_i}_{\text{accounting profits}} .$$

Entrepreneurs earn zero utility net of entry costs, and no economic profits remain in the economy.

The baseline version of this microfoundation generates, for each seller j, a uniform price wedge $\frac{\sigma_j}{\sigma_j - 1}$ over marginal cost of production. This environment is allocationally equivalent to one in which producers set prices equal to marginal costs, but all buyer i's of good j incur proportional cost $\chi_{ij} = \frac{\sigma_j}{\sigma_j - 1}$, which is deadweight loss in terms of the consumption good. Under this transformation, the microfoundation maps into the accounting convention, used in the main text, that using factor inputs does not directly generate deadweight losses.

Lastly, note that one can always microfound buyer-seller-pair specific imperfections χ_{ij} using markups, by generalizing the aggregator in (A.2) to be buyer-specific aggregator functions, causing sellers in sector j to charge buyer-specific markups $\frac{\sigma_{ij}}{\sigma_{ij} - 1}$ and, correspondingly, proportional deadweight losses that vary across buyer-seller pairs.

A.2.4 Marshallian Externality

Under the previous microfoundation, firms' entry effectively imposes negative externalities upon one another through the aggregator in (A.2). I now show that non-negative imperfection wedges can also represent Marshallian externalities, under which firms impose positive spillovers to each other and underproduce relative to the first-best. Conceptually, the sign restriction $\chi_{ij} \geq 0$ does not assume the direction of spillovers but instead assumes that, holding input-prices constant, sectoral output prices are *higher* when imperfections are present than if they are not.

Specifically, consider again the two-stage entry game, in which a large measure of potential entrants choose whether to incur disutility cost κ_i to enter each sector. Upon entry, each firm x in sector i competitively produces an identical good i according to the production function

$$q_i(x) = z_i \left(\frac{Q_i}{N_i}\right)^{1-\alpha_i} F_i\left(\ell_i(x), \left\{m_{ij}(x)\right\}\right)^{\alpha_i},$$

where $F_i(\cdot)$ features CRTS and $\alpha_i \in (0,1)$. N_i is again the measure of firms in sector i. The term $\left(Q_i/N_i\right)^{1-\alpha_i}$ represents Marshallian externalities; it captures the component of productivity that each firm takes as exogenous but nevertheless depends on the average firm output in the sector. The exponent $(1-\alpha_i)$ controls the strength of Marshallian externality. In any equilibrium, firms within a sector make identical production decisions, and sectoral production emits a CRTS representation despite positive externalities. Firms underproduce relative to the first-best, and despite being price-takers, firms earn accounting profits because they take $\frac{Q_i}{N_i}$ as exogenous while optimizing over a strictly concave production function. There is no pure economic profit due to disutility entry cost. In equilibrium, firms set prices to be private marginal costs and are higher than sectoral marginal costs, thereby generating a seller-specific sectoral wedge that is common to all buyers, $\chi_{ij} \equiv \left(1-\alpha_j\right)/\alpha_j$. As with markups, this environment is allocationally equivalent to one in which producers set prices equal to sectoral marginal costs, and buyers incur deadweight losses proportional to χ_{ij}, thus mapping into the accounting convention in the main text.

A.3 Multiple Factors

The baseline model features a single, composite factor L in fixed supply. This, together with the CRTS assumption, implies an important property in input-output economies: demand changes and variations in production quantity do not affect production costs. Consequently, policy interventions' price effects can be fully summarized by local elasticities, a key property underlying Lemma 1 and my subsequent results. This property is first noted by Samuelson (1951) as the "no substitution" theorem, derived when there is a single factor in the economy.

Now suppose there are multiple factors, $L_1, \cdots, L_K$. To extend my results to this environment, an important assumption is that all factors must enter production only through an aggregator $H\left(L_1, \cdots, L_K\right)$ that is common across sectors. When this is the case, one can re-define a composite factor through the aggregator $L \equiv H\left(L_1, \cdots, L_K\right)$ and define the price W of L as the price index of the aggregator. Relative prices of various factors $L_1, \cdots, L_K$ are unaffected by production quantities; the "no substitution" theorem holds, and so do my subsequent results. On the other hand, if intermediate sectors use factors in varying intensities, then relative factor prices will be affected by production quantities. In this case, resource reallocation induced by policy interventions will generate indirect effects on all prices, and the size of these effects depends on parametric production structures in all sectors of the economy; from this perspective, "no substitution" and my subsequent results fail to hold.

A.4 Imperfections $\neq$ Iceberg Costs

In this appendix, I further distinguish between market imperfections and iceberg costs. I first solve for closed-form allocations in a simple example, and I compare allocations under market imperfections with those under iceberg costs. I then show that distortion centrality is always equal to one in iceberg economies, implying that policy interventions have no first-order effects.

Comparing Allocations Under Market Imperfections and Iceberg Costs

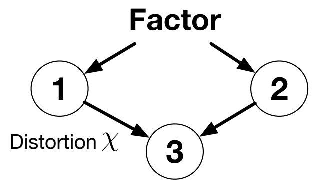
I compare a decentralized economy with imperfections to an economy with iceberg costs, maintaining the same wedge size across the two economies. From the set of cost-minimization problems (3) and (5), it is easy to see that equilibrium prices coincide under these economies. In what follows, I provide closed-form solutions to show:

1. Factor allocations in the imperfection economy do not coincide with those under the first-best. By redistributing resources, policy interventions can raise output Y in the imperfection economy.

2. Factor allocations in the iceberg economy coincide with those under the first best, and output cannot be improved by interventions.

To maximize transparency, I exposit through a simple Cobb-Douglas example.

First, consider an economy with imperfections. Sectors 1 and 2 produce linearly from the factor and sell their entire output to sector 3, which faces imperfections $\chi > 0$ for buying input 1 and no imperfection regarding input 2. The consumption good is produced linearly from good 3.



Following the notations in the paper,

$$\text{(production functions)} \quad Q_1 = L_1, \;\; Q_2 = L_2, \;\; Q_3 = M_{31}^{\alpha} M_{32}^{1-\alpha}, \;\; Y^G = Y_3,$$

$$\text{(market clearing conditions)} \quad Q_1 = M_{31}, \;\; Q_2 = M_{32}, \;\; Q_3 = Y_3, \;\; L_1 + L_2 = L, \;\; Y = Y^G - \chi P_1 M_{31}. \tag{A.3}$$

Absent interventions, factor allocations and output in this distorted economy follow

$$L_1 = \frac{\alpha}{\alpha + (1-\alpha)(1+\chi)} L, \quad L_2 = \frac{(1-\alpha)(1+\chi)}{\alpha + (1-\alpha)(1+\chi)} L.$$

$$Y^G = L_1^{\alpha} L_2^{1-\alpha}, \quad Y = L_1^{\alpha} L_2^{1-\alpha} \left(1 - \frac{\alpha\chi}{1+\chi}\right).$$

Compare these with allocations and output in a first-best economy, without any wedges:

$$L_1^* = \alpha L, \quad L_2^* = (1-\alpha) L, \quad Y^* = (L_1^*)^{\alpha} (L_2^*)^{1-\alpha}.$$

I make two observations. First, relative to the first-best, imperfections cause factor inputs to be underallocated to sector 1 and, conversely, overallocated to sector 2 ($L_1 < L_1^*$, $L_2 > L_2^*$). Second, output under imperfections is lower than the first-best output $Y^* > Y$. The decline in output is due to two effects from imperfections: $Y^* > Y^G$ due to misallocation, and $Y^G > Y$ due to deadweight losses associated with quasi-rents.

Now consider policy interventions that reallocate factor inputs across sectors. Specifically, factor inputs in sector 1 receive a proportional subsidy $\frac{1}{1-\tau_{1L}} = \alpha + (1-\alpha)(1+\chi)$; those in sector 2 receive $\frac{1}{1-\tau_{2L}} = \frac{\alpha+(1-\alpha)(1+\chi)}{1+\chi}$. Under these policies, factor allocations become aligned with those under the first-best, and output becomes

$$Y^G(\tau_{1L},\tau_{2L}) = (L_1^*)^\alpha (L_2^*)^{1-\alpha} = Y^*, \quad Y(\tau_{1L},\tau_{2L}) = Y^*\left(1-\frac{\alpha\chi}{1+\chi}\right).$$

The subsidies correct factor input misallocations and consequently improve both Y and Y^G. In fact, in this example, Y^G under the subsidies prescribed above becomes equal to the first-best output, because misallocation is eliminated entirely. Nevertheless, $Y < Y^G$, as subsidies do not counteract deadweight losses. Note that the level of these subsidies is chosen to align with distortion centrality, following Proposition 3.

Interventions can improve output in the imperfection economy because the size of deadweight losses depends on relative prices. In this example, the total deadweight losses are $\chi P_1 Q_1$ and always equal to a constant fraction ($\frac{\alpha\chi}{1+\chi}$) of gross output. By manipulating relative prices and reallocate productive resources, the level of deadweight losses can be minimized if allocative inefficiency is corrected.

Next, consider an iceberg economy, in which χ units of good 1 are lost for every unit delivered to sector 3 as a production input. The relevant equations become

$$\text{(production functions)} \quad Q_1 = L_1,\ \ Q_2 = L_2,\ \ Q_3 = M_{31}^\alpha M_{32}^{1-\alpha},\ \ Y^{IB} = Y_3,$$

$$\text{(market clearing conditions)} \quad Q_1 = (1+\chi)M_{31},\ \ Q_2 = M_{32},\ \ Q_3 = Y_3,\ \ L_1 + L_2 = L, \qquad \text{(A.4)}$$

where Y^{IB} is the output of consumption good, and there is no distinction between gross and net output in an iceberg economy. Note how iceberg wedge χ enters market-clearing conditions differently than do imperfections (compare A.3 and A.4).

Equilibrium allocations follow:

$$L_1^{IB} = \alpha L, \quad L_2^{IB} = (1-\alpha)L, \quad Y^{IB} = \left(\frac{L_1^{IB}}{1+\chi}\right)^\alpha (L_2^{IB})^{1-\alpha}.$$

Factor allocations are therefore efficient this iceberg economy. Iceberg cost is isomorphic to a negative technology shocks $\left(\frac{1}{1+\chi}\right)$ to sector 1, and there is no room for interventions to improve output.

In fact, one can show that aggregate consumption coincides in an iceberg economy and in an imperfection economy without interventions:[19]

$$Y^* > Y^G > Y = Y^{IB}.$$

[19] That $Y = Y^{IB}$ follows from 1) absent interventions, $Y = WL$ in the imperfection economy, and $Y^{IB} = W^{IB}L$ in the iceberg economy and 2) iceberg costs and my imperfections wedges have identical effects on equilibrium prices, $W = W^{IB}$.

As discussed earlier, the decline in output $Y^* > Y$ in an imperfection economy is due to two effects: $Y^* > Y^G$ due to misallocation and $Y^G > Y$ due to deadweight losses associated with quasi-rents. By contrast, the entire output loss in an iceberg economy ($Y^* > Y^{IB}$) is due to deadweight losses associated with iceberg costs, and there is no misallocation of resources.

