A Network Approach to Geopolitics*

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Abstract

We develop a tractable framework for analyzing geopolitics as a network of coordination and strategic influence. Countries choose geopolitical actions that trade off domestic interests against incentives to align with connected countries. Prior to policy choice, they can make costly bilateral investments to influence one another, with effects that propagate directly and indirectly through the geopolitical network. We show that the equilibrium mapping from domestic interests to geopolitical actions admits a spectral representation, allowing international disagreements to be decomposed into latent geopolitical cleavages. We further establish that the covariance matrices of domestic interests and geopolitical actions identify the underlying geopolitical network through its eigencomponents. We implement the framework using data on countries' domestic interests and geopolitical actions. The inferred network is low-dimensional, aligns with external measures of geopolitical ties, and recovers well-known historical cleavages, from the Cold War division between the Western and Soviet blocs to the contemporary U.S.–China divide. Counterfactual exercises show that geopolitical actions are determined not only by domestic interests and national capabilities, but also by countries' positions in the geopolitical network and by the direct and indirect propagation of strategic influence.

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1 Introduction

Geopolitics is inherently a network phenomenon. Countries' actions are interdependent: each country's geopolitical choices depend not only on its own interests, but also on the actions of other countries with which it is connected through alliances, trade and financial ties, ideological affinities, and security relationships. Countries also seek to shape one another's behavior through strategic influence, and such influence is rarely confined to a single bilateral relationship. States can exert influence directly on one another, but also indirectly by influencing third countries whose responses propagate throughout the broader geopolitical network. As a result, the consequences of any geopolitical action depend not only on the two countries directly involved, but also on how the rest of the network responds. Even the decisions of a hegemon are shaped by the strategic positioning of its allies and rivals. A sanction is effective only if others enforce it; a military threat is credible only if allies coordinate behind it; and a diplomatic initiative succeeds or fails depending on whether pivotal states align, hedge, or defect.

The importance of geopolitical networks extends across both military and non-military domains, and across both historical and contemporary episodes. In the military domain, Russia's invasion of Ukraine was not simply a bilateral conflict between Russia and Ukraine. It quickly became a problem of coalition formation and coordination across Europe, North America, and beyond, involving sanctions, military assistance, energy reallocation, and diplomatic positioning. The same network logic also characterized many of the defining confrontations of the Cold War—including the Berlin Airlift, the Korean and Vietnam Wars, and the Cuban Missile Crisis—where outcomes depended on the alignment and responses of broader geopolitical blocs rather than the actions of any two countries alone. Network interdependence is equally central in non-military settings. Contemporary US export controls on advanced semiconductors matter because key supplier states coordinate complementary restrictions, while maritime disputes in the South China Sea depend critically on whether regional states coordinate their positions or fragment under competing economic and security ties. Across these diverse settings, geopolitical actions reverberate beyond the countries directly involved because influence propagates through networks of aligned and competing states.

This paper develops a tractable framework for analyzing geopolitics from that network perspective, both theoretically and empirically. Viewing geopolitics through the lens of networks changes how we think about power. In a purely dyadic world, power is the direct leverage over another state. In a networked world, power is also positional. Countries can become influential because they occupy central locations in coalitions, broker between blocs, control chokepoints, or act as pivotal swing states. Our network perspective also changes how we think about polarity. Rather than defining unipolarity or multipolarity solely as rankings of national capabilities, it treats polarity as a property of the system: a question of how influence, coalition formation, and coordination are organized across the network. Finally, it offers a new way to understand why some issues generate broad international convergence while others produce sharp polarization. Polarization is especially likely when disagreement aligns with major geopolitical cleavages, so that connected countries reinforce one another's positions and cross-bloc divisions become amplified.

We make two main contributions. First, we develop a tractable theoretical model of strategic investments in geopolitical networks. Countries choose policies that trade off domestic interests against incentives to align with the interests of neighbors in a geopolitical network. Before policies are chosen, countries can make costly bilateral strategic investments to influence one another's policy choices. This allows us to introduce both the interdependence in actions across the geopolitical networks and the ability to influence other countries' actions via strategic investment. We allow all countries to make such investments, such that our network structure encompasses unipolar, multipolar or nonpolar worlds. Since influence propagates through the network, bilateral influence has cascading effects: shifting one country's policy changes equilibrium policies throughout the system, so a country may affect another indirectly by influencing third parties. Second, we implement our model quantitatively using data on countries' political interests (as measured by democratic orientation) and geopolitical actions (as measured by UN General Assembly voting). We use the assumption that the cross-country covariance matrices of actions and interests share the same principal components to identify the unobserved geopolitical network using its underlying eigencomponents. We validate our inferred geopolitical network using a range of empirical proxies, and show that our estimated eigencomponents capture interpretable geopolitical cleavages, such as the Cold War division between the Western and Soviet blocs.

We begin by describing the setup of our framework. Our theoretical model considers a world that consists of N countries connected by a weighted graph or geopolitical network $\boldsymbol{G}$. This geopolitical network captures bilateral considerations for coordination in foreign policy, which may reflect alliances, economic dependence, or other strategic linkages. For a given international issue, each country i has a domestic interest θ_i and chooses a geopolitical action a_i. The geopolitical action is the outcome of a network "beauty-contest" coordination game: countries balance alignment with their own domestic interest and alignment with the actions of their network neighbors. Prior to the coordination game, each country engages in costly bilateral strategic influence to shift other countries' incentives for geopolitical actions. Country i may strategically influence j not because i wants to coordinate with j, but because doing so indirectly influences a third country k—who coordinates with j either directly or indirectly through the network—with whom i seeks alignment. The global equilibrium is a fixed point of these bilateral choices, where each country internalizes the full cascading impact of its influence on geopolitical actions across the geopolitical network.

We provide a closed-form characterization of the equilibrium profile of geopolitical actions and bilateral influence. The model admits a unique equilibrium, in which each country's equilibrium action is a convex combination of all countries' domestic interests. This equilibrium is "conservative": across countries, the average geopolitical actions and average domestic interests coincide; and net geopolitical influence sums to zero across countries. These properties formalize a basic feature of multilateral influence: strategic pressure reallocates incentives across countries but, in aggregate, does not create net persuasion "out of thin air."

