## Industrial Policies in Production Networks

Ernest Liu

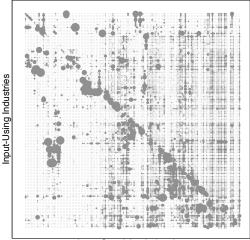
## Motivation

- Industrial policies: selective intervention into key economic sectors
- Widely adopted today and in the past
  - examples: Japan, Korea, Taiwan, China
  - tax incentives, subsidized credit, direct state involvement
  - despite many arguments against industrial policies
    - how can we trust the bureaucrats?
- ▶ How to conduct industrial policies *if we must*? Not well understood
  - important to consider linkages across sectors (Hirschman 1958)
- This paper:
  - analyze policy interventions in production networks
  - use the framework to evaluate industrial policies

# Korea's Heavy-Chemical Industry Drive (1973-1979)

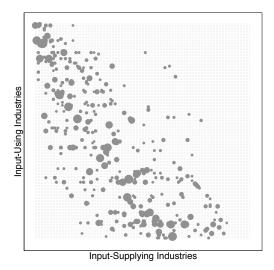
promoted six broad "strategic" sectors:

> steel, non-ferrous metals, shipbuilding, machinery, electronics, petrochemicals

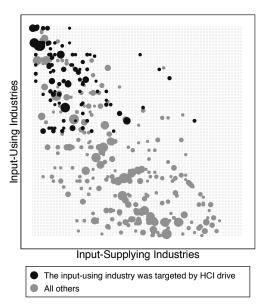


Input-Supplying Industries

## Korea's input-output table in 1970 — transformed



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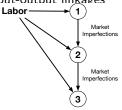


## Model and stylized example

- Rep. consumer, exogenous factor supply L, a unique consumption good
- **S** intermediate sectors, CRTS production with input-output linkages

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- Example: three intermediate sectors:
  - upstream (sector 1):  $Q_1 = z_1 L_1$
  - midstream (sector 2):  $Q_2 = z_2 F_2 (L_2, M_{21})$
  - downstream (sector 3):  $Q_3 = z_3 F_3 (L_3, M_{32})$
  - final good is produced linearly from good 3



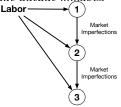
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Analyze how policies can improve efficient under market imperfections

- example: intermediate inputs are subject to credit constraints

$$P_{i} = \min_{\ell_{i}, m_{i,i-1}, k_{i}} \left( P_{i-1}m_{i,i-1} + W\ell_{i} + rk_{i} \right)$$
  
s.t.  $z_{i}F_{i}(\ell_{i}, m_{i,i-1}) \ge 1, \qquad \delta_{i}P_{i-1}m_{i,i-1} \le k_{i}$ 



## Sectoral allocations in decentralized economy

Start with the decentralized economy: no intervention

► Let  $\sigma_i (\equiv \frac{\partial \ln F_i(L_i, M_{i,i-1})}{\partial \ln M_{i,i-1}})$  denote equilibrium elasticity on intermediate inputs

Imperfections distort sectoral expenditure shares:

$$P_i M_{i,i-1} = \frac{\sigma_i}{1 + \chi_{i,i-1}} P_i Q_i;$$

in the example, distortion wedge is  $\chi_{i,i-1} = r\delta_i$ 

Assume the distortion payments are deadweight losses

- interest payments are "quasi-rent"

## Influence, sales, and distortion centrality

Sectoral influence  $\mu_i \equiv \frac{d \ln Y}{d \ln z_i}$ : an elasticity measure of sectoral importance

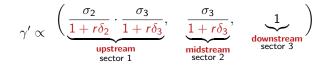
 $\mu' \propto \begin{pmatrix} \sigma_2 \sigma_3 & , & \sigma_3 & , & 1 \\ \underset{\text{sector 1}}{\text{upstream}} & \underset{\text{sector 2}}{\text{midstream}} & , & 1 \\ \underset{\text{sector 3}}{\text{downstream}} \end{pmatrix}$ 