To summarize, the following observations hold more broadly in arbitrary networks with CRTS production functions:

1. Market imperfections and iceberg costs generate identical effects on equilibrium prices and aggregate consumption.
2. The two formulations generate different sectoral and factor allocations.
3. The imperfection economy is inefficient, and interventions can raise aggregate consumption by affecting equilibrium prices. The iceberg economy is efficient.

The reason the two formulations differ in terms of sectoral and input allocations is as follows. While both formulations depress input demand, iceberg costs do so by raising the technological production cost of inputs, and demand remain undistorted given prices. By contrast, rather than changing inputs' technological costs, market imperfections distort input demand given prices. Hence, by affecting prices, subsidies can have first-order aggregate effects.

Allocations cannot be solved in closed-form in a nonparametric production network. Nevertheless, observation #1 follows from cost-minimizations (3) and (5), while observation #2 can be demonstrated from the two equations in (12). Below, I demonstrate observation #3 by showing that influence and Domar weights are always equal; thus, Lemma 2 and Proposition 1 imply that, starting from an iceberg economy, policy subsidies have no first-order effects on aggregate output, consistent with the First Welfare Theorem.

Distortion Centrality Is Always One In Iceberg Economies Recall that influence is defined as $\mu' \equiv \beta' (I - \Sigma)^{-1}$. Moreover, Lemma 1 holds, and influence captures factor price elasticity with respect to sectoral TFP shocks.

I now show that sectoral Domar weights, defined as $\gamma_i \equiv \frac{P_i Q_i}{WL}$, are always equal to influence. I start from the market clearing condition for good j in an iceberg economy:

$$Q_j = Y_j + \sum_i M_{ij} \left(1 + \chi_{ij}\right), \tag{A.5}$$

where χ_{ij} represents the proportional loss in good j during transportation to sector i. In the iceberg economy, expenditure shares on inputs are always equal to production elasticities; hence, $P_j M_{ij} \left(1 + \chi_{ij}\right) = \sigma_{ij} P_i Q_i$ for all i and j. By multiplying both sides of (A.5) by $\frac{P_j}{WL}$ and applying the substitution for

$P_j M_{ij}\left(1+\chi_{ij}\right)$, one derives

$$\frac{P_j Q_j}{WL} = \frac{P_j Y_j}{WL} + \sum_i \sigma_{ij} \frac{P_i Q_i}{WL}.$$

In the iceberg economy, $WL = Y^{IB}$; thus, $P_j Y_j / WL = \beta_j$. In matrix notation, the set of equations becomes

$$\begin{aligned} \gamma' &= \beta' + \gamma' \Sigma \\ &= \beta' (I - \Sigma)^{-1}, \end{aligned}$$

establishing that $\xi_i \equiv \mu_i / \gamma_i = 1$ for all i and that policy interventions have no first-order effect on output in iceberg economies.

A.5 Extension: Subsidies Counteract Deadweight Losses

In my model, subsidies redistribute resources but do not counteract deadweight losses. Specifically, given subsidy τ_{ij}, deadweight losses are $\chi_{ij} P_j M_{ij}$ and not $\chi_{ij}\left(1-\tau_{ij}\right) P_j M_{ij}$: the former scales with transactions' *market value*, whereas the latter scales with transactions' *subsidized value*. In the main text, I adopt the first formulation because it effectively subjects policy spending to the same imperfections faced by market-based transactions, thereby isolating only the reallocative effects of policy interventions. Under the latter "subsidized value" formulation, the government has an advantage over private market participants and can directly reduce deadweight losses in the economy by subsidizing all inputs. Policy expenditures' social value in the latter formulation is $SV_{ij} = \xi_i \times \left(1+\chi_{ij}\right)$ and consists of two parts: 1) the reallocative effect captured by sectoral distortion centrality, and 2) the efficiency gained by directly canceling out sectoral imperfections. My model in the main text isolates the first effect. An earlier version of this paper (available upon request) adopts the alternative formulation.

Tables A.1 and A.2 demonstrate that my findings in the main text are qualitatively robust under the alternative formulation, but, because policy spending has an additional advantage in canceling out imperfections, the quantitative welfare gains from policy interventions become even larger. Specifically, Table A.1 reproduces regressions in Tables 11 and 12, replacing the main variable of interest on the right-hand-side from the benchmark distortion centrality measure to the product between estimated distortion centrality and imperfection wedges under various specifications. Because estimated imperfection wedges necessarily include noise and estimation errors, I use the upstreamness measure from Antràs et al. (2012) as an instrument for the main right-hand-side variable in order to correct for attenuation bias in the regressions. All regressions include the full set of sectoral control variables, but only coefficients on the main variable of interest are reported. Table A.2 conducts policy evaluations in China under the alternative formulation. Results show that all three sectoral intervention categories (credit markets, tax policies, and funds to SOEs) generated positive aggregate gains, and the gains are somewhat larger relative to results reported in the main text's Table 13 (note that this exercise is unaffected by the potential attenuation bias and thus needs no correction).

Table A.1: Reduced form coefficients under the alternative specification in Appendix A.5

Specification	Effective Interest Rate	Debt Ratio	Tax Break	Effective Tax Rate	SOE Share of Value-Added
B1	-0.946	2.465	3.503	-1.582	8.650
B2	-1.115	2.907	4.132	-1.866	10.20
B3	-0.870	2.269	3.225	-1.456	7.962
B4	-1.466	3.823	5.432	-2.453	13.41

Table A.2: Policy evaluation under the alternative specification in Appendix A.5

	Aggregate gains ($\Delta \ln Y$) by intervention (in percentage points)			
Specification	Subsidized credit	Tax incentive	SOEs	Total
B1	5.60	2.05	4.14	11.79
B2	3.17	1.19	2.03	6.40
B3	1.78	0.70	1.00	3.48
B4	2.07	0.80	1.49	4.36

A.6 Normative Implications and the Role of A Social Welfare Function

The normative interpretation of my positive characterizations in Propositions 1 and 2 implicitly assumes a social welfare function $U(C,G)$ that places equal marginal value on private and public consumption (for instance, $U(C,G) = C + G$). This is without loss of generality when the government has unrestricted access to lump-sum taxes, which enable one-for-one transfers between private and public consumption. That said, my results are still useful when lump-sum taxes are restricted: the generic welfare impact of a subsidy τ_{ik} financed by marginally cutting back G is

$$\left.\frac{dU/d\tau_{ik}}{dG/d\tau_{ik}}\right|_{\tau=\mathbf{0},\,T\text{ constant}} = -\frac{\partial U}{\partial G} + \frac{\partial U}{\partial C} \times SV_{ik}.$$

The social value of policy expenditures, as characterized in Proposition 1, directly translates into a preference ordering over policy instruments under $U(\cdot)$.

A.7 Solving For the Example in Section 2.2

The elasticity and expenditure share matrices in the example are, respectively,

$$\Sigma = \begin{bmatrix} 0 & 0 & 0 \\ \sigma_2 & 0 & 0 \\ 0 & \sigma_3 & 0 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\sigma_2}{1+\chi_2} & 0 & 0 \\ 0 & \frac{\sigma_3}{1+\chi_3} & 0 \end{bmatrix}$$

The consumption share vector is $\beta' = (0,0,1)$, and the sectoral factor share is $\omega_L' = (1, (1-\sigma_2), (1-\sigma_3))$. Thus, influence follows $\mu' = \beta'(I-\Sigma)^{-1} = (\sigma_2\sigma_3, \sigma_3, 1)$, and Domar weight follows

$$\gamma' = c_1 \cdot \left(\frac{\sigma_2\sigma_3}{(1+\chi_2)(1+\chi_3)}, \frac{\sigma_3}{1+\chi_3}, 1 \right)$$

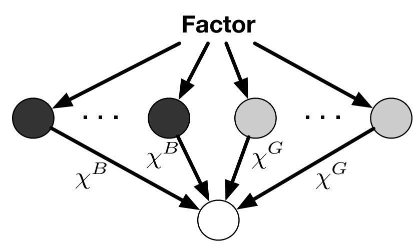
for some constant c_1 such that factor income share sums to one: $\gamma'\omega_L = 1$. Factor allocation in the economy follows

$$(L_1, L_2, L_3) = c_2 \cdot \left(\frac{\sigma_2\sigma_3}{(1+\chi_2)(1+\chi_3)}, \frac{\sigma_3}{1+\chi_3}(1-\sigma_2), 1-\sigma_3 \right)$$

for some constant c_2 that ensures the factor market clearing condition holds with $L_1 + L_2 + L_3 = L$.

A.8 Another Example Network

To gain further intuitions for Proposition 4, consider another example with three types of intermediate sector: black (B), grey (G), and white (W). B's and G's produce from the factor and supply to a single white sector. The white produces from black and grey inputs with imperfections χ^B and χ^G, respectively, and feeds into the final sector.



Imperfections depress white's demand for black and grey inputs. The relative distortion centrality of the black and grey sectors depends solely on relative imperfections faced by these inputs' buyer—the white sector—and is not affected by other fundamentals such as productivities: $\chi^B > \chi^G \iff \xi^B > \xi^G$. When demand for black inputs is more severely depressed, too little factor is allocated to B's and too much to G's; in this case, subsidizing B's generates higher social value because the policy channels productive resources from sectors that are too large to ones that are too small.

B Proofs

B.1 Derivation for Domar weight

The market clearing condition for good j is

$$Q_j = Y_j + \sum_{i=1}^{N} M_{ij}.$$

By multiplying both sides by P_j/WL and substituting for Domar weight $\gamma_j \equiv \frac{P_jQ_j}{WL}$, consumption expenditure share $\beta_j \equiv \frac{P_jY_j}{Y^G}$, and intermediate expenditure share $\omega_{ij} \equiv \frac{P_jM_{ij}}{P_iQ_i}$, we obtain

$$\gamma_j = \frac{Y^G}{WL}\beta_j + \sum_{i=1}^{N} \omega_{ij}\gamma_i$$

or, in matrix notation,

$$\gamma' = \frac{Y^G}{WL} \cdot \beta' (I - \Omega)^{-1}. \tag{B.1}$$

Factor payments in sector i is

$$WL_i = WL\gamma_i\omega_{iL}$$

By the labor market clearing condition, it must be the case that

$$\begin{aligned} \sum_{i=1}^{N} WL_i &= WL\sum_{i=1}^{N} \gamma_i\omega_{iL} \\ \Longleftrightarrow \sum_{i=1}^{N} \gamma_i\omega_{iL} &= 1, \end{aligned}$$

which pins down the proportionality constant $\frac{Y^G}{WL}$ in equation (B.1), deriving

$$\gamma' = \frac{\beta' (I - \Omega)^{-1}}{\beta' (I - \Omega)^{-1} \omega_L}.$$

B.2 Proof of Lemma 1

The equilibrium prices $\{P_i\}$ and W form the fixed point in the system of equations implied by the cost-minimization problems (3) and (5). Totally differentiating these equations, we obtain

$$d\ln P_i = -d\ln z_i + \sigma_{iL} d\ln W + \sum_{j=1}^{N} \sigma_{ij} d\ln P_j$$

$$0 = \sum_{j=1}^{N} \beta_j d\ln P_j$$

or in matrix form,

$$d\ln \mathbf{P} = (I - \Sigma)^{-1} (-d\ln \mathbf{z} + \sigma_L \cdot d\ln W) \tag{B.2}$$

$$0 = \beta' d\ln \mathbf{P}. \tag{B.3}$$

A key property used to derive results in this paper is that production elasticities $\{\sigma_L, \Sigma, \beta\}$ are sufficient statistics to characterize how prices respond to economic shocks, such as the productivity shocks summarized in equation (B.2) and (B.3): one needs not know how these production elasticities change in response to productivity shocks. This follows from the fact that, even with market imperfections and policy subsidies in the economy, producers engage in cost-minimization; accordingly, the envelope theorem can be applied.