We next theoretically characterize the structural features of international issues that drive either polarization or convergence in geopolitical actions across countries. We show that the equilibrium mapping from heterogeneous domestic interests to geopolitical actions is a spectral function of the geopolitical network. This implies a property of "spectral decoupling": when domestic interests are decomposed along the network's eigenvectors—which we interpret as latent geopolitical cleavages—the equilibrium scales each dimension of conflict independently. Issues that align with large eigenvalues are those where disagreements are small across network neighbors but large across blocs. We show that these issues generate a geopolitical echo-chamber effect, amplifying cross-bloc frictions into stark policy polarization. Conversely, issues associated with small eigenvalues are neutralized by local coordination motives, leading to highly convergent policy outcomes.

We characterize the geometry of strategic influence by introducing a "contestability index," which is issue-dependent and measures the degree of misalignment in the foreign influence directed at any given country. At one extreme, a country may face perfectly monolithic foreign pressure, where all foreign actors push its policy in a unified direction. At the other extreme, a country may face a geopolitical tug-of-war, where foreign actors exert competing, contradictory pressures that partially cancel each other out. Our index formally identifies these highly contested "swing states" and shows how a country's status as a battleground for influence depends on the interaction between its network position and the specific international issue at hand.

We transition from the analysis of a single international issue to an environment in which countries repeatedly play the coordination and strategic influence game over a fixed geopolitical network $\boldsymbol{G}$, with domestic interests $\boldsymbol{\theta}$ for each international issue drawn from a fixed distribution of global geopolitical shocks. This stochastic arrival generates variation in both domestic interests and equilibrium geopolitical actions $\boldsymbol{a}$. We establish a structural identification result that the eigenvalues and eigenvectors of the geopolitical network (and hence the entire geopolitical network) are recoverable from the observed data on domestic interests and geopolitical actions. We make a graph stationarity assumption that the set of latent issues that determine the strength of geopolitical connections between countries is the same as those that generate differences in domestic interests across countries. Under this assumption, the cross-country covariance matrices of domestic interests, $\boldsymbol{\Sigma}_\theta$, and geopolitical actions, $\boldsymbol{\Sigma}_a$, both share the same eigenbasis as the geopolitical network ($\boldsymbol{G}$). Within the structure of our model, the responsiveness of geopolitical actions relative to domestic interests for each eigenvector identifies the corresponding eigenvalue, as long as there is variation in domestic interests along that cleavage.

Our theoretical identification result motivates an empirical implementation based on estimating the joint principal components of the covariance matrices of $\boldsymbol{\theta}$ and $\boldsymbol{a}$. We use two widely used sources of cross-country political data. We measure the cross-country covariance in domestic interests (Σ_θ) using bilateral similarities in the country-year indicators from V-Dem, aggregating 33 variables that measure domestic interests and political institutions. We measure geopolitical actions ($\boldsymbol{a}$) using UN General Assembly votes. We treat $\boldsymbol{G}$ as evolving slowly and implement the joint diagonalization separately by decade. The resulting decomposition is strongly low-dimensional: using only the top five eigencomponents, the reconstructed uncentered second-moment matrices achieve R^2 above 95% across the sample. The first eigenvector is a constant that corresponds to complete agreement among all countries. The second eigenvector—the leading nontrivial geopolitical cleavage—tracks major shifts in the structure of international politics. In the 1960s, it aligns closely with the Cold War division between the Western and Soviet blocs. During the 1970s, however, the dominant cleavage shifts to a divide between the major powers—including the United States and the Soviet Union—and the developing world, reflecting the growing political salience of the "Third World." Notably, China aligns with the major-power bloc rather than the developing world, despite its self-identification as a leader of the Third World. In the 1980s, the principal cleavage returns to the East–West divide before shifting sharply after the collapse of the Soviet Union. Since the 1990s, it has aligned with a geopolitical cleavage in which the United States and Western Europe occupy one pole, China the other, and many countries in the Global South lie in between.

Next, we use the estimated eigenvectors and eigenvalues to construct an inferred geopolitical network for each decade. The inferred dyadic links reflect a range of empirical proxies for geopolitical connections between countries used in the economics and political science literature, including bilateral trade, geographical distance and indicators of common religion, language and historical colonizers. Based on this network, we demonstrate that our model's predictions for strategic influence between countries are robustly associated with state visits, separate data that is not used in the quantification of our model. We show three additional facts about the geopolitical network: *(i)* the geopolitical network exhibits substantial asymmetries —countries' outward connections (how much they care about other countries) can differ substantially from their inward connections (how much other countries care about them); *(ii)* while geopolitical networks are typically stable over time, large-scale geopolitical alignments such as the fall of the Iron Curtain can occur; and *(iii)* the cascade of strategic influence through indirect connections in the network makes a substantial contribution to the overall strategic influence of countries, particularly for middle and small powers.

Our framework provides a tractable platform for undertaking counterfactuals to assess the role of domestic interests and network connections in shaping geopolitical actions. We illustrate this with two exercises. First, we examine how the global geopolitical landscape would change if China's domestic interests converged to those of the United States. We find that China's geopolitical actions remain constrained by its network relationships, while the shift propagates throughout the network, moving many of China's partners toward the United States despite no change in their own domestic interests. Second, we measure geopolitical power by asking how changes in each country's domestic interests affect its own actions and those of others. The United States and China stand out in their ability to translate domestic interests into geopolitical actions, while China exerts the greatest influence on the actions of other countries. Together, these exercises show that geopolitical outcomes are determined not only by domestic interests or national capabilities, but also by countries' positions in the broader geopolitical network.

Our paper is related to several strands of existing research. First, we connect with recent research on geoeconomics, including Becko and O'Connor (2024), Clayton et al. (2024), Broner et al. (2024), Kleinman et al. (2024), Liu and Yang (2025), and Clayton et al. (2026).[1] Much of this research assumes a distinction between a hegemon and other individual nations. In contrast, we allow theoretically and empirically for a rich geopolitical network, in which whether or not a hegemon emerges is an endogenous outcome, and countries can be connected by alliances.