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Sectoral sales share  $\gamma_i = \frac{p_i Q_i}{Y}$ : a measure of equilibrium sector size



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$$\gamma' \propto \begin{pmatrix} \frac{\sigma_2}{1+r\delta_2} \cdot \frac{\sigma_3}{1+r\delta_3}, & \frac{\sigma_3}{1+r\delta_3}, & \frac{1}{1+r\delta_3}, \\ \frac{\text{upstream}}{\text{sector 1}}, & \frac{\sigma_3}{1+r\delta_3}, & \frac{1}{\text{downstream}} \end{pmatrix}$$

Definition. Distortion centrality is influence over sales

$$\xi_i \equiv \mu_i / \gamma_i.$$

- ln efficient economies,  $\xi_i = 1$
- Upstream has the highest distortion centrality

## Introducing a government

• Consider sector-specific input subsidies  $\tau_{ij}$ , for j = 1, ..., S, L

- subsidies expand sectoral expenditures, but cost government resources

$$(1 - \tau_{ij} + \chi_{ij}) P_i M_{ij} = \sigma_{ij} P_i Q_i$$

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► Government budget constraint:  $G_{public consumption} + B_{subsidies} = T_{lump-sum tax}$   $- B \text{ is the total subsidy payments:} B \equiv \sum_{i=1}^{S} \left( \sum_{j=1}^{S} \tau_{ij} P_j M_{ij} + \tau_i^L W L_i \right)$ 

• Aggregate output is Y = C + G

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Lemma. The elasticity of aggregate output w.r.t. subsidy  $au_{ij}$  is

$$\left. \frac{d\ln Y}{d\tau_{ij}} \right|_{\tau=\mathbf{0}} = \underbrace{\frac{\sigma_{ij}}{1+\chi_{ij}}}_{\substack{\text{expenditure} \\ \text{share}}} \left( \underbrace{\mu_i}_{\substack{\text{ifluence}}} - \underbrace{\gamma_i}_{\substack{\text{sales}}} \right) \quad \text{for } j = 1, ..., S, L.$$

A reduced-form formula for non-parametric and ex-ante counterfactuals!

• Decomposing changes in aggregate consumption: dY = dC + dG

**Definition:** the social value of policy expenditure on input subsidy  $\tau_{ij}$  is

$$SV_{ij} \equiv -\frac{dC/d au_{ij}}{dG/d au_{ij}}\Big|_{\text{hold }T \text{ constant, } au=0}$$

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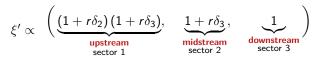
$$SV_{ij} \equiv -\frac{dC/d au_{ij}}{dG/d au_{ij}}\Big|_{\text{hold }T \text{ constant, } au=0}$$

- a general equilibrium spending multiplier; "bang for the buck"

Theorem. Sectoral distortion centrality  $\xi_i$  is a sufficient statistic for the social value of marginal policy spending into the sector:

$$SV_{ij} = \xi_i$$
 for all  $j = 1, \dots, S, L$ .

Interpretation: subsidize upstream!

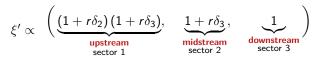


 two intuitions: subsidizing upstream 1) indirectly relaxes constraints downstream; 2) pushes resources towards efficient allocations

Policy should not target the most important / large / distorted sectors

- ranking by  $\xi$  is reversed to the ranking by *influence* or *sales* 

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Policy should not target the most important / large / distorted sectors
 – ranking by ξ is reversed to the ranking by influence or sales

#### Result applies to other policy instruments

- example: subsidies to credit  $u_i$  and to intermediates are isomorphic

$$(1 + (\mathbf{r} - \mathbf{u}_i) \,\delta_i) \, P_i M_{i,i-1} = \sigma_i P_i Q_i$$

- always better to channel credit to more upstream firms!
- cross-sector dispersion in interest rate  $\neq$  misallocation

Proposition. Distortion centrality averages to one:

$$\mathbb{E}\left[\xi\right] \equiv \sum_{i \in S} \xi_i \cdot \omega_i^L = 1, \text{ with } \omega_i^L \equiv L_i/L.$$

The aggregate gain from selective sectoral intervention is

$$\frac{\Delta Y}{Y} = Cov(\xi_i, s_i) + O\left(\max_i s_i^2\right);$$

where  $s_i$  is government spending per value-added in sector *i*.