From equations (B.2) and (B.3), deriving Lemma 1 is a matter of re-arranging and solving for $d\ln W/d\ln z_i$. In particular, equation (B.2) implies

$$\frac{d\ln \mathbf{P}}{d\ln z_i} = (I-\Sigma)^{-1}\left(\sigma_L \cdot \frac{d\ln W}{d\ln z_i} - \mathbf{e}_i\right)$$

where $\mathbf{e}_i$ is the vector with i-th element equal to one and zero otherwise; plug this into the equation (B.3),

$$\frac{d\ln W}{d\ln z_i} = \frac{\beta'(I-\Sigma)^{-1}\cdot \mathbf{e}_i}{\beta'(I-\Sigma)^{-1}\sigma_L} = \mu_i.$$

The last equality follows from the fact that the numerator is simply the influence vector, and the denominator is equal to one under constant-returns-to-scale. Specifically, CRTS implies production elasticities within each sector must sum to one: $\sum_j \sigma_{ij} + \sigma_{iL} = 1$, which, in matrix notation,

$$\Sigma\mathbf{1} + \sigma_L = \mathbf{1} \iff (I-\Sigma)^{-1}\sigma_L = \mathbf{1},$$

hence

$$\beta'(I-\Sigma)^{-1}\sigma_L = \beta'\mathbf{1} = \sum_i \beta_i = 1.$$

Furthermore, note that market imperfections and subsidies affect prices in ways similar to input-augmenting productivity shocks; thus, we have the following Lemma.

Lemma 4. $\frac{d\ln W}{d\tau_{ij}} = \mu_i\omega_{ij}$ *for all* $i = 1,\ldots,N$ *and* $j = 1,\ldots,N,L$.

B.3 Proof of Lemma 2

Using the income accounting identity (10), we derive

$$\begin{aligned}\frac{d\ln Y}{d\tau_{ij}} &= \frac{1}{Y}\left(\frac{dWL}{d\tau_{ij}} - \frac{d\left(\sum_{i=1}^N S_i\right)}{d\tau_{ij}}\right) \\ &= \frac{1}{Y}\left(WL\cdot\mu_i\omega_{ij} - \frac{d\left(\sum_{k=1}^N\left(\sum_{n=1}^N \tau_{kn}P_nM_{kn} + \tau_{kL}WL_k\right)\right)}{d\tau_{ij}}\right),\end{aligned}$$

in which I have applied Lemma 4 from Appendix B.2 to derive $\frac{dWL}{d\tau_{ij}} = WL \cdot \mu_i \omega_{ij}$ and used the definition of policy spending S_i to expand the second term.

Note that $d \ln Y / d\tau_{ij}$ generically cannot be characterized by reduced-form sufficient statistics and depends instead on structural features of the economy, which govern how production elasticities respond to policy shocks τ_{ij}. The difficulty, as explained in the main text, arises because of the indirect budgetary effects in the second term below:

$$\frac{d\left(\sum_{i=1}^{N} S_i\right)}{d\tau_{ij}} = \underbrace{P_j M_{ij}}_{\text{direct impact from targeted input}} + \underbrace{\sum_{k,n=1}^{N} \tau_{kn} \frac{d\left(P_n M_{kn}\right)}{d\tau_{ij}} + \sum_{k=1}^{N} \tau_{kL} \frac{dWL_k}{d\tau_{ij}}}_{\text{indirect effects due to endogenous changes in network structure}} .$$

However, in the decentralized economy, these indirect effects are zero; hence

$$\left.\frac{d \ln Y}{d\tau_{ij}}\right|_{\tau=\mathbf{0}} = \frac{1}{Y}\left(WL \cdot \mu_i \omega_{ij} - P_j M_{ij}\right).$$

Further, note that even though output differs generically from total factor income ($Y = WL - \sum_i S_i$), they coincide in the decentralized economy, when subsidy expenditures are zero. Accordingly, the equation above can be further simplified to:

$$\left.\frac{d \ln Y}{d\tau_{ij}}\right|_{\tau=\mathbf{0}} = \omega_{ij}\left(\mu_i - \gamma_i\right),$$

as desired. Note that the same derivation applies to value-added subsidies $\{\tau_{iL}\}_{i=1}^{N}$, establishing also that $\left.\frac{d \ln Y}{d\tau_{iL}}\right|_{\tau=\mathbf{0}} = \omega_{iL}\left(\mu_i - \gamma_i\right)$.

B.4 Proof of Proposition 1

From the proof of Lemma 2, we see that

$$\frac{dWL}{d\tau_{ij}} = WL \cdot \mu_i \omega_{ij}, \quad \left.\frac{\sum_{i=1}^{N} S_i}{d\tau_{ij}}\right|_{\tau=\mathbf{0}} = P_j M_{ij} = WL \cdot \gamma_i \omega_{ij}.$$

When lump-sum taxes T are held constant, the first term $dWL/d\tau_{ij}$ captures the marginal gain in private consumption per unit of subsidy τ_{ij}, and the second term $d\left(\sum_{i=1}^{N} S_i\right)/d\tau_{ij}$ captures the required marginal deduction in public consumption per unit subsidy. The social value of policy expenditures is defined as their ratio in the decentralized economy, thus

$$\begin{aligned} SV_{ij} &\equiv -\left.\frac{dC/d\tau_{ij}}{dG/d\tau_{ij}}\right|_{\tau=\mathbf{0},T\text{ constant}} \\ &= \left.\frac{dWL/d\tau_{ij}}{d\left(\sum_{i=1}^{N} S_i\right)/d\tau_{ij}}\right|_{\tau=\mathbf{0},T\text{ constant}} \\ &= \xi_i, \text{ as desired.} \end{aligned}$$

B.5 Proof of Proposition 2

Under constant returns to scale, production elasticities within each sector must sum to one: $\sum_j \sigma_{ij} + \sigma_{iL} = 1$, which, in matrix notation, implies

$$\Sigma\mathbf{1} + \sigma_L = \mathbf{1} \iff (I-\Sigma)^{-1}\sigma_L = \mathbf{1},$$

so that $\mu'\sigma_L = \beta'(I-\Sigma)^{-1}\sigma_L = \beta'\mathbf{1} = 1$. This equality can be rewritten as

$$\sum_{i=1}^{N}\mu_i\sigma_{iL} = \sum_{i=1}^{N}\frac{\mu_i}{\gamma_i}\gamma_i\sigma_{iL} = 1. \tag{B.4}$$

I have chosen the accounting convention that using factor inputs does not directly incur deadweight losses, so that total factor endowment accurately represents the total value of resources. Accordingly, at the decentralized economy, factor allocations follow

$$\frac{WL_i}{WL} = \frac{P_iQ_i\cdot\sigma_{iL}}{WL} = \gamma_i\sigma_{iL},$$

reflecting the fact that production elasticities with respect to factor input is equal to factor expenditure share. Equation (B.4) can therefore be re-written as

$$\sum_{i=1}^{N}\xi_i\frac{L_i}{L} = 1,$$

establishing that distortion centrality averages to one when weighted by sectoral value-added.

Next, consider the aggregate impact of subsidies $\{\tau_{ij},\tau_{iL}\}$. It follows from Lemma 2 that, as a first-order expansion around the decentralized economy, $\Delta\ln Y \equiv \frac{Y|_{\{\tau_{ij},\tau_{iL}\}} - Y|_{\tau\equiv\mathbf{0}}}{Y|_{\tau\equiv\mathbf{0}}}$ can be characterized as

$$\begin{aligned}
\Delta \ln Y &\approx \sum_{i=1}^{N}\left(\sum_{j=1}^{N}\frac{d\ln Y}{d\tau_{ij}}\tau_{ij}+\frac{d\ln Y}{d\tau_{iL}}\tau_{iL}\right) \\
&= \sum_{i=1}^{N}(\mu_i-\gamma_i)\left(\sum_{j=1}^{N}\tau_{ij}\omega_{ij}+\tau_{iL}\omega_{iL}\right) \\
&= \sum_{i=1}^{N}(\xi_i-1)\left(\sum_{j=1}^{N}\tau_{ij}\gamma_i\omega_{ij}+\tau_{iL}\omega_{iL}\right)
\end{aligned}$$

From the definition of sectoral policy spending in equation (6), we see

$$s_i \equiv S_i/WL_i \equiv \left(\sum_{j=1}^{N}\tau_{ij}\gamma_i\omega_{ij}+\tau_{iL}\omega_{iL}\right)\frac{L}{L_i}$$

Hence

$$\Delta \ln Y \approx \sum_{i=1}^{N}(\xi_i-1)\,s_iL_i/L$$

The covariance between ξ_i and s_i, using relative sectoral value-added as the distribution, can be written as

$$\begin{aligned}
Cov(\xi_i,s_i) &\equiv \mathbb{E}[\xi_i s_i]-\mathbb{E}[\xi_i]\,\mathbb{E}[s_i] \\
&= \sum_{i=1}^{N}\xi_i s_i L_i/L-\sum_{i=1}^{N}s_iL_i/L \\
&= \sum_{i=1}^{N}(\xi_i-1)\,s_iL_i/L,
\end{aligned}$$

thereby establishing that $\Delta \ln Y \approx Cov(\xi_i,s_i)$, as desired.

The corollary follows directly from properties of bivariate regressions.