Second, we contribute to research on networks in economics, including Wolinsky and Jackson (1996), Jackson (2008), Acemoglu et al. (2012), Chaney (2014), Oberfield (2018), Goyal (2023), Elliott et al. (2022), Galeotti et al. (2024), Taschereau-Dumouchel (2024), and Cerreia-Vioglio et al. (2026). Our model builds on the theory literature on network games and beauty contests, including Morris and Shin (2002), Galeotti et al. (2010), and Allen et al. (2006). We apply these theoretical techniques to geopolitics. We extend a network coordination game to include an influence stage of endogenous strategic investments that distort coordination incentives.

Third, our empirical approach contributes to the econometric literature on the structural identification of networks, including Graham (2017), Mele (2017), Paula (2020), and Fernández-Val and Chen (2021). We provide conditions under which the cross-country covariances of domestic interests and geopolitical actions identify the unobserved geopolitical network.

Fourth, our work builds on the existing literature on geopolitics. This literature includes approaches emphasizing central locations (following Mackinder 1904); rimland locations (after Spykman 1944); hegemonic power and imperial overstretch (e.g., Mahan 1890 and Kennedy 1987); competition between competing ideologies (as during the Cold War in Gaddis 2005); armed conflict (e.g., Fearon 1995); chokepoints (e.g., Farrell and Newman 2023 and Fishman 2025); resource competition (e.g., Miller 2022); and spheres of influence (e.g., Allison 2020).

The remainder of the paper is structured as follows. Section 2 sets up the model and derives our main theoretical results regarding equilibrium influence, spectral decoupling, and contestability. Section 3 introduces our data and implements our methodology empirically. Section 4 provides fur-

[1]For recent reviews, see Clayton et al. (2025) and Mohr and Trebesch (2025).

ther evidence on the key empirical features of the geopolitical network. Section 5 undertakes counterfactuals to explore the role of domestic interests and network connections in shaping geopolitical actions. Section 6 summarizes our conclusions.

2 Model

We consider a world that consists of N countries indexed by $i, j \in \{1, \ldots, N\}$. The countries are connected by a weighted graph that can be represented by the matrix $\boldsymbol{G}$, where we use boldface throughout the paper to denote vectors (lowercase) and matrices (uppercase). The elements of this matrix $G_{ij} \in \mathbb{R}_{\geq 0}$ capture the magnitude of the political connections between countries i and j. The representative domestic policy-maker in each country (the national government) has a domestic interest $\theta_i \in \mathbb{R}$ and can take a geopolitical action $a_i \in \mathbb{R}$.

We interpret $\boldsymbol{G}$ as the geopolitical network. It captures bilateral considerations for coordination in foreign policy. It could reflect strategic alliances (e.g., G_{ij} is large between allies), economic dependence (e.g., G_{ij} is increasing in bilateral trade), or other considerations for coordination (e.g., US and Russia deliberating over a nuclear deal). We view $\boldsymbol{G}$ as reflecting medium to long-run geopolitical relationships between countries.

We interpret $\boldsymbol{\theta}$ as the heterogeneous domestic interests of countries over a specific international issue. For example, countries around the world may react differently to the capture of President Maduro of Venezuela or President Trump's threat of invading Greenland. We begin by analyzing the equilibrium for a given $\boldsymbol{\theta}$. Later we allow for multiple international issues, each of which corresponds to different $\boldsymbol{\theta}$, by assuming that $\boldsymbol{\theta}$ is drawn from a distribution across issues.

We interpret $\boldsymbol{a}$ as the geopolitical actions that countries adopt in response to the international issue. These geopolitical actions are influenced not only by domestic interests but also by global considerations of coordinating geopolitical actions with other nations. We model the choice of geopolitical actions $\boldsymbol{a}$ as the solution of a beauty-contest coordination game over the network, where each country i chooses its geopolitical action (a_i) to balance its alignment with its own domestic interests (θ_i) and the geopolitical actions of its network neighbors (a_j for $j \neq i$).

Prior to the coordination game, each country i can engage in costly strategic influence (ω_{ij}) to alter any other country j's incentives to choose geopolitical actions. The game proceeds in two stages for each issue $\boldsymbol{\theta}$. In the influence stage, countries observe $\boldsymbol{\theta}$ and choose bilateral strategic influence. In the coordination stage, countries choose geopolitical actions, taking bilateral strategic influence as given. We characterize how bilateral strategic influence ($\boldsymbol{\Omega}$) and geopolitical actions ($\boldsymbol{a}$) depend on the geopolitical network ($\boldsymbol{G}$) and domestic interests $\boldsymbol{\theta}$. The game has perfect information: each country knows the domestic interests ($\boldsymbol{\theta}$), strategic influence ($\boldsymbol{\Omega}$) and geopolitical network connections ($\boldsymbol{G}$) for all countries.

In what follows, we first introduce a special case of the model with only the coordination stage without any bilateral strategic influence. We then develop the full model with both the influence and coordination stages. In our baseline model, we assume for expositional simplicity that the geopolitical network is undirected ($\boldsymbol{G}$ is symmetric) and regular (every row of $\boldsymbol{G}$ sums to the same value). We denote the common row sum of $\boldsymbol{G}$ by $1-\alpha$, where $\alpha \in (0,1)$, and we assume throughout that the geopolitical network is strongly connected.[2] In a later subsection, we extend our baseline model to introduce asymmetries in the geopolitical network.

2.1 Model without Strategic Influence

In the special case of the model without strategic influence, countries play a beauty-contest coordination game over the geopolitical network. The representative policy-maker in each country i chooses its geopolitical action a_i to minimize its misalignment with its domestic interest (θ_i) and the geopolitical actions of its network neighbors (a_j), taking the geopolitical actions chosen by other countries as given. We assume that country i solves the following problem:

$$\max_{a_i} \left\{ -\left[\alpha \left(a_i - \theta_i\right)^2 + \sum_{j=1}^{N} G_{ij} \left(a_i - a_j\right)^2 \right] \right\}, \tag{1}$$

where the bilateral geopolitical links (G_{ij}) capture the extent to which countries i and j want their geopolitical actions to align.[3]

Taking the first-order condition with respect to a_i, we obtain country i's best-response function:

$$a_i = \alpha\theta_i + \sum_{j=1}^{N} G_{ij} a_j. \tag{2}$$

Noting that $\alpha + \sum_j G_{ij} = 1$, each country's best response is a weighted average of its own domestic interest (θ_i) and the geopolitical actions of its network neighbors (a_j). In vector form, this best-response function can be written as:

$$\boldsymbol{a} = \alpha\boldsymbol{\theta} + \boldsymbol{G}\boldsymbol{a}. \tag{3}$$

The equilibrium in geopolitical actions corresponds to the fixed point in which each country's best-response is consistent with the best-responses of all other countries:

$$\boldsymbol{a} \equiv \alpha \left(\boldsymbol{I} - \boldsymbol{G}\right)^{-1} \boldsymbol{\theta}. \tag{4}$$

[2] That is, for any i, j, there exists $k > 0$ where $\left[\boldsymbol{G}^k\right]_{ij} > 0$.