## Welfare evaluation and counterfactual

Let

- sd be the standard deviation of  $\xi_i$
- $\bar{\xi}_i \equiv \xi_i/sd$ : distortion centrality standardized to unit variance

Corollary. Consider the bivariate regression

$$\mathbf{s}_i = \alpha + \beta \cdot \bar{\xi}_i + \epsilon_i,$$

each observation is a sector and is weighted by sectoral value-added. Then

$$\frac{\Delta Y}{Y} \approx sd \cdot \beta.$$

Intuitively,

- high sd: more dispersion in  $\xi$ , more scope for welfare-enhancing policies
- high  $\beta$ : spendings are better targeted to high- $\xi$  sectors

## Constrained-optimal subsidies

Earlier results are non-parametric and local

Global, constrained-optimal results depend on parametric assumptions

► Given the set of policy instruments *P* available to the planner:

$$\frac{dY}{d\tau_{ij}} = \frac{d\left(WL - B\right)}{d\tau_{ij}} = 0 \quad \text{ for } \tau_{ij} \in \mathcal{P}.$$

Proposition. Under Cobb-Douglas, the optimal value-added subsidies follow

$$\frac{1}{1-\tau_i^L} \propto \xi_i.$$

## Distortion centrality in general production networks

Let  $\omega_{ij}$  be the fraction of good j that is sold to sector i:  $\omega_{ij} \equiv \frac{M_{ij}}{Q_i}$ 

- captures the importance of *i* as a buyer of good *j*; define  $\omega_i^F$  similarly

**Proposition.** (Distortion Centrality). For scalar  $\delta = \frac{WL}{Y}$ ,

$$\xi_j = \delta \cdot \omega_j^{\mathsf{F}} + \sum_{i \in S} \xi_i \cdot (1 + \chi_{ij}) \cdot \omega_{ij}$$

or in matrix form ( $\mathbf{D} \equiv [1 + \chi_{ij}]$ ),

$$\xi' \propto \left(\omega^{\mathsf{F}}\right)' \left(I - \mathbf{D} \circ \Omega\right)^{-1},$$

with Leontief inverse  $(I - \mathbf{D} \circ \Omega)^{-1} = I + \mathbf{D} \circ \Omega + (\mathbf{D} \circ \Omega)^{2} + \cdots$ 

Empirical applications: evaluate policies and compute welfare gains
 – challenge: computing ξ requires knowledge of distortions D

### Hierarchical networks

**Definition.** A network  $\Omega$  has the *hierarchical* property if sectors can be ordered as  $1, 2, \ldots, S$  such that it has non-increasing partial column sums:

$$\sum_{k=1}^{K} \omega_{ik} \geq \sum_{k=1}^{K} \omega_{jk} \quad \text{for all } i < j \text{ and } K \leq S.$$

In hierarchical networks, sector *i* is said to be *upstream* to sector *j* whenever i < j.

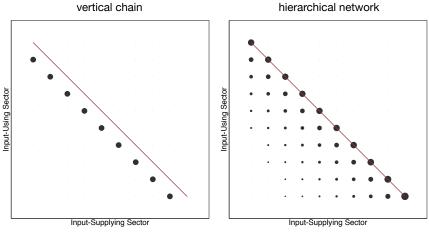
**Proposition**. Consider a hierarchical network  $\Omega$ . Case 1 (Stochastic). If for all  $i \neq j$ ,  $\chi_{ij}$ 's are i.i.d. conditional on  $\{\chi_{ii}\}_{i=1}^{5}$ , and  $\chi_{ij} \geq \chi_{ii}$  almost surely, then

$$\mathbb{E}\left[\xi_{i}\right] \geq \mathbb{E}\left[\xi_{j}\right]$$
 for all  $i < j$ .