B.6 Proof of Proposition 3

The solution to the problem $\left\{\arg\max_{\{\tau_{iL}\}_{i=1}^{N}} Y\right\}$ can be characterized by the set of first-order conditions $\left\{\frac{dY}{d\tau_{iL}}=0\right\}$. To derive $dY/d\ln\tau_{iL}$, note

$$\frac{dY}{d\tau_{iL}}=\frac{dWL}{d\tau_{iL}}-\frac{d\left(\sum_{k=1}^{N}S_k\right)}{d\tau_{iL}} \tag{B.5}$$

We perform a substitution to express $dS_k/d\tau_{iL}$ using $d\left(S_k/WL\right)/d\tau_{iL}$ and $dWL/d\tau_{iL}$, noting that

$$dS_k/d\tau_{iL} = WL\frac{d\left(S_k/WL\right)}{d\tau_{iL}} + S_k\frac{d\ln WL}{d\tau_{iL}}$$

Substitute for $dS_k/d\tau_{iL}$ in equation (B.5),

$$\begin{aligned}
\frac{dY}{d\tau_{iL}} &= \frac{dWL}{d\tau_{iL}} - \frac{WL\left(\frac{\sum_{k=1}^{N} d(S_k/WL)}{d\tau_{iL}} + \frac{d\ln WL}{d\tau_{iL}}\sum_{k=1}^{N}\frac{S_k}{WL}\right)}{d\tau_{iL}} \\
&= WL\left\{\frac{d\ln WL}{d\tau_{iL}}\left(1-\sum_{k=1}^{N}\frac{S_k}{WL}\right) - \frac{\sum_{k=1}^{N} d\left(S_k/WL\right)}{d\tau_{iL}}\right\} \\
&= WL\left\{\mu_i\omega_{iL}\left(1-\sum_{k=1}^{N}\frac{S_k}{WL}\right) - \frac{\sum_{k=1}^{N} d\left(S_k/WL\right)}{d\tau_{iL}}\right\} \qquad \text{(B.6)}
\end{aligned}$$

the last equality follows from Lemma 4 in Appendix B.2. The second term inside the curly brackets of (B.6) can also be written as

$$\frac{\sum_{k=1}^{N} d\left(S_k/WL\right)}{d\tau_{iL}} = \frac{d\left(\sum_{k=1}^{N}\left(\sum_{n=1}^{N}\tau_{kn}\gamma_k\omega_{kn} + \tau_{kL}\gamma_k\omega_{kL}\right)\right)}{d\tau_{iL}}$$

To characterize this, note that under Cobb Douglas, $\frac{d\omega_{kn}}{d\tau_{iL}} = 0$ for all $kn \neq iL$, and the numerator in Domar weight $\gamma' \equiv \frac{\beta'(I-\Omega)^{-1}}{\beta'(I-\Omega)^{-1}\omega_L}$ is invariant to value-added subsidies; thus,

$$\begin{aligned}
\frac{d\gamma_k}{d\tau_{ij}} &= -\gamma_k \cdot \frac{d\ln\beta'\left(I-\Omega\right)^{-1}\omega_L}{d\tau_{ij}} \\
&= -\gamma_k\frac{\gamma_i\omega_{iL}}{1-\tau_{iL}}
\end{aligned}$$

and

$$\frac{d\tau_{iL}\omega_{iL}}{d\tau_{iL}} = \frac{\omega_{iL}}{1-\tau_{iL}}.$$

Using these expressions, we can simplify the second term inside the curly brackets of (B.6):

$$\begin{aligned}
\frac{\sum_{k=1}^{N} d\left(S_k/WL\right)}{d\tau_{iL}} &= \frac{d\left(\sum_{k=1}^{N}\left(\sum_{n=1}^{N}\tau_{kn}\gamma_k\omega_{kn} + \tau_{kL}\gamma_k\omega_{kL}\right)\right)}{d\tau_{iL}} \\
&= \frac{\gamma_i\omega_{iL}}{1-\tau_{iL}} - \frac{\gamma_i\omega_{iL}}{1-\tau_{iL}}\sum_{k=1}^{N}\left(\sum_{n=1}^{N}\tau_{kn}\gamma_k\omega_{kn} + \tau_{kL}\gamma_k\omega_{kL}\right) \\
&= \frac{\gamma_i\omega_{iL}}{1-\tau_{iL}}\left(1-\frac{\sum_{k=1}^{N}S_k}{WL}\right) \qquad \text{(B.7)}
\end{aligned}$$

The first-order condition (setting the RHS of equation B.6 to zero) can therefore be written as

$$0 \quad = \quad \mu_i \omega_{iL} \left(1 - \sum_{k=1}^{N} \frac{S_k}{WL}\right) - \frac{\gamma_i \omega_{iL}}{1-\tau_{iL}} \left(1 - \frac{\sum_{k=1}^{N} S_k}{WL}\right)$$

The solution features

$$\frac{1}{1-\tau_{iL}} = \frac{\mu_i}{\gamma_i},$$

as desired.

To see that the same levels of subsidies also solve $\left\{\arg\max_{\{\tau_{iL}\}_{i=1}^{N}} Y^G\right\}$, use the definition $Y \equiv Y^G - \Pi$ and the income accounting identity $Y \equiv WL - \sum_{k=1}^{N} S_k$ to obtain

$$\frac{dY^G}{d\tau_{iL}} = \frac{dWL}{d\tau_{iL}} + \frac{d\Pi}{d\tau_{iL}} - \frac{d\left(\sum_{k=1}^{N} S_k\right)}{d\tau_{iL}}.$$

We apply the following substitutions

$$\frac{dS_k}{d\tau_{iL}} = WL \frac{d\left(S_k/WL\right)}{d\tau_{iL}} + S_k \frac{d\ln WL}{d\tau_{iL}}, \quad \frac{d\Pi}{d\tau_{iL}} = WL \frac{d\left(\Pi/WL\right)}{d\tau_{iL}} + \Pi \frac{d\ln WL}{d\tau_{iL}}$$

to obtain:

$$\begin{aligned} \frac{dY^G}{d\tau_{iL}} &= WL\left(\frac{d\ln WL}{d\tau_{iL}} + \left(\frac{d\left(\Pi/WL\right)}{d\tau_{iL}} + \frac{\Pi}{WL}\frac{d\ln WL}{d\tau_{iL}}\right) - \left(\frac{\sum_{k=1}^{N} d\left(S_k/WL\right)}{d\tau_{iL}} + \frac{d\ln WL}{d\tau_{iL}}\sum_{k=1}^{N}\frac{S_k}{WL}\right)\right) \\ &= WL\left(\frac{d\ln WL}{d\tau_{iL}}\left(1 + \frac{\Pi}{WL} - \sum_{k=1}^{N}\frac{S_k}{WL}\right) + \frac{d\left(\Pi/WL\right)}{d\tau_{iL}} - \frac{\sum_{k=1}^{N} d\left(S_k/WL\right)}{d\tau_{iL}}\right) \end{aligned}$$

Following the same steps that preceded equation (B.7), we can show

$$\frac{\sum_{k=1}^{N} d\left(S_k/WL\right)}{d\tau_{iL}} \quad = \quad \frac{\gamma_i \omega_{iL}}{1-\tau_{iL}} \left(1 - \frac{\sum_{k=1}^{N} S_i}{WL}\right)$$

$$\frac{d\left(\Pi/WL\right)}{d\tau_{iL}} = -\frac{\gamma_i \omega_{iL}}{1-\tau_{iL}} \frac{\Pi}{WL}$$

Hence the first-order condition is

$$0 = \left(\mu_i \omega_{iL}\left(1 + \frac{\Pi}{WL} - \sum_{k=1}^{N}\frac{S_k}{WL}\right) - \frac{\gamma_i \omega_{iL}}{1-\tau_{iL}}\left(1 + \frac{\Pi}{WL} - \frac{\sum_{k=1}^{N} S_i}{WL}\right)\right)$$

with solution

$$\frac{1}{1-\tau_{iL}} = \frac{\mu_i}{\gamma_i},$$

as desired.

B.7 Proof of Proposition 4

Let $\delta \equiv \frac{WL}{Y^G}$. We write out both influence μ_j and Domar weight γ_j in their scalar form:

$$\mu_j = \beta_j + \sum_{i=1}^{N} \mu_i \sigma_{ij},$$

$$\gamma_j = \beta_j/\delta + \sum_{i=1}^{N} \gamma_i \omega_{ij},$$

where $\beta_j \equiv \frac{P_j Y_j}{Y^G}$ is the consumption expenditure share, $\omega_{ij} \equiv \frac{P_j M_{ij}}{P_i Q_i}$ is the intermediate expenditure share, and σ_{ij} is the intermediate production elasticity. Dividing both influence by the Domar weight of sector j, we get

$$\begin{aligned} \xi_j &= \frac{\beta_j}{\gamma_j} + \sum_{i=1}^{N} \frac{\mu_i \sigma_{ij}}{\gamma_j} \\ &= \frac{P_j Y_j / Y^G}{P_j Q_j / WL} + \sum_{i=1}^{N} \frac{\mu_i}{\gamma_i} \frac{\sigma_{ij}}{\omega_{ij}} \cdot \frac{\gamma_i}{\gamma_j} \omega_{ij} \\ &= \theta_j^F \cdot \delta + \sum_{i=1}^{N} \xi_i \left(1 + \chi_{ij} - \tau_{ij}\right) \theta_{ij}, \end{aligned}$$

where, recall, $\theta_j^F \equiv \frac{Y_j}{Q_j}$ is the fraction of good j used to produce the consumption good, and $\theta_{ij} \equiv \frac{M_{ij}}{Q_j}$ is the fraction of good j used to produce good i. In deriving the last equality, we have used the property that $\frac{\gamma_i}{\gamma_j}\omega_{ij} = \frac{P_i Q_i}{P_j Q_j} \frac{P_j M_{ij}}{P_i Q_i} = \theta_{ij}$ and that intermediate elasticities and expenditure shares relate by $\frac{\sigma_{ij}}{\omega_{ij}} = \left(1 + \chi_{ij} - \tau_{ij}\right)$.

Let $\mathbf{D} \equiv \left[\chi_{ij}\right]$ be the matrix of market imperfections. In the decentralized economy, $\tau_{ij} = 0$; thus, distortion centrality can be written in matrix form as:

$$\xi' = \delta \cdot \left(\theta^F\right)' \left(I - \left(\Theta + \mathbf{D} \circ \Theta\right)\right)^{-1},$$

as desired.