[3] Although the summation in the second term includes the own country $i = j$, the own-country entries in this second term are necessarily equal to zero (since $a_i = a_j$ for $i = j$).

2.2 Model with Strategic Influence

The full model includes both the influence and coordination stages of the game. In the influence stage, each country i can choose to engage in costly bilateral strategic influence to change the incentives of any country j to choose geopolitical actions. After strategic influence has occurred, the game moves on to the coordination stage, in which each country i chooses its geopolitical action a_i, taking as given the geopolitical actions of its network neighbors (a_j) and bilateral strategic influence (ω_{ij}). We assume that country i's payoff takes the following form:

$$\begin{aligned} U_i(\boldsymbol{a}, \boldsymbol{\Omega}) \quad = \quad & -\left[\alpha\left(a_i-\theta_i\right)^2+\sum_{j=1}^N G_{ij}\left(a_i-a_j\right)^2\right] \\ & -\kappa \sum_{k=1}^N \omega_{ki}^2 \\ & +2 \sum_{m=1}^N \omega_{im}\left(a_i-\theta_i\right), \end{aligned} \tag{5}$$

where $\omega_{ki} \in \mathbb{R}$ is the influence exerted by country i on country k—the first subscript indicates the country receiving influence and the second subscript indicates country exerting influence—and $\boldsymbol{\Omega} \equiv [\omega_{ki}]$ is the matrix of bilateral influence.

The first line in (5) is the country i's payoff from the network coordination game. The second line is country i's convex cost of exerting strategic influence. The parameter $\kappa > 1/\alpha$ is a cost shifter for strategic influence. Strategic influence can be either positive or negative. In both cases, the costs of strategic influence increase quadratically with its distance from zero. The third line is the payoff from receiving foreign influence ω_{im} exerted by each country m. Receiving positive net strategic influence ($\sum_{m=1}^N \omega_{im} > 0$) raises the marginal benefit of choosing an action a_i greater than domestic interests θ_i. Conversely, receiving negative net strategic influence ($\sum_{m=1}^N \omega_{im} < 0$) increases the incentive to choose an action a_i lower than domestic interests θ_i.

Therefore, strategic influence is valuable to country i insofar as it shifts its actions away from what it would choose based purely on domestic interests. This formulation of the impact of strategic influence on actions using domestic interests as the benchmark has several convenient properties. First, it implies translation invariance in $\boldsymbol{\theta}$: adding a constant c to every country's domestic interest θ_i shifts every equilibrium action by c without changing welfare, since the deviation $a_i - \theta_i$ is invariant to uniform translations. Second, this specification permits self-influence—country i can exert influence on itself ($\omega_{ii} > 0$)—which captures the idea that a country may invest resources to entrench its own policy stance. Our concavity assumption $\kappa > 1/\alpha$ ensures that the returns to self-influence are bounded. Incorporating self-influence allows us to apply linear algebra tools without separately dealing with the diagonals $\{\omega_{ii}\}$ of the influence matrix $\boldsymbol{\Omega}$. Finally, because θ_i is exogenous to country i, the term in $\sum_{m=1}^N \omega_{im}\theta_i$ in the third line does not affect country i's best-response choice of actions in the coordination stage, nor does it enter the payoff of any influencing country m. Therefore, it only affects the equilibrium through country i's choice of self-influence ω_{ii}.

To illustrate the impact of strategic influence on countries' objectives, we can re-write country i's payoff in equation (5) in the following form:

Lemma 1. *The payoff of country i in equation (5) can be rewritten as:*

$$U_i\left(\boldsymbol{a}, \boldsymbol{\Omega}\right) = -\left[\alpha\left(a_i - \left(\theta_i + \frac{\sum_{m=1}^{N}\omega_{im}}{\alpha}\right)\right)^2 + \sum_{j=1}^{N} G_{ij}\left(a_i - a_j\right)^2\right] + \frac{\left(\sum_{m=1}^{N}\omega_{im}\right)^2}{\alpha} - \kappa\sum_{k=1}^{N}\omega_{ki}^2. \quad (6)$$

Comparing equations (6) and (1), the net strategic influence received by country i acts like a shift in its domestic interests from θ_i to $\theta_i + \sum_{m=1}^{N}\omega_{im}/\alpha$, with net transfers of $\left(\sum_{m=1}^{N}\omega_{im}\right)^2/\alpha$.

2.3 Equilibrium

We solve the game by backward induction. We first solve for equilibrium geopolitical actions in the second stage of the game taking bilateral influence as given. We then solve for equilibrium bilateral strategic influence in the first-stage of the game.

2.3.1 Equilibrium at the Coordination Stage

In the coordination stage, country i chooses its geopolitical action (a_i) to maximize its payoff (6), taking as given bilateral strategic influence ($\boldsymbol{\Omega}$) and the geopolitical actions of all other countries (a_j). The first-order condition or best-response function for country i takes a similar form as in equation (2), but modified by the impact of net strategic influence:

$$a_i = \alpha\theta_i + \sum_{j=1}^{N} G_{ij}a_j + \sum_{m=1}^{N}\omega_{im}. \quad (7)$$

In vector form, this best-response function can be written as:

$$\boldsymbol{a} = \alpha\boldsymbol{\theta} + \boldsymbol{G}\boldsymbol{a} + \boldsymbol{\Omega}\mathbf{1}. \quad (8)$$