Case 2 (Deterministic). If  $\mathbf{D} \circ \Omega$  also satisfies the hierarchical property, then

 $\xi_i \geq \xi_j$  for all i < j.

## A hierarchical network



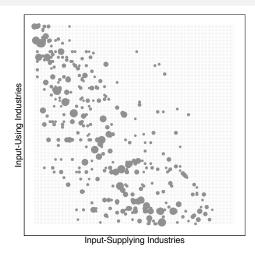
#### hierarchical network

## Korea's input-output table in 1970



Input-Supplying Industries

# Korea's input-output table in 1970 — sectors ordered by $\xi^{10\%}$

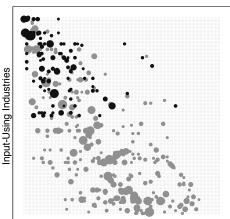


Testing for hierarchical property: among >1 million unique inequalities,

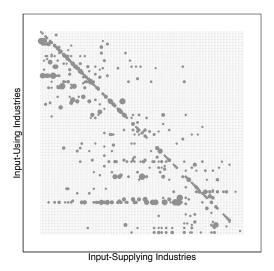
- 84% holds true (90% if small violations <0.01 are tolerated)

# South Korea in the 1970s promoted sectors with high distortion centrality

- "Heavy-Chemical Industry Drive" (1973-1979): promoted six broad "strategic" sectors:
  - steel, non-ferrous metals, shipbuilding, machinery, electronics, petrochemicals



## Input-output table of China in 2007



Testing for hierarchical property: among >1 million unique inequalities,

- 85% holds true (90% if small violations <0.01 are tolerated)

# $\xi_{j}^{10\%}$ : distortion centrality with constant distortion $\chi_{ij}=0.1$

## $\xi_i^{10\%}$ : distortion centrality with constant distortion $\chi_{ij} = 0.1$

	Average correlation with benchmark $\xi_i^{10\%}$					
	South Ke	orea in 1970	China in 2007			
Panel A: Simulated $\chi_{ij}$ 's	Pearson's r	Spearman's $\rho$	Pearson's r	Spearman's $\rho$		
N(0.1,0.1)	0.95	0.93	0.99	0.99		
<i>U</i> [0, 0.1]	0.98	0.97	1	1		
<i>Exp</i> (0.1)	0.95	0.94	0.98	0.99		
Panel B: Estimated Distortions						
De Loecker and Warzynski	-	-	1.00	1.00		
Foreign firms as controls	-	-	0.97	0.98		
Rajan and Zingales	0.98	0.97	0.98	0.97		
Self-reported financial costs	-	-	0.92	0.92		
Sectoral profit share	0.91	0.91	0.99	0.98		
$\xi^{10\%}$ with open-economy adjustments	0.95	0.93	0.98	0.94		
Sales	-0.20	-0.32	-0.40	-0.16		
"Upstreamness" by Antras et al. (2012)	0.96	0.96	0.98	0.97		

## Korea's HCI industries

HCI industries have higher simulated distortion centralities!

			Average $\xi_i$ of	% sectors with $\xi_i > 1$	
	$\xi$ Specification	sd (ξ)	HCI sectors	HCI	non-HCI
	Benchmark	0.09	1.16	100%	47.8%
B3	Rajan and Zingales	0.06	1.12	100%	47.0%
B5	Sectoral profit share	0.16	1.28	100%	45.1%
A3	N(0.1,0.1)	0.09	1.17	100%	47.7%
A7	<i>U</i> [0,0.2]	0.09	1.16	100%	47.7%
A8	<i>Exp</i> (0.1)	0.10	1.17	100%	47.7%

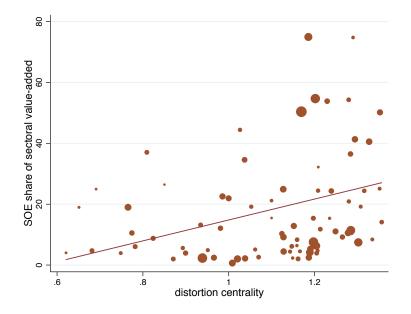
#### Which Chinese industries have high / low distortion centralities?