B.8 A Few Properties of Hierarchical Matrices

I use the term "hierarchical" to describe a square matrix if the matrix has non-increasing partial column sums. That is, a $N \times N$ square matrix $A \equiv \left[a_{ij}\right]$ is hierarchical iff

$$\sum_{k=1}^{K} a_{ki} \geq \sum_{k=1}^{K} a_{kj} \quad \text{for all } i < j \text{ and } K \leq N.$$

Lemma 5. *Let A be a hierarchical matrix. If $\{b_m\}_{m=1}^{N}$ is a non-increasing and non-negative sequence, then*

$$\sum_{m=1}^{N} a_{mi} b_m \geq \sum_{m=1}^{N} a_{mj} b_m \text{ for all } i<j.$$

Proof. Note

$$\begin{aligned}
\sum_{m=1}^{N} a_{mi} b_m &= \sum_{m=1}^{N} a_{mi} b_N + \sum_{m=1}^{N-1} a_{mi}\left(b_m - b_N\right) \\
&= \sum_{m=1}^{N} a_{mi} b_N + \sum_{m=1}^{N-1} a_{mi}\left(b_{N-1} - b_N\right) + \sum_{m=1}^{N-2} a_{mi}\left(b_m - b_{N-1}\right) \\
&\vdots \\
&= b_N\left(\sum_{m=1}^{N} a_{mi}\right) + \sum_{k=1}^{N-1}\left(\left(b_{N-k} - b_{N-(k-1)}\right)\sum_{m=1}^{N-k} a_{mi}\right) \\
&\geq b_N\left(\sum_{m=1}^{N} a_{mj}\right) + \sum_{k=1}^{N-1}\left(\left(b_{N-k} - b_{N-(k-1)}\right)\sum_{m=1}^{N-k} a_{mj}\right) \\
&= \sum_{m=1}^{N} a_{mj} b_m,
\end{aligned}$$

the inequality follows from the fact that A is hierarchical and that $b_{N-k} - b_{N-(k-1)} \geq 0$ for all $k = 1,\ldots,N-1$. □

Lemma 6. *If A and B are two $N \times N$ non-negative hierarchical matrices, then $C \equiv B \times A$ is hierarchical.*

Proof. By definition, $C_{mi} = \sum_{k=1}^{N} B_{mk} A_{ki}$. Let $i<j$. Note

$$\begin{aligned}
\sum_{m=1}^{M} C_{mi} &= \sum_{m=1}^{M}\sum_{k=1}^{N} B_{mk} A_{ki} \\
&= \sum_{k=1}^{N}\left[\left(\sum_{m=1}^{M} B_{mk}\right) A_{ki}\right] \\
&\geq \sum_{k=1}^{N}\left[\left(\sum_{m=1}^{M} B_{mk}\right) A_{kj}\right] \\
&= \sum_{m=1}^{M} C_{mj}.
\end{aligned}$$

The inequality follows from the fact that $\left(\sum_{m=1}^{M} B_{mk}\right)$ forms a non-increasing sequence in k; accordingly, we can apply Lemma 5. Because $\sum_{m=1}^{M} C_{mi} \geq \sum_{m=1}^{M} C_{mj}$ for any $i<j$ and M, we conclude that

C is hierarchical, as desired. $\square$

B.9 Proof of Lemma 3

Let Θ be a hierarchical input-output demand matrix. Let $U \equiv \mathbf{1}'(I-\Theta)^{-1}$ be the upstreamness measure. The goal is to show $U_i \geq U_j$ for all $i < j$.

First, note that $\theta_j^F + \sum_{i=1}^N \theta_{ij} = 1$; that is, the share of good j supplied as an input to the consumption good and other intermediate goods should sum to one. Stacking the equations for all intermediate goods into vector form,

$$\begin{aligned} \left(\theta^F\right)' + \mathbf{1}'\Theta &= \mathbf{1}' \\ \iff \left(\theta^F\right)'(I-\Theta)^{-1} &= \mathbf{1}'. \end{aligned}$$

Let η be the vector with j-th element $\eta_j \equiv \sum_{i=1}^N \theta_{ij}$. The fact that Θ is hierarchical implies that $\eta_i \geq \eta_j$. for all $i < j$. The upstreamness measure can be re-written as

$$\begin{aligned} U &= \mathbf{1}' + \eta'(I-\Theta)^{-1} \\ &= \mathbf{1}' + \eta' \sum_{k=0}^{\infty} \Theta^k. \end{aligned}$$

U_i can be written as $1 + \sum_{k=0}^{\infty} \sum_{m=1}^N \eta_m \left[\Theta^k\right]_{mi}$. We now apply the Lemmas in Appendix B.8. Lemma 6 implies Θ^k is hierarchical for all k. Furthermore, Lemma 5 implies $\sum_{m=1}^N \eta_m \left[\Theta^k\right]_{mi} \geq \sum_{m=1}^N \eta_m \left[\Theta^k\right]_{mj}$ for all k and $i < j$. Thus, we have $U_i \geq U_j$, as desired.

B.10 Proof of Proposition 5

The model in my paper assumes market imperfection wedges χ_{ij}'s are all non-negative. We now prove a slightly stronger version of Proposition 5: under case 2, we model market imperfections as random, and we exchange the strict non-negativity of χ_{ij}'s for the weaker condition that imperfections are *on average* non-negative.

Proposition 6. *Consider a hierarchical production network with input-output demand matrix Θ.*

Case 1. *(Deterministic imperfections) If $\mathbf{D} \circ \Theta$ is non-negative and hierarchical, then*

$$\xi_i \geq \xi_j \text{ for all } i < j \text{ in the decentralized economy.}$$

Case 2. *(Random imperfections) Suppose Θ is lower-triangular. If cross-sector imperfections $\{\chi_{ij}\}$ are i.i.d. and $\mathbb{E}^\chi\left[\chi_{ij}\right] \geq 0$, then*

$$\mathbb{E}^\chi\left[\xi_i\right] \geq \mathbb{E}^\chi\left[\xi_j\right] \text{ for all } i < j \text{ in the decentralized economy,}$$

where the expectation is taken with respect to the distribution of χ_{ij}'s.

Proof of Case 1 (deterministic imperfections)

The plan is to derive a matrix representation of distortion centrality—different from the representation in Proposition 4—so that we can easily apply Lemmas 5 and 6 to show that the distortion centrality is non-increasing under the assumption that $\mathbf{D} \circ \Theta$ is hierarchical.

Note that $\mu' = \beta' (I - \Sigma)^{-1}$ and $\delta \cdot \gamma' = \beta' (I - \Omega)^{-1}$, where $\delta \equiv \frac{WL}{Y^G}$. We have

$$\begin{aligned} \mu' (I - \Sigma) &= \delta \cdot \gamma' (I - \Omega) \\ \iff \mu' - \delta \cdot \gamma' &= \mu' \Sigma - \delta \cdot \gamma' \Omega. \end{aligned}$$

We write out the j-th entry of the equation above and divide both sides by $\delta \cdot \gamma_j$:

$$\begin{aligned} \frac{\mu_j - \delta \cdot \gamma_j}{\delta \cdot \gamma_j} &= \sum_{i=1}^{N} \frac{\mu_i \sigma_{ij} - \delta \cdot \gamma_i \omega_{ij}}{\delta \cdot \gamma_i} \frac{\gamma_i}{\gamma_j} \\ &= \sum_{i=1}^{N} \frac{\mu_i \sigma_{ij} - \delta \cdot \gamma_i \sigma_{ij} + \delta \cdot \gamma_i \left(\sigma_{ij} - \omega_{ij}\right)}{\delta \cdot \gamma_i} \frac{\gamma_i}{\gamma_j} \end{aligned}$$

In the decentralized economy, $\sigma_{ij} = \omega_{ij} \left(1 + \chi_{ij}\right)$; thus,

$$\begin{aligned} \frac{\mu_j - \delta \cdot \gamma_j}{\delta \cdot \gamma_j} &= \sum_{i=1}^{N} \frac{(\mu_i - \delta \gamma_i)\, \omega_{ij} \left(1 + \chi_{ij}\right) + \delta \cdot \gamma_i \omega_{ij} \chi_{ij}}{\delta \cdot \gamma_i} \frac{\gamma_i}{\gamma_j} \\ &= \sum_{i=1}^{N} \frac{(\mu_i - \delta \gamma_i)\, \theta_{ij} \left(1 + \chi_{ij}\right) + \delta \cdot \gamma_i \theta_{ij} \chi_{ij}}{\delta \cdot \gamma_i} \end{aligned}$$

where we have used the substitution that $\omega_{ij} \frac{\gamma_i}{\gamma_j} = \frac{P_j M_{ij}}{P_i Q_i} \frac{P_i Q_i}{P_j Q_j} = \frac{M_{ij}}{Q_j} \equiv \theta_{ij}$. In matrix form, we have

$$\begin{aligned} \frac{\xi'}{\delta} - \mathbf{1}' &= \left(\frac{\xi'}{\delta} - \mathbf{1}'\right) (\Theta + \mathbf{D} \circ \Theta) + \mathbf{1}' (\mathbf{D} \circ \Theta) \\ &= \mathbf{1}' (\mathbf{D} \circ \Theta) \left(I - (\Theta + \mathbf{D} \circ \Theta)\right)^{-1} \\ &= \mathbf{1}' (\mathbf{D} \circ \Theta) \sum_{k=0}^{\infty} (\Theta + \mathbf{D} \circ \Theta)^k \end{aligned}$$

Because the sum of two hierarchical matrices is hierarchical, $\Theta + \mathbf{D} \circ \Theta$ is hierarchical. Lemma 6 implies $(\mathbf{D} \circ \Theta)(\Theta + \mathbf{D} \circ \Theta)^k$ is hierarchical for any k, and so is $(\mathbf{D} \circ \Theta) \sum_{k=0}^{\infty} (\Theta + \mathbf{D} \circ \Theta)^k$. Premultiplying by $\mathbf{1}'$ generates the vector of column sums, which form a non-decreasing sequence by the hierarchical property; hence, $\xi_i \geq \xi_j$ for all $i < j$, as desired.

Proof of Case 2 (random imperfections)

Suppose Θ is hierarchical and lower-triangular, i.e. $\theta_{ij} = 0$ for all $j \geq i$. Let $d_i \equiv \frac{1}{\delta}\xi_i$, $\theta_{N+1,i} \equiv \theta_i^F$, and $\bar{\chi} \equiv \mathbb{E}[\chi] \geq 0$ to simplify notation; we now use induction to show $\mathbb{E}[d_i] \geq \mathbb{E}\left[d_j\right]$ for all $i < j$.

From Theorem (4) and the fact that Θ is lower-triangular, we have $d_N = \theta_N^F = 1$, and

$$
\begin{aligned}
\mathbb{E}[d_{N-1}] &= \theta_{N+1,N-1} + \theta_{N,N-1} d_N (1 + \mathbb{E}[\chi_{N,N-1}]) \\
&= 1 + \theta_{N,N-1}\bar{\chi} \\
&\geq \mathbb{E}[d_N].
\end{aligned}
$$

We next apply mathematical induction: suppose we know $\mathbb{E}[d_i] \geq \mathbb{E}\left[d_j\right]$ for all $s+1 \leq i < j \leq N$; we now show $\mathbb{E}[d_s] \geq \mathbb{E}[d_{s+1}]$.