Therefore, relative to the pure beauty-contest game in equation (3), country m's strategic influence shifts the target country i's geopolitical action by ω_{im}, taking the actions of all other countries as given. Moving $\boldsymbol{G}\boldsymbol{a}$ to the left-hand side, and multiplying both sides by $\left(\boldsymbol{I} - \boldsymbol{G}\right)^{-1}$, we can write equilibrium actions in the coordination stage as an explicit function of domestic interests and influence:

$$\boldsymbol{a} = \left(\boldsymbol{I} - \boldsymbol{G}\right)^{-1}\left(\alpha\boldsymbol{\theta} + \boldsymbol{\Omega}\mathbf{1}\right). \quad (9)$$

The ik-th entry of the matrix $\left(\boldsymbol{I} - \boldsymbol{G}\right)^{-1}$ captures how country k's domestic interest (θ_k) and net strategic influence received ($[\boldsymbol{\Omega}\mathbf{1}]_k = \sum_{j=1}^{N}\omega_{kj}$) affect the equilibrium geopolitical actions chosen by country i. Note that this matrix inverse has the following Neumann series representation:

$(\boldsymbol{I}-\boldsymbol{G})^{-1} = \boldsymbol{I}+\boldsymbol{G}+\boldsymbol{G}^2+\cdots$. In this representation, each successive power of the matrix $\boldsymbol{G}$ captures a higher-order network connection: $\boldsymbol{G}_{ik}$ is the connection between i and k, $\boldsymbol{G}^2_{ik} = \sum_{j=1}^{N} G_{ij}G_{jk}$ is the indirect connection between i and k through a common neighbor j, and so on. Therefore, foreign influence on country k affects country i's action if they are either directly or indirectly connected through the network.

2.3.2 Equilibrium at the Influence Stage

In the influence stage, country i chooses its bilateral strategic influence $\{\omega_{ki}\}_k$ to maximize its objective $U_i(\boldsymbol{a}(\boldsymbol{\Omega}), \boldsymbol{\Omega})$. In the first-order condition for country i's optimal choice of strategic influence, the total derivative of its objective with respect to its bilateral strategic influence (ω_{ki}) can be decomposed into the following three terms:

$$0 = \frac{\mathrm{d}U_i}{\mathrm{d}\omega_{ki}} = \underbrace{\frac{\partial U_i}{\partial a_i}\frac{\partial a_i}{\partial \omega_{ki}}}_{\text{effect through } a_i} + \underbrace{\sum_{j\neq i}\frac{\partial U_i}{\partial a_j}\frac{\partial a_j}{\partial \omega_{ki}}}_{\text{effect through } a_j,\, j\neq i} + \underbrace{\left.\frac{\partial U_i}{\partial \omega_{ki}}\right|_{\boldsymbol{a}}}_{\text{direct effect}} \tag{10}$$

The first term on the right-hand side is the indirect effect of ω_{ki} on country i's payoff through the change in i's own action a_i. Because a_i is chosen optimally at the coordination stage, this term vanishes by the envelope theorem. The second term is the indirect effect through changes in other countries' actions a_j ($j \neq i$). Since country i does not control these actions, the envelope theorem does not apply, and this second term does not zero out. The third term is the direct effect of ω_{ki} on country i's payoff holding all actions fixed, which consists of the marginal cost of strategic influence ($-2\kappa\,\omega_{ki}$), and the marginal benefit of self-influence $2(a_i-\theta_i)$ for $k=i$.

Evaluating the non-zero terms in the first-order condition (10) yields the best-response function for country i's strategic influence towards country k:

$$\kappa\,\omega_{ki} = \sum_{j=1}^{N}\left[(\boldsymbol{I}-\boldsymbol{G})^{-1}\right]_{jk} G_{ij}\,(a_i-a_j) + \mathbf{1}_{k=i}\,(a_i-\theta_i)\,. \tag{11}$$

The left-hand side is the marginal cost of the strategic influence exerted by country i on country k. The right-hand side is the marginal benefit of this strategic influence. The first term on the right-hand side captures country i's incentive to influence country k to take a higher action, taking into account the equilibrium propagation of higher a_k through the network. Country i's payoff is higher when its neighbors' actions are closer to its own, as captured by the gap $(a_i - a_j)$. If this gap is positive, country i benefits from a higher a_j, and vice versa. The object $\left[(\boldsymbol{I}-\boldsymbol{G})^{-1}\right]_{jk}$ in this first term captures how influencing country k raises country j's action in the coordination equilibrium, accounting for both the direct effect on k and all indirect network propagation. The object G_{ij} in this first term captures how important country j's action is for country i's payoff. Summing across countries j, we obtain country i's overall incentive to influence country k, taking into account the full network of policy coordination.

The second term on the right-hand side arises only for self-influence ($k = i$) and reflects the direct benefit of shifting country i's own action further from its domestic interest θ_i. Intuitively, when $a_i > \theta_i$—that is, when the geopolitical coordination environment pushes i to act above its domestic interest—additional self-influence raises a_i further, amplifying the deviation from θ_i with marginal benefit $(a_i - \theta_i)$. This second term is absent for $k \neq i$, since the influence benefit of $\sum_{m=1}^{N} \omega_{im} (a_i - \theta_i)$ in the third line of country i's payoff in equation (5) depends on ω_{ki} only through ω_{ii}, as discussed earlier.

Summing across i in the best-response function (11), and noting that $\sum_{i=1}^{N} \mathbf{1}_{k=i} (a_i - \theta_i) = a_k - \theta_k$, we can write the net strategic influence received by country k as:

$$\kappa \sum_{i=1}^{N} \omega_{ki} = \sum_{i=1}^{N} \sum_{j=1}^{N} G_{ij} (a_i - a_j) \left[(\boldsymbol{I} - \boldsymbol{G})^{-1} \right]_{jk} + a_k - \theta_k. \tag{12}$$

Since $\boldsymbol{G}$ is symmetric with a constant row sum of $1 - \alpha$, we have $\sum_{i=1}^{N} G_{ij} (a_i - a_j) = [\boldsymbol{G}\boldsymbol{a}]_j - (1 - \alpha) a_j$, which enables us to re-write this relationship as:

$$\begin{aligned} \kappa \boldsymbol{\Omega} \mathbf{1} &= \boldsymbol{a} - \boldsymbol{\theta} + (\boldsymbol{I} - \boldsymbol{G})^{-1} [\boldsymbol{G}\boldsymbol{a} - (1 - \alpha) \boldsymbol{a}], \\ &= \alpha (\boldsymbol{I} - \boldsymbol{G})^{-1} \boldsymbol{a} - \boldsymbol{\theta}. \end{aligned} \tag{13}$$