Top 10	ξ	Bottom 10	ξ
Coke making	1.36	Canned food products	0.62
Nonferrous metals and alloys	1.35	Dairy products	0.65
Ironmaking	1.35	Other miscellaneous food products	0.68
Ferrous alloy	1.33	Condiments	0.69
Steelmaking	1.33	Drugs	0.77
Metal cutting machinery	1.32	Meat products	0.77
Chemical fibers	1.31	Grain mill products	0.78
Electronic components	1.30	Liquor and alcoholic drinks	0.81
Specialized industrial equipments	1.30	Vegetable oil products	0.82
Basic chemicals	1.29	Tobacco	0.83

# $\xi_i$ predicts availability of credit and low taxes

	Interest Rate	Debt Ratio	Tax Break	Effec. Tax Rate	
	(1)	(2)	(3)	(4)	
$\xi_{i}^{10\%}$	-0.987***	2.726***	2.911**	-1.589***	
	(0.223)	(0.622)	(1.412)	(0.431)	
Capital intensity	-0.425**	-0.390	0.759	-0.253	
	(0.199)	(0.556)	(1.263)	(0.385)	
Lerner index	-0.0247	0.146	-0.559	0.0958	
	(0.173)	(0.481)	(1.092)	(0.333)	
Fixed cost of entry	-0.0273	0.511	-0.559	-0.643	
	(0.204)	(0.568)	(1.290)	(0.394)	
Export intensity	-0.682***	0.284	2.824**	-0.375	
	(0.172)	(0.487)	(1.105)	(0.337)	
adj. <i>R</i> <sup>2</sup>	0.301	0.231	0.097	0.176	
# Obs.	79	79	79	79	

# More SOEs in high- $\xi$ sectors



Outcome variable: SOEs' Share of Sectoral Value-Added in 2007

			SOEs established after year T			
	All SOEs in 2007		T = 2000	<i>T</i> = 2001	T = 2002	
	(1)	(2)	(3)	(4)	(5)	
$\xi_{i}^{10\%}$	7.577**	7.808***	2.960***	2.549***	2.123***	
	(2.963)	(2.834)	(1.059)	(0.886)	(0.725)	
Capital intensity		0.914	0.774	0.717	0.602	
		(2.535)	(0.947)	(0.792)	(0.649)	
Lerner index		$-4.622^{**}$	$-2.191^{***}$	$-1.997^{***}$	$-1.611^{***}$	
		(2.193)	(0.820)	(0.685)	(0.561)	
Fixed cost of entry		6.974***	2.042**	1.632**	1.245*	
		(2.590)	(0.968)	(0.809)	(0.663)	
Export intensity		-5.660**	-2.013**	-1.810**	$-1.484^{**}$	
		(2.218)	(0.829)	(0.693)	(0.568)	
adj. R <sup>2</sup>	0.066	0.290	0.269	0.284	0.276	
# Obs.	79	79	79	79	79	

# Policy Evaluation: Aggregate Gains $(\Delta Y/Y)$

		Δ	$\Delta Y/Y$ in percentage points			
Distortion centrality specification	$sd\left(\xi ight)$	Credit	Taxes	SOEs	Total	
Benchmark ( $\xi^{10\%}$ )	0.22	1.69	0.64	1.27	3.60	
De Loecker and Warzynski	0.42	3.07	1.19	2.39	6.65	
Foreign firms as controls	0.25	1.69	0.67	1.16	3.51	
Rajan and Zingales	0.11	1.01	0.36	0.65	2.02	
Sectoral profit share	0.17	1.20	0.47	0.95	2.62	

### Counterfactual Gains

• Consider alternative policy target  $\overline{\gamma}$ , with counterfactual policy interventions  $\tilde{s}_i$ :

$$\tilde{\mathbf{s}}_i = \alpha + \beta \cdot \bar{\gamma}_i + \mathbf{u}_i, \quad \mathbf{u} \perp \xi, \gamma.$$