By independence of χ_{ij}'s, we know $\mathbb{E}[d_n(1+\chi_{n,m})] = \mathbb{E}[d_n](1+\bar{\chi})$ for all $m < n$. By Proposition (4),

$$
d_{s+1} = \theta_{N+1,s+1} + \sum_{n=s+2}^{N} \theta_{n,s+1} d_n (1 + \chi_{n,s+1})
$$

$$
d_s = \theta_{N+1,s} + \sum_{n=s+1}^{N} \theta_{n,s} d_n (1 + \chi_{n,s})
$$

Taking expectation over d_{s+1}, we get

$$
\begin{aligned}
\mathbb{E}[d_{s+1}] &= \theta_{N+1,s+1} + \sum_{n=s+2}^{N} \theta_{n,s+1}\mathbb{E}[d_n](1+\bar{\chi}) \\
&= \theta_{N+1,s+1} + \sum_{n=s+1}^{N} \theta_{n,s+1}\mathbb{E}[d_n](1+\bar{\chi}) \\
&\leq \theta_{N+1,s} + \sum_{n=s+1}^{N} \theta_{n,s}\mathbb{E}[d_n](1+\bar{\chi}) \\
&= \theta_{N+1,s} + \sum_{n=s+1}^{N} \theta_{n,s}\mathbb{E}[d_n(1+\chi_{n,s})] \\
&= \mathbb{E}[d_s].
\end{aligned}
$$

The second equality follows from the fact that the matrix Θ is lower triangular (thus $\theta_{s+1,s+1} = 0$). The inequality follows from Lemma 5, exploiting the fact that
$\{\mathbb{E}[d_{s+1}](1+\bar{\chi}), \mathbb{E}[d_{s+2}](1+\bar{\chi}), \cdots, \mathbb{E}[d_N](1+\bar{\chi}), 1\}$ forms a non-increasing sequence and that Θ

is a lower-triangular hierarchical matrix. The third equality follows from the fact that d_n is independent from $\chi_{n,s}$ for all $n \geq s+1$ (d_n is not a function of imperfections within sector n). Hence, by induction, we have $\mathbb{E}[d_i] \geq \mathbb{E}\left[d_j\right]$ for all $i < j$, as desired.

C Empirical Exercise: Data and Estimation Strategies

C.1 Data Sources and Variable Construction

My empirical analysis relies on four data sources. My first source is the 1970 input-output table of South Korea, which is published by the Bank of Korea, translated from Korean into English, and then digitized into a machine-readable format by Nathaniel Lane, who graciously shared the data with me. My second source is the 2007 Chinese input-output table, published by the National Bureau of Statistics of China.

The third dataset I use is the Chinese Annual Survey of Manufacturing (ASM), an extensive yearly survey of Chinese manufacturing firms, also published by the National Bureau of Statistics of China. The ASM is weighted towards medium and large firms, and includes all Chinese manufacturing firms with total annual sales of more than 5 million RMB (approximately $800,000), as well additional state-owned firms with lower sales. The years covered include 1998 through 2007. The data provide detailed information on production, including real and nominal output, assets, number of workers, wages, inputs, public ownership, foreign ownership, and sales revenue. The dataset also contains information on manufacturing firms' balance sheets, including external liabilities, interest payments, and production subsidies received from the government. The ASM dataset is well-studied in the literature; for instance, detailed descriptions of the data appear in Brandt et al. (2012) and Du et al. (2014). I use this panel data for production function estimation, and I extract policy variables including debt-to-capital ratio (defined as total external liabilities relative to total assets), effective interest rates (ratio of interest payments to total external liabilities, conditioned on reporting positive interest payments), government subsidies, and SOEs' sectoral value-added shares from the 2007 cross-section. Following Hsieh and Song (2015), I identify SOEs as either firms that are legally registered as state-owned or firms for which the state is the controlling shareholder.

My fourth data source is the administrative enterprise income tax records, which contain detailed tax rate information for individual manufacturing firms. The data are collected by the Chinese State Administration of Taxation, which is China's counterpart to the IRS and is responsible for collecting tax, auditing, and supervising tax incentive programs. Descriptive statistics of the administrative tax records can be found in Chen et al. (2018). From the 2008 edition of this data, I compute effective corporate income tax rates for each firm and extract information on whether a firm received tax incentives and tax breaks in the fiscal year.

All policy variables extracted from the two firm-level datasets have outliers removed at 1% above and below. Sectors in both ASM and the administrative tax records are coded at the four-digit CIC (Chinese industrial codes) level, which I harmonize to the 135 sector input-output tables.

C.2 Estimating Market Imperfections

I estimate market imperfections using five strategies:

1. Specification B1 follows the methodology and code of De Loecker and Warzynski (2012). The strategy first estimates sectoral-level translog production functions using the "control function" approach (Olley and Pakes (1996), Levinsohn and Petrin (2003), Ackerberg et al. (2015)) and then recovers firm-level "wedges" between production elasticities and expenditure shares over flexible inputs, which I then average to sector-level.

2. Specification B2 employs the approach outlined by Gandhi, Navarro, and Rivers (Gandhi et al.), who estimate production elasticities from dynamic panels by exploiting first-order conditions over flexible inputs. Compared to the "control function" approach, this strategy requires less stringent assumptions about information sets and timing of production decisions, but it does require a sample of "control" firms that are not subject to market imperfections and choose input quantities to equate input expenditure shares to production elasticities. I use foreign firms (defined as firms with over 50% foreign equity share) operating in China as the control group, under the assumption that foreign firms are subject to fewer market imperfections (such as financial and contracting frictions) than domestic firms in China. Based on foreign firms' dynamic input choices, I estimate flexible translog production functions, from which I recover "wedges" between production elasticities and expenditure shares for private firms.

3. Specification B3 relies on Rajan and Zingales (1998)'s measure of external financial dependence and applies to both South Korea and China. The measure captures the fraction of external financing required for capital expenditures in U.S. industries and is likely a lower bound on financial dependence in the two developing economies that I study. To convert the measure into proportional wedges over input costs, I winsorize the measure from below at zero (because financial costs should not be negative) and then interact with the prevailing interest rate in the respective economies: 12.33% for China in 2007, based on the highest sectoral average interest rate from the manufacturing survey data, as reported in Table 9, and 22% for South Korea in 1970, based on International Financial Statistics released by the International Monetary Fund.[20]

4. Specification B4 uses accounting profits' sectoral revenue share (i.e. Lerner index) recorded in the national IO tables as the sectoral wedge; the estimates are available for both economies.

Table C.1 reports summary statistics for market imperfections across specifications. The vast majority of sectoral wedges are positive; this is not mechanical except for B3. For instance, because specification B2 estimates production functions from foreign firms that operate in China, positive wedges in

[20] International Monetary Fund, Interest Rates, Discount Rate for Republic of Korea [INTDSRKRM193N], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/INTDSRKRM193N.

this case reflect the fact that foreign firms use inputs more intensively then domestic, non-state-owned firms, suggesting the latter face greater market imperfections. Likewise, specification B1 estimates production functions based on the Olley-Pakes method and does not mechanically guarantee $\chi \geq 0$. Table C.1 therefore supports my assumption that $\chi \geq 0$. Moreover, by Proposition 5, upstream sectors tend to have higher distortion centrality in expectation as long as the market imperfection wedges are non-negative on average, which holds true under all specifications.[21]

Table C.1: Mean and dispersion of estimated market imperfections

		South Korea in 1970			China in 2007	
Specifications	mean(χ)	$sd\,(\chi)$	% sectors with $\chi > 0$	mean(χ)	$sd\,(\chi)$	% sectors with $\chi > 0$
Benchmark measure $\xi_i^{10\%}$	0.1	0	100%	0.1	0	100%
B1 De Loecker and Warzynski	-	-	-	0.18	0.08	100%
B2 Foreign firms as controls	-	-	-	0.11	0.13	86%
B3 Rajan and Zingales	0.07	0.05	100%	0.05	0.03	100%
B4 Sectoral profit share	0.18	0.17	100%	0.10	0.10	96%

When constructing distortion centrality, I winsorize the estimated wedges from below at zero—in accordance with my modeling assumption—and, because B1-B3 only recover wedges for manufacturing sectors, I specify wedges in the missing sectors to be the average wedge among manufacturing sectors. Table C.2 reports distortion centrality's range and standard deviation across estimated specifications, and Table C.3 reports the pair-wise correlations. Table C.4 shows the high correlations are insensitive to either winsorization or to setting wedges in missing sectors to zero. Table C.5 conducts policy evaluation using additional specifications of distortion centrality. The first row computes distortion centrality using sectoral wedges specified as the sum of wedges from B3 and B4—the goal is to capture both financial frictions and markups. Based on this specification, sectoral policies in China generate 5.01% aggregate gains, which is of comparable magnitude to estimates based on B1 and B2. The remaining rows rerun policy evaluation using specifications B1–B4 without adjusting international trade, i.e., distortion centrality are computed under the assumption that China is a closed economy. The table shows that adjusting for trade does not significantly change my quantitative policy evaluation results.

[21] A potential issue with strategies B1 and B2 is that they confound government interventions as part of market imperfections, measuring $\chi - \tau$ as χ. First, this suggests that wedges are positive even net of subsidies, further supporting my assumption that $\chi \geq 0$. Second, I argue and present evidence in Section 4.3 and D that my findings are quantitatively robust to the mismeasurement.

Table C.2: Dispersion and range of estimated distortion centrality

Specifications		South Korea in 1970			China in 2007		
		sd(ξ)	min	max	sd(ξ)	min	max
Benchmark measure $\xi_i^{10\%}$		0.08	0.92	1.41	0.22	0.56	1.47
B1	De Loecker and Warzynski	-	-	-	0.42	0.19	1.96
B2	Foreign firms as controls	-	-	-	0.25	0.51	1.60
B3	Rajan and Zingales	0.06	0.93	1.25	0.11	0.78	1.27
B4	Sectoral profit share	0.16	0.81	2.31	0.17	0.64	1.37

Table C.3: Pair-wise correlation of distortion centrality based on estimated imperfections

	South Korea		China			
	B3	B4	B1	B2	B3	B4
B1	-	-	1			
B2	-	-	0.97	1		
B3	1	0.90	0.98	0.95	1	
B4	0.90	1	0.99	0.95	0.97	1

Table C.4: Correlation between baseline and alternative specifications

Specifications		no winsorization		setting $\chi = 0$ for missing sectors	
		South Korea	China	South Korea	China
B1	De Loecker and Warzynski	-	1.00	-	0.99
B2	Foreign firms as controls	-	0.95	-	0.98
B3	Rajan and Zingales	1.00	1.00	0.95	0.99
B4	Sectoral profit share	1.00	1.00	1.00	1.00

D Empirical Exercise: Robustness and Additional Results

D.1 Robustness: Data Aggregation

In Section 4.3, I acknowledge the possibility of a mismatch between the level of aggregation in IO tables and the level of product differentiation at which my theory applies, either because a sectoral aggregator over varieties fails to exist or because data are mismeasured due to firms operating across industries and conducting multi-stage production in house.[22] While I cannot conclusively verify distortion centrality stability when underlying product differentiation is finer than the data available, I can indeed conduct the robustness check in reverse, testing the stability when I use *even coarser* data

[22]Orr (2018) documents a high fraction of vertically integrated firms in Indian manufacturing sectors. Conceptually, vertical integration could arise endogenously to minimize frictions associated with inter-firm trade (Williamson (1985)), but such joint-production arrangements could also generate new sources of market imperfections—for instance, through low-powered incentives (Grossman and Hart (1986)).

Table C.5: Evaluating sectoral interventions in modern-day China using an additional specification

Distortion centrality specification	$sd(\xi)$	Aggregate gains ($\Delta \ln Y$) by intervention (in percentage points): Subsidized credit	Tax incentive	SOEs	Total
Based on the sum of wedges from B3 and B4	0.31	2.39	0.90	1.72	5.01
Without accounting for international trade					
B1	0.36	2.01	1.04	2.14	5.19
B2	0.21	1.08	0.57	0.97	2.61
B3	0.10	0.84	0.32	0.57	1.73
B4	0.16	0.90	0.43	0.91	2.24

than those available.