Therefore, the net strategic influence received by country k is proportional to the gap between the network-weighted sum of equilibrium actions, $\alpha \left[(\boldsymbol{I} - \boldsymbol{G})^{-1} \boldsymbol{a} \right]_k = \alpha \left(a_k + [\boldsymbol{G}\boldsymbol{a}]_k + [\boldsymbol{G}^2 \boldsymbol{a}]_k + \cdots \right)$, and its domestic interest θ_k. Country k receives positive net influence whenever the average equilibrium action of itself, its neighbors, its neighbors' neighbors, and so on collectively exceeds θ_k / α. Therefore, the ability of a country to obtain transfers through strategic influence depends on its domestic interests relative to equilibrium actions in the network. Country k receives net influence to take a higher action if its domestic interest is below the equilibrium actions of itself, its neighbors, its neighbors' neighbors, and so on.

Substituting for the equilibrium actions from the coordination game $\boldsymbol{a}(\boldsymbol{\Omega})$ in equation (9), we obtain the following closed-form solutions for equilibrium net influence and geopolitical actions in terms of domestic interests (θ) and the geopolitical network ($\boldsymbol{G}$).

Proposition 1. *The equilibrium vectors of net influence and geopolitical actions satisfy:*

$$\boldsymbol{\Omega} \mathbf{1} = - \left(\kappa (\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I} \right)^{-1} \left((\boldsymbol{I} - \boldsymbol{G})^2 - \alpha^2 \boldsymbol{I} \right) \boldsymbol{\theta} \tag{14}$$

$$\boldsymbol{a} = \left(\kappa (\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I} \right)^{-1} (\boldsymbol{I} - \boldsymbol{G}) (\kappa \alpha - 1) \boldsymbol{\theta} \tag{15}$$

Moreover, as $\kappa \to \infty$*,* $\boldsymbol{\Omega}\mathbf{1} \to \mathbf{0}$ *and* $\boldsymbol{a} \to \alpha \left(\boldsymbol{I} - \boldsymbol{G}\right)^{-1} \boldsymbol{\theta}$*; and as* $\kappa \downarrow 1/\alpha$*,*

$$\boldsymbol{a} \to \bar{\theta}\,\mathbf{1}, \qquad \boldsymbol{\Omega}\mathbf{1} \to \alpha \left(\bar{\theta}\,\mathbf{1} - \boldsymbol{\theta}\right),$$

where $\bar{\theta} \equiv \frac{1}{N}\sum_i \theta_i$*.*

Proof. See Online Appendix A. $\square$

To interpret Proposition 1, recall that $\kappa > 1/\alpha$ is the cost shifter of influence. Note also that the expression for equilibrium actions in this proposition admits a power-series representation:

$$\boldsymbol{a} = (\kappa\alpha - 1) \sum_{n=0}^{\infty} c_n \left(\frac{\boldsymbol{G}}{1-\alpha}\right)^n \boldsymbol{\theta}, \tag{16}$$

where $c_n > 0$ is a decreasing sequence defined by the recursion

$$c_0 = \frac{1}{\kappa - \alpha}, \qquad c_1 = \frac{(1-\alpha)(\kappa+\alpha)}{(\kappa-\alpha)^2}, \qquad c_n = \frac{1-\alpha}{\kappa-\alpha}\left(2\kappa\, c_{n-1} - \kappa\,(1-\alpha)\, c_{n-2}\right) \quad \forall\, n \geq 2. \tag{17}$$

In this power-series representation, $\boldsymbol{G}/(1-\alpha)$ is row-stochastic, and $(\boldsymbol{G}/(1-\alpha))^n\,\boldsymbol{\theta}$ is the weighted average of the domestic interests of country i's n-step network neighbors. The coefficient c_n determines the weight placed on network distance n. This weight is decreasing in network distance, so that closer neighbors receive greater weight.

To gain further understanding for Proposition 1, it is useful to consider the two limits of the influence cost shifter (κ). As influence becomes infinitely costly ($\kappa \to \infty$), net influence vanishes and the game converges back to the beauty contest without influence. As influence becomes as cheap as feasible—κ converges to $1/\alpha$ from above ($\kappa \downarrow 1/\alpha$)—equilibrium actions converge to the unweighted global average $\bar{\theta}$. Intuitively, when influence is cheap, countries can push each other toward a common action, which is pinned down by the equal-weighted average of domestic interests.[4] Net influence acts as a corrective force: countries whose domestic interests lie above the global average ($\theta_k > \bar{\theta}$) receive negative net influence that pulls their actions down, while those with domestic interests below the global average receive positive net influence that pushes their actions up.

For any interior value of influence costs ($\kappa \in (1/\alpha, \infty)$), the two equilibrium forces of beauty-contest coordination and strategic influence are both active, and the equilibrium of the game smoothly interpolates between these two limits. Although countries exert strategic influence, its magnitude is strictly attenuated relative to that in the lower limit. A reduction in the cost of influence (a lower κ) reduces the differences in equilibrium geopolitical actions across countries. As

[4] In this limit, equilibrium actions are disproportionately determined by distant linkages: the coefficient c_n converges to a strictly positive constant for large n, so that distant neighbors receive non-vanishing weight.

strategic influence becomes cheaper, the resulting web of bilateral influence pushes countries further away from their idiosyncratic domestic interests and toward the network consensus $\bar{\theta}$. To formally establish this comparative static, and precisely characterize how strategic influence reshapes the cross-country dispersion of actions, we use a spectral decomposition of the geopolitical network, as developed in Section 2.4 below.

2.3.3 Properties of the Equilibrium

We now summarize a number of convenient properties of the equilibrium in Proposition 1.