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$$\tilde{s}_i = \alpha + \beta \cdot \bar{\gamma}_i + u_i, \quad u \perp \xi, \gamma.$$

Then aggregate gains under the counterfactual can be capture by  $\lambda$ :

$$\bar{\gamma}_i = \mathbf{c} + \lambda \cdot \bar{\xi}_i + \nu_i, \quad \nu \perp \gamma.$$

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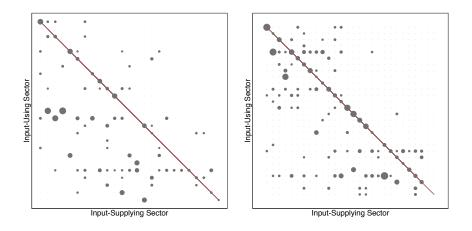
$$\bar{\gamma}_i = \mathbf{c} + \lambda \cdot \bar{\xi}_i + \nu_i, \quad \nu \perp \gamma.$$

	Gains relative to real-world interventions ( $\lambda$ )				
Specification for $\xi$	ξ10%	DLW	Foreign	RZ	LI
Real-world interventions	100%	100%	100%	100%	100%
Counterfactual policy target					
Sales $\gamma$	-39.5%	-38.6%	-33.5%	-41.3%	-43.6%
Consumption share $eta$	-71.1%	-69.5%	-69.3%	-71.3%	-72.6%
Export intensity	31.4%	29.8%	28.3%	38.9%	30.5%
Sectoral value-added	-36.2%	-36.3%	-31.7%	-37.3%	-36.4%
Interm. exp. share	37.2%	35.9%	31.6%	41.2%	33.3%
Optimal assignment	148.1%	153.4%	166.8%	147.0%	151.5%

## Coarse IO tables

Hierarchical property survives with coarse IO tables:

figures show 25 sectors for South Korea and 28 for China



	Average correlation with benchmark $\xi_i^{10\%}$				
	South Korea in 1970		China in 2007		
Number of sectors (S)	Pearson	Spearman	Pearson	Spearman	
$S^{SK}=54,\ S^{CN}=57$	0.97	0.96	0.97	0.99	
$S^{SK}=25,~S^{CN}=28$	0.94	0.91	0.97	0.94	
$S^{SK}=16,\ S^{CN}=17$	0.94	0.88	0.98	0.95	

### Conclusion

**Distortion centrality**: the ratio between sectoral influence and sales share

- a sufficient statistic for social value of sectoral spending
- can be used to assess welfare impact of sectoral intervention
- Distortions accumulate upstream through backward demand linkages
  - distortion centrality is stable in hierarchical networks
- Many arguments against industrial policies:
  - theory abstracts away from practical aspects of policy implementation and political economy factors
- Yet, evidence suggests that certain aspects of Korean and Chinese industrial strategy might be motivated by a desire to subsidize sectors that create positive network effects

## Relation to Hulten (1978)

Income accounting identity in a decentralized equilibrium:

 $Y^G - \Pi = Y = WL - B$ 

Hulten's theorem: In efficient economies,

$$\frac{d\ln Y^G}{d\ln z_i} = \frac{d\ln Y}{d\ln z_i} = \gamma_i.$$

- generically,

$$\frac{d \ln Y^G}{d \ln z_i} \neq \frac{d \ln Y}{d \ln z_i} \neq \frac{d \ln WL}{d \ln z_i} = \mu_i \neq \gamma_i.$$

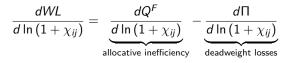
Thus Hulten's theorem fails for two reasons:

- influence does not equal to sales (well-known in the literature)
- elasticity of distortion and subsidy payments does not move proportionally with factor payments

- Distortions are not equivalent to iceberg costs
  - similar effect on aggregate output, different efficiency implications
  - quantity of output losses depend on relative prices  $\implies$  room for intervention