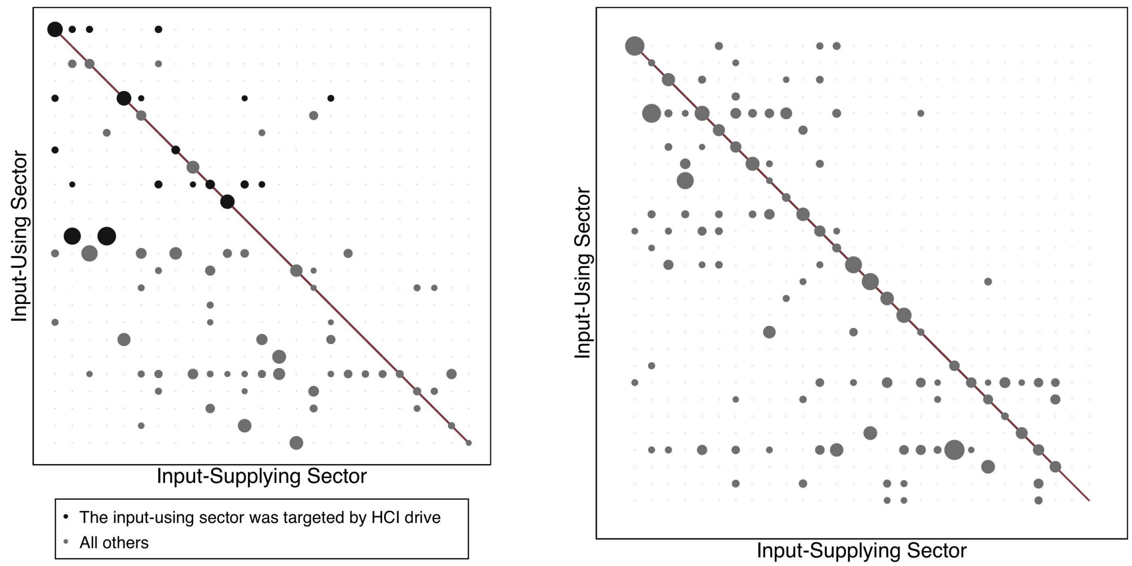
To this end, I merge sectors and progressively create coarser sectoral partitions over several iterations, and I re-compute distortion centrality using the collapsed IO tables at each iteration. For South Korea, the original, disaggregated table has 148 sectors, and I create collapsed tables with 54, 25, and 16 sectors. For China, the original table has 135 sectors and the collapsed tables have 57, 28, and 17 sectors. When merging sectors, standardized codes are followed whenever possible. 54 sectors in South Korea and 57 sectors in China correspond to two-digit sectors; 25 and 28 sectors correspond closely to sectoral definitions in the World Input-Output Data; the coarsest partition (16 and 17 sectors) only differentiates broad sectors (e.g. textiles, chemicals, metals, non-metals, machines). The number of sectors in collapsed IO tables differs slightly across the two economies due to differences in their disaggregated industrial codes.

Table D.1: Distortion centrality based on coarse IO tables correlates highly with benchmark measure

	Average correlation with benchmark $\xi_i^{10\%}$			
	South Korea in 1970		China in 2007	
Number of sectors (N)	Pearson's r	Spearman's ρ	Pearson's r	Spearman's ρ
$N^{SK}=54$, $N^{CN}=57$	0.95	0.96	0.97	0.99
$N^{SK}=25$, $N^{CN}=28$	0.94	0.91	0.97	0.94
$N^{SK}=16$, $N^{CN}=17$	0.94	0.88	0.98	0.95

Table D.1 shows that, at all aggregation levels, benchmark distortion centrality computed from the collapsed IO tables almost perfectly correlates with the benchmark measure computed from the original, disaggregated tables. The stability is once again due to the hierarchical property of the

Figure D.1: Collapsed IO demand matrices are hierarchical in South Korea (left) and China (right)



collapsed IO tables. Figure D.1 visualizes the IO demand matrix for collapsed tables with 25 and 28 sectors for South Korea and China, respectively. Comparing figures 2 and D.1 reveals an interesting, fractal-like property of these networks: linkages across broad sector categories seem to follow a hierarchical structure, as do linkages within each broad category and across more narrowly defined sectoral definitions.

D.2 Robustness: Policy-Induced Measurement Errors

In Section 4.3, I acknowledge the possibility of policy-induced measurement errors in the IO demand matrix Θ and market imperfections χ. I argue that these errors are second-order, and, if anything, correcting for errors in χ would strengthen my findings.[23] I now quantitatively verify these intuitions in the Chinese context by correcting for policy-induced measurement errors using actual policy variations. For simplicity, I compute sector-specific policy wedges τ_i as the total policy spending in each sector relative to sectoral revenue. I normalize τ's to mean zero, assume τ_i applies equally to all inputs in sector i, and correct separately and jointly for errors in Θ and χ.[24] Correspondingly, I

[23]Policy-induced errors in χ can arise in specifications B1 and B2 because they misattribute imperfections net of subsidies $(\chi-\tau)$ as true imperfections χ. Specifications B3 and B4 are, in principle, not subject to this issue; nevertheless, my robustness tests below can be seen as sensitivity analysis with respect to systematic errors in χ.

[24]Distortion centrality in the main text is computed as

$$\xi' \propto \left(\theta^F\right)' \left(I - \Theta \circ (1+\chi)\right)^{-1}$$

Error correction for χ simply replaces χ in the formula with $\chi-\tau$. Because high-ξ sectors tend to receive more subsidies, this correction effectively raises wedges in high-ξ sectors and lowers wedges in low-ξ sectors.

Correcting for subsidies in Θ is more involved. I abuse the notation and let $1+\chi-\tau$ be the $N\times N$ matrix with ij-th element $1+\chi_{ij}-\tau_{ij}$, and let $\frac{1+\chi-\tau}{1+\chi}$ be the matrix with elements $\frac{1+\chi_{ij}-\tau_{ij}}{1+\chi_{ij}}$. Let variables with tilde-overhead denote pre-intervention variables, and those without are post-intervention variables. I maintain the assumption that elasticities are recompute every distortion centrality specification and replicate the welfare evaluation exercise using the corrected measure.

Table D.2: Policy evaluations are robust to policy-induced endogeneity and specification errors

	Distortion centrality specification	Total output gain ($\Delta \ln Y$) in percentage points			
		Uncorrected	With Correction		
			Θ	χ	Both
	Benchmark ($\xi^{10\%}$)	3.60	3.58	3.85	3.69
B1	De Loecker and Warzynski	6.64	6.61	6.96	6.76
B2	Foreign firms as controls	3.47	3.43	3.74	3.55
B3	Rajan and Zingales	2.02	2.01	2.25	2.09
B4	Sectoral profit share	2.62	2.61	2.87	2.70

Table D.2 uses the corrected measures to recalculate total output gain $\Delta \ln Y$ from sectoral interventions. As a comparison, the first column reports gains based on uncorrected measures, reproducing the last column of Table 13. Results show that aggregate gains remain quantitatively unchanged, and, as my discussion suggests, output gains become slightly stronger when specification errors in χ are corrected. These results also empirically validate that second-order errors due to policy endogeneity are indeed small.

Table D.3 reports cross-sector means of gross policy expenditure as a share of sectoral revenue and value-added. Total policy spending on subsidized credit, tax incentives, and funds to SOEs account for, on average, 6.14% of sectoral revenue and 23.1% of sectoral value-added.

D.3 Stress-Test: Systematic Errors in Market Imperfections

My evidence suggests that industrial policy in both South Korea and China favored sectors with high distortion centrality over those with low distortion centrality. Can these findings be driven by systematic specification errors in imperfections? I demonstrate that false positives are unlikely given the hierarchical nature of these production networks.

Intuitively, false positives arise when ξ is biased negatively for downstream sectors, meaning χ must be underspecified for purchasing downstream goods as production inputs, and, conversely, over-

locally policy-invariant, i.e., $\mu_i = \tilde{\mu}_i$. One would like to measure $\tilde{\xi}_i \equiv \mu_i/\tilde{\gamma}_i$ for welfare calculations but can only measure $\xi = \mu_i/\gamma_i$ in the data. Following the proof for Proposition 4, one can show that the vectors with elements μ_i/γ_i and $\tilde{\gamma}_i/\gamma_i$ respectively follow

$$[\mu_i/\gamma_i]' \propto \left(\theta^F\right)' \left(I - \Theta \circ (1+\chi-\tau)\right)^{-1}$$

$$[\tilde{\gamma}_i/\gamma_i]' \propto \left(\theta^F\right)' \left(I - \Theta \circ \left(\frac{1+\chi-\tau}{1+\chi}\right)\right)^{-1}.$$

Hence, given imperfections and subsidies, pre-intervention distortion centrality $\mu_i/\tilde{\gamma}_i$ can be recovered by first taking the element-wise ratio between the two vectors $[\mu_i/\tilde{\gamma}_i]$ and $[\gamma_i/\tilde{\gamma}_i]$ and then normalizing so that it averages to one across sectors.

Table D.3: Policy spending as a share of sectoral revenue

	Average ratio between policy expenditure and sectoral revenue	Average ratio between policy expenditure and sectoral value-added
Subsidized credit	3.14%	11.85%
Tax incentive	1.04%	3.90%
SOEs	1.96%	7.39%
Total	6.14%	23.1%

specified for purchasing upstream goods. Such misspecification is a priori counterintuitive because, in contrast to capital goods produced by upstream sectors, downstream sectors tend to produce consumption goods that are more tradable, less durable, and generally less frequently used as intermediate inputs. Most importantly, the scope of false positives is severely limited in hierarchical networks, as imperfections over buying downstream goods eventually accumulate into upstream sectors' distortion centrality.

I demonstrate these intuitions by conducting exercises that are specifically designed to put stress on my empirical findings.

South Korea For each specification of imperfections $\{\chi_{ij}\}$, I assume true imperfections $\tilde{\chi}$ follow

$$\tilde{\chi}_{ij} = \begin{cases} (1-\kappa)\,\chi_{ij} & \text{if sector } j \text{ is HCI,} \\ (1+\kappa)\cdot\chi_{ij} & \text{if sector } j \text{ is non-HCI.} \end{cases}$$

I correct for specification errors and correspondingly re-compute distortion centrality using $\tilde{\chi}$. The parameter $\kappa \in [0,1]$ tunes the degree of misspecification. The case $\kappa = 1$ is designed to minimize HCI distortion centrality, by doubling imperfections for non-HCI inputs and setting imperfections over HCI inputs to zero. Under this case, HCI sectors can have high distortion centrality only by supplying to other high-distortion-centrality sectors. The specification errors are therefore chosen to maximize the scope for false positives.

Table D.4 shows that HCI sectors' distortion centrality remain consistently above one for the entire range of $\kappa \in [0,1]$, even in the extreme case of $\kappa = 1$. My finding that HCI sectors tend to have higher distortion centrality is therefore unlikely to be driven by specification errors.