Proposition 2. *The equilibrium has the following properties.*

1. *The equilibrium is unique.*

2. *Each country's equilibrium action is a convex combination over the domestic interests of all countries.*

3. *The equilibrium is translation invariant to $\boldsymbol{\theta}$: adding a constant c to all θ_i's results in a new equilibrium in which all actions are also shifted by c, without any change in payoffs.*

4. *The equilibrium is conservative: the average geopolitical action and the average domestic interest across countries coincide: $\mathbf{1}'\boldsymbol{a} = \mathbf{1}'\boldsymbol{\theta}$.*

5. *The equilibrium strategic influence nets out to zero: $\mathbf{1}'\boldsymbol{\Omega}\mathbf{1} = 0$.*

Proof. See Online Appendix B. □

Our model features a unique equilibrium despite the rich network interactions between countries. The linearity of the best-response function (11) in domestic interests implies that each country's equilibrium geopolitical action is a convex combination of the domestic interests of all countries. Our assumption that the return to strategic influence depends on the gap between geopolitical actions and domestic interests ensures that the equilibrium is translation invariant to domestic interests. Therefore, adding a constant to all domestic interests simply results in a new equilibrium, in which all actions are also shifted by c, without any change in payoffs. Equilibrium geopolitical actions are also conservative, in the sense that the average geopolitical action and average domestic interest coincide. Finally, equilibrium strategic influence is positive for some bilateral pairs and negative for others, but averages out to zero across all bilateral pairs.

2.4 Spectral Decomposition and Geopolitical Cleavage

In reality, some international issues can lead to a stark divergence in geopolitical actions across countries. An example is Russia's invasion of Ukraine. A western coalition led by NATO unified to provide military aid to Kyiv and formally condemned Moscow at the UN. In contrast, Russia's strategic allies—including China, Iran, and Belarus—absorbed western pressure and continued to supply

Russia with critical economic and military support. Other international issues can induce aligned policy responses. Examples include coordinated disaster relief (e.g., the 2023 Turkey–Syria earthquake) or localized peacekeeping efforts (e.g., anti-piracy operations in the Gulf of Aden).

2.4.1 Policy Divergence and Convergence

In our model, equilibrium geopolitical actions in Proposition 1 depend on bilateral influence (in the influence stage) and global coordination (in the coordination stage), as determined by the domestic interests for a specific international issue ($\boldsymbol{\theta}$) and the geopolitical network ($\boldsymbol{G}$). We now formally investigate the underlying characteristics of domestic interests ($\boldsymbol{\theta}$) that determine whether there is policy divergence or convergence across countries. We use a spectral decomposition of the geopolitical network $\boldsymbol{G}$ to provide this characterization:

$$\boldsymbol{G} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^\top \qquad \left(\text{equivalently, } \boldsymbol{G} = \sum_{k=1}^{N} \lambda_k \boldsymbol{u}_k \boldsymbol{u}_k^\top\right), \tag{18}$$

where $\boldsymbol{\Lambda}$ is the diagonal matrix of $\boldsymbol{G}$'s eigenvalues $\lambda_k \in \mathbb{R}$, and each column of $\boldsymbol{U}$ is an associated eigenvector $\boldsymbol{u}_k$. Since $\boldsymbol{G}$ is real and symmetric, this spectral decomposition necessarily exists. Furthermore, the symmetry of $\boldsymbol{G}$ implies that the vectors are orthonormal, $\boldsymbol{U}^{-1} = \boldsymbol{U}^\top$, and all eigenvalues are real. We rank these eigenvalues in decreasing order.

We next note that any vector of domestic interests ($\boldsymbol{\theta}$) can be projected onto $\boldsymbol{G}$'s eigenbasis:

$$\boldsymbol{\theta} = \sum_{k=1}^{N} \widetilde{\theta}_k \boldsymbol{u}_k, \qquad \widetilde{\boldsymbol{\theta}} \equiv \boldsymbol{U}^\top \boldsymbol{\theta}. \tag{19}$$

That is, any vector $\boldsymbol{\theta}$ of domestic interests across countries can be written as a linear combination of $\boldsymbol{G}$'s eigenvectors $\{\boldsymbol{u}_k\}$, where $\widetilde{\theta}_k$ are the coordinates after the projection.

Each eigenvector captures a particular axis of polarization, which can be interpreted as a latent geopolitical cleavage—a specific arrangement of domestic interests across countries. We now show that each latent geopolitical cleavage translates into a natural notion of disagreements among network neighbors. We define the following network disagreement measure:

$$\mathcal{S}(\theta) \equiv \frac{1}{2} \sum_{i,j} G_{ij} \left(\theta_i - \theta_j\right)^2,$$

which captures how dissimilar domestic interests are averaged across network neighbors.

Proposition 3. *The network disagreement of an eigenvector $\boldsymbol{u}_k$ is $\mathcal{S}(\boldsymbol{u}_k) = 1 - \alpha - \lambda_k$. Moreover,*

$$\boldsymbol{u}_1 = \underset{\|\boldsymbol{\theta}\|=1}{\arg\min}\, \mathcal{S}(\boldsymbol{\theta}) \propto \mathbf{1}, \qquad \lambda_1 = 1 - \alpha.$$

$$\boldsymbol{u}_k = \underset{\boldsymbol{\theta} \perp \{\boldsymbol{u}_j\}_{j<k}, \|\boldsymbol{\theta}\|=1}{\arg\min} \mathcal{S}(\boldsymbol{\theta}) \textit{ for all } k > 1.$$

Proof. See Online Appendix C. □

We use Proposition 3 to characterize the extent of policy divergence or convergence given domestic interests ($\boldsymbol{\theta}$). First, the network disagreement of an eigenvector $\boldsymbol{u}_k$ is directly and negatively tied to its associated eigenvalue λ_k. Second, the eigenvector with the largest eigenvalue is a constant vector, and each subsequent eigenvector $\boldsymbol{u}_k$ is the vector of domestic interests, among those orthogonal to all prior eigenvectors, that minimizes network disagreement.

Third, because the eigenvectors are pairwise orthogonal, each eigenvector $\boldsymbol{u}_k$ averages to zero across countries for all $k > 1$ (since $\mathbf{1}'\boldsymbol{u}_k = 0$). Therefore, each eigenvector captures a geopolitical cleavage—a particular configuration of disagreements in domestic interests across countries. Those with positive entries in $\boldsymbol{u}_k$ are on one side of the cleavage while those with negative entries are on the other side of the cleavage.