- Distortions are not equivalent to iceberg costs
  - similar effect on aggregate output, different efficiency implications
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- Compare efficient, distortion, and iceberg economies:

$$Y^* > Y^G > Y = Y^{IB}$$



- iceberg costs: only deadweight losses  $(Y^* Y^{IB})$ , no allocative inefficiency
- distortions: allocative inefficiency ( $Y^* Y^G > 0$ )
- Policy instruments can improve allocative efficiency

Example: two intermediate sectors, vertical network

$$Q_1 = L_1, \quad Q_2 = L_2^{\alpha} M_{21}^{1-\alpha}, \quad Q^F = Q_2$$

- Sector 2 faces a wedge  $(1 + \chi)$  when purchasing good 1
- Under iceberg formulation,
  - market clearing condition:

$$M_{21} = \frac{Q_1}{1+\chi}, \quad C = Q^F$$

- equilibrium factor allocations and aggregate consumption:

$$L_1 = \alpha L, \quad L_2 = (1 - \alpha) L, \quad C = L \alpha^{\alpha} (1 - \alpha)^{1 - \alpha} \left(\frac{1}{1 + \chi}\right)^{1 - \alpha}$$

- productivity loss  $\left(\frac{1}{1+\gamma}\right)^{1-\alpha}$  is entirely due to deadweight losses
- labor input is allocated efficiently

- Under distortion formulation,
  - market clearing condition:

$$M_{21}=Q_1, \quad C=Q^F-\chi P_1 M_{21}$$

- equilibrium factor allocations and aggregate consumption:

$$\mathcal{L}_{1} = \frac{\alpha}{\alpha + \frac{1-\alpha}{1+\chi}} \mathcal{L}, \qquad \mathcal{L}_{2} = \frac{\frac{1-\alpha}{1+\chi}}{\alpha + \frac{1-\alpha}{1+\chi}} \mathcal{L}$$

$$\mathcal{C} = \mathcal{L} \underbrace{\left(\frac{\alpha}{\alpha + \frac{1-\alpha}{1+\chi}}\right)^{\alpha} \left(\frac{\frac{1-\alpha}{1+\chi}}{\alpha + \frac{1-\alpha}{1+\chi}}\right)^{1-\alpha}}_{\text{allocative inefficiency}} \underbrace{\left(1 - \frac{\chi}{1+\chi}\left(1-\alpha\right)\right)}_{\text{deadweight losses}}$$

$$= \mathcal{L}\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} \underbrace{\left(\frac{1}{1+\chi}\right)^{1-\alpha}}_{\text{total productivity large}}$$

total productivity loss

- total productivity loss is the same as under iceberg costs
- but labor input is allocated efficiently  $\implies$  room for policy!

Optimal labor subsidies should be

$$\frac{1}{1-\tau_1} = \frac{\alpha}{\alpha / \left(\alpha + \frac{1-\alpha}{1+\chi}\right)}, \quad \frac{1}{1-\tau_2} = \frac{1-\alpha}{\left(\frac{1-\alpha}{1+\chi}\right) / \left(\alpha + \frac{1-\alpha}{1+\chi}\right)}$$

- elasticities over equilibrium expenditure shares on labor!



### Microfoundations

Marshallian externality:

$$Q_{i} = \int_{0}^{N_{i}} q_{i}\left(\nu\right) d\nu, \quad q_{i}\left(\nu\right) = z_{i} \left(\frac{Q_{i}}{N_{i}}\right)^{1-\alpha_{i}} F_{i}\left(\ell_{i}, \{m_{ij}\}\right)^{\alpha_{i}}$$

Negative production externality:

$$Q_{i} = \left(\int_{0}^{N_{i}} q_{i}\left(\nu\right)^{\frac{\sigma-1}{\sigma}} d\nu\right)^{\frac{\sigma}{\sigma-1}}, \quad q_{i}\left(\nu\right) = z_{i}N_{i}^{-\frac{1}{\sigma-1}}F_{i}\left(\ell_{i}, \{m_{ij}\}\right)^{\alpha_{i}}$$