Table D.4: $\xi > 1$ for HCI sectors is robust to specification errors

	Degree of misspecification, κ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	Benchmark	1.16	1.14	1.13	1.12	1.10	1.09	1.07	1.06	1.05	1.04	1.02
B3	Rajan and Zingales	1.12	1.11	1.10	1.09	1.08	1.06	1.05	1.04	1.03	1.02	1.01
B4	Sectoral profit share	1.28	1.25	1.23	1.20	1.18	1.15	1.13	1.10	1.08	1.05	1.03

China For each version of distortion centrality, I hypothetically assume the corresponding market imperfections are underspecified by 10 percentage points for purchasing downstream goods (i.e. those produced by sectors with below-median distortion centrality) and are symmetrically overspecified for purchasing upstream goods. These errors are specifically assigned to maximize the scope of false positives, and the magnitude of errors is liberally chosen to be significantly higher than the full range of policy variations (see Table 9). Correcting for these hypothetical errors should significantly weaken total gains in output, but any positive gains should be seen as very conservative lower bounds and can be used to vindicate my findings from being false positives. I redo policy evaluations using the corrected measures, reported in Table D.5. Results show that, after corrections, the variance of distortion centrality significantly decreases across specifications; consequently, welfare gains are smaller. Yet output gains remain consistently positive even after accounting for significant specification errors. This stress test lends credence to my earlier inference.

Table D.5: Qualitative conclusion survives stress-testing

Distortion centrality specification		$sd(\xi)$	Aggregate gains ($\Delta Y/Y$) by intervention (in percentage points): Subsidized credit	Tax incentive	SOEs	Total
	Benchmark ($\xi^{10\%}$)	0.09	0.18	0.07	0.11	0.36
B1	De Loecker and Warzynski	0.25	1.28	0.51	1.01	2.80
B2	Foreign firms as controls	0.15	0.60	0.22	0.36	1.18
B3	Rajan and Zingales	0.05	0.21	0.07	0.10	0.39
B4	Sectoral profit share	0.09	0.32	0.11	0.24	0.67

D.4 Additional Robustness Results for China

Table D.2 reports policy evaluations after correcting distortion centrality measures for policy-induced endogeneity in Θ, χ, or both; Table D.5 reports welfare evaluations after correcting for hypothetical extreme measurement errors in χ (underspecified by 10 percentage points for purchasing downstream goods and symmetrically overspecified by 10 percentage points for purchasing upstream goods). In Table D.6, I report various distortion centrality measures' Pearson correlation coefficients before and after these error corrections. Columns 1 through 3 show that ξ remains almost perfectly correlated after correcting for endogeneity in Θ, χ, or both, using subsidies computed from real-world policy spending. The last column shows that distortion centrality corrected for extreme errors in χ remains highly correlated with the uncorrected measures.

Table D.7 replicates selected reduced-form regressions from Tables 11 and 12, using the "upstreamness" measure from Antràs et al. (2012) as an instrument variable for the benchmark distortion centrality measure. All specifications in Table D.7 include the full set of controls. Results show that coefficients on policy outcomes remain quantitatively unchanged from OLS to IV specifications.

Table D.8 replicates selected reduced-form regressions from Tables 11 and 12, using various estimated distortion centrality as the main right-hand-side variables and adding the corresponding estimated sectoral imperfections as a control variables. All specifications include the full set of other sectoral controls. I report coefficient only on the main variable of interest, i.e. distortion centrality. For instance, the entry in column 1 row 1 should be read as "one standard deviation increase in distortion centrality specification B1 is associated with a 0.58 percentage points decrease in sectoral effective interest rate". Results show that coefficients on policy outcomes remain quantitatively unchanged, with few exceptions.

Table D.6: Policy evaluations are robust to policy-induced endogeneity and specification errors

		Pearson correlation with uncorrected measures			
		Correcting for policy-induced errors in			Correcting for extreme errors in χ
Distortion centrality specification		Θ	χ	Both	
	Benchmark ($\xi^{10\%}$)	1.0000	0.9982	0.9987	0.7238
B1	De Loecker and Warzynski	1.0000	0.9994	0.9996	0.9552
B2	Foreign firms as controls	1.0000	0.9983	0.9991	0.9220
B3	Rajan and Zingales	1.0000	0.9940	0.9949	0.6686
B4	Sectoral profit share	1.0000	0.9973	0.9978	0.8523

Table D.7: Replicating Tables 11 and 12 using "upstreamness" as instrument variable

	Effective Interest Rate	Debt Ratio	Tax Break	Effective Tax Rate	SOEs' Share of Value-Added
	(1)	(2)	(3)	(4)	(5)
$\xi_i^{10\%}$	-0.958^{***}	2.679^{***}	2.812^{**}	-1.591^{***}	6.589^{***}
	(0.216)	(0.603)	(1.369)	(0.418)	(2.752)

Table D.8: Replicating Tables 11 and 12 using estimated distortion centrality as main explanatory variable and adding the corresponding estimated imperfections as control variables

Specification	Effective Interest Rate	Debt Ratio	Tax Break	Effective Tax Rate	SOE Share of Value-Added
B1	−0.675	2.385	1.218	−1.325	7.633
B2	−0.902	2.777	1.940	−1.463	4.131
B3	−0.835	2.377	2.079	−1.122	4.339
B4	−1.004	2.771	2.593	−1.399	8.647

D.5 Additional Tables and Figures

Table D.9: Distortion centrality is highly correlated across specifications (cont'd)

	Average correlation between the benchmark $\xi_i^{10\%}$ and simulated distortion centrality			
	South Korea in 1970		China in 2007	
Distribution of χ_{ij}'s	Pearson	Spearman	Pearson	Spearman
Constant distortion				
$\chi_{ij} = 0.15$	1.00	1.00	0.99	0.99
$\chi_{ij} = 0.2$	1.00	1.00	0.99	0.99
Log-Normal				
log-$N(0.09, 0.05)$	0.98	0.97	0.99	0.99
log-$N(0.15, 0.05)$	0.99	0.99	0.99	0.99
log-$N(0.15, 0.1)$	0.97	0.97	0.98	0.99
Normal				
$N(0.05, 0.05)$	0.99	0.98	0.99	1.00
$N(0.1, 0.05)$	0.95	0.93	0.99	0.99
$N(0.2, 0.05)$	1.00	0.99	0.98	0.98
$N(0.2, 0.1)$	0.98	0.98	1.00	1.00
Truncated Normal				
$\left(max\left\{0, \text{Norm}\left(\mu, \sigma^2\right)\right\}\right)$				
$\mu = 0.05,\ \sigma^2 = 0.05$	0.97	0.95	1.00	1.00
$\mu = 0.05,\ \sigma^2 = 0.1$	0.94	0.93	0.99	0.99
$\mu = 0.15,\ \sigma^2 = 0.1$	0.98	0.97	0.99	0.99
$\mu = 0.15,\ \sigma^2 = 0.2$	0.94	0.95	0.97	0.98
Uniform				
$U[0, 0.3]$	0.98	0.98	0.99	0.99
$U[0, 0.4]$	0.98	0.98	0.98	0.98
Exponential				
Scale $= 0.05$	0.95	0.94	1.00	1.00
Scale $= 0.2$	0.91	0.93	0.92	0.94

Note: This table reports the average Pearson and Spearman-rank correlation between the benchmark distortion centrality $\xi_i^{10\%}$ and simulated distortion centrality. The benchmark distortion centrality constructed by assuming imperfections $\chi_{ij} \equiv 0.1$ for all i, j. Simulated distortion centrality is constructed by randomly and independently drawing imperfections χ_{ij} drawn from the listed distribution. Each specification is simulated 10,000 times, and the correlation between $\xi_i^{10\%}$ and simulated distortion centrality, averaged over 10,000 draws, is reported.

Figure D.2: Visualizing IO demand matrix Θ (sectors sorted by original 3-digit industrial codes)
Left: South Korea in 1970; Right: China in 2007

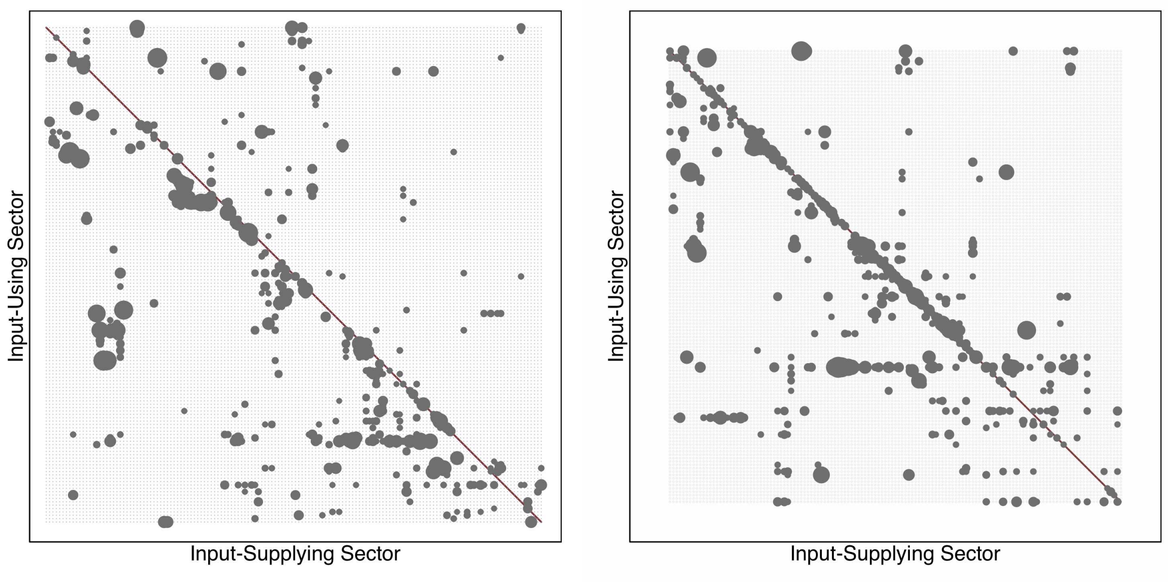


Table D.10: Three-digit industries targeted by the Heavy-Chemical Industry drive

Industry Name	Industry Name
Sulfuric acid and hydrochloric acid	Ferroalloys
Carbide	Steel rolling
Caustic soda products	Pipe and plated steel
Industrial compressed gases	Steel casting
Other inorganic basic chemicals	Non-ferrous metals
Petrochemical based products	Primary non-ferrous metal products
Acyclic intermediate	Construction metal products
Cyclic intermediate	Other metal products
Other organic basic chemicals	Prime movers, boilers
Chemical fertilizer	Machine tool
Pesticides	Special industry machinery
Synthetic resin	General purpose machinery and equipment
Chemical Fiber	General machinery parts
Explosives	Industrial electrical machinery and apparatus
Paints	Electronics and telecommunications equipment
Other chemical products	Other electrical equipment
Petroleum products	Shipbuilding and ship repair
Pig iron	Railroad transportation equipments
Crude iron	Cars and parts