Fourth, for all $k > 1$, Proposition 3 shows that the geopolitical cleavages with large eigenvalues capture international issues where neighbors in tightly-connected geopolitical blocs (with high G_{ij}) tend to have similar domestic interests, whereas the disagreements across blocs (with low G_{ij}) can be large. Conversely, the cleavages with small or negative eigenvalues capture issues where neighbors in tightly-connected geopolitical blocs (with high G_{ij}) tend to have dissimilar domestic interests, whereas the disagreements across blocs (with low G_{ij}) can be small.

We now show that geopolitical cleavages with large eigenvalues tend to translate into divergent policy outcomes, whereas those with small eigenvalues tend to lead to policy convergence. We use the result from Proposition 1 that the equilibrium geopolitical action profile $\boldsymbol{a}$ can be characterized as a linear transformation of the domestic interests $\boldsymbol{\theta}$: $\boldsymbol{a} = \mathcal{L}(\boldsymbol{G})\,\boldsymbol{\theta}$, where $\mathcal{L}(\boldsymbol{G}) \equiv \left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^2 - \alpha\boldsymbol{I}\right)^{-1}\left(\boldsymbol{I}-\boldsymbol{G}\right)\left(\kappa\alpha - 1\right)$.

Proposition 4. *The operator $\mathcal{L}(\boldsymbol{G})$, mapping from domestic interests to geopolitical actions, is a spectral function of $\boldsymbol{G}$: it shares the same eigenbasis as $\boldsymbol{G}$. This implies spectral decoupling: when domestic interests are decomposed along $\boldsymbol{G}$'s eigenvectors, the equilibrium mapping scales each dimension of cleavages independently without spilling over into others:*

$$\boldsymbol{a} = \sum_{\ell} L\left(\lambda_\ell\right)\widetilde{\theta}_\ell \boldsymbol{u}_\ell, \qquad L\left(\lambda\right) \equiv \frac{\left(1-\lambda\right)\left(\kappa\alpha - 1\right)}{\kappa\left(1-\lambda\right)^2 - \alpha};$$

or, equivalently, projecting the equilibrium action profile onto $\boldsymbol{G}$'s eigenbasis $\widetilde{\boldsymbol{a}} \equiv \boldsymbol{U}^\top\boldsymbol{a}$, we have:

$$\widetilde{a}_\ell = L\left(\lambda_\ell\right)\widetilde{\theta}_\ell.$$

The scaling factor $L(\cdot)$ is non-negative, bounded above by one ($0 \leq L(\cdot) \leq 1$), and is strictly increasing in the eigenvalue λ_ℓ ($L'(\cdot) > 0$), implying that eigenvectors with larger eigenvalues translate into larger differences in geopolitical actions. Finally, the scaling factor $L(\cdot)$ is increasing in κ.

Proof. See Online Appendix D. □

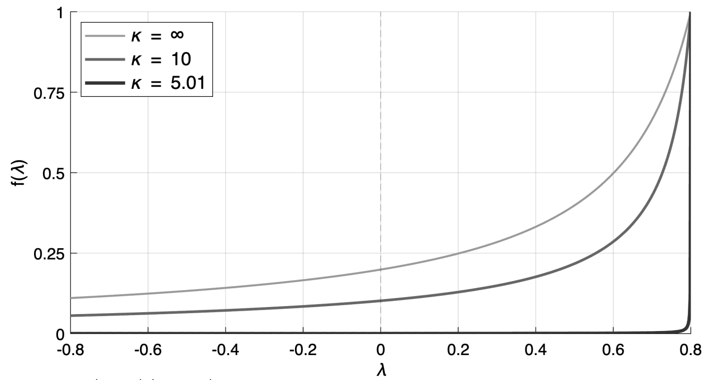
Proposition 4 establishes that the complex interplay of strategic influence and network coordination resolves into a tractable linear operator $\mathcal{L}(\boldsymbol{G})$ that shares the eigenbasis of $\boldsymbol{G}$ and is diagonalizable by the same latent cleavages. A key implication of this result is that equilibrium geopolitical actions exhibit a spectral decoupling along the network's axes of geopolitical cleavage. When domestic interests are decomposed along these eigenvectors, the equilibrium mapping scales each cleavage independently, without spilling over to the other cleavages.

Proposition 4 also shows that the scaling factor, which translates domestic interests into geopolitical actions, depends only on the eigenvalue of the geopolitical cleavage. International disagreements associated with large eigenvalues (where disagreements are small between geopolitically-close countries and large between geopolitically-distant countries) tend to translate into large differences in geopolitical actions. This divergence is driven by a geopolitical echo-chamber effect: because the coordination game incentivizes alignment with immediate neighbors, connected countries mutually reinforce each other's stances, amplifying cross-bloc frictions into stark policy polarization. Conversely, disagreements associated with small eigenvalues (where disagreements are large between geopolitically-close countries and small between geopolitically-distant countries) are neutralized by local coordination incentives, leading to convergent policy outcomes.

Finally, Proposition 4 also demonstrates that a lower cost of influence systematically lowers polarization in equilibrium geopolitical actions. As strategic influence becomes cheaper, the resulting web of bilateral pressure pushes countries further away from their idiosyncratic domestic interests and toward the network consensus.

In Figure 1, we illustrate these results numerically, using the property that the scaling factor from domestic interests to geopolitical actions ($L(\lambda)$) depends only on the network row sum (α) and influence costs (κ). We plot the value of this scaling factor as a function of the geopolitical cleavage's eigenvalue (λ) for $\alpha = 0.2$ and three different values of κ. As $\kappa \to \infty$, $L(\lambda) = \frac{\alpha}{1-\lambda}$, which is the scaling factor of the beauty-contest game without strategic influence: larger eigenvalues map into larger policy divergence, but all cleavages contribute. As κ decreases, geopolitical cleavages associated with small eigenvalues become disproportionately attenuated in their contribution to geopolitical actions, while those with large eigenvalues remain largely intact. As κ continues to decline towards $1/\alpha$, all cleavages but the Perron vector (i.e., the eigenvector with eigenvalue $1-\alpha$) cease to matter: $L(\lambda) \to 0$ for every $\lambda < 1-\alpha$, while $L(1-\alpha) \to 1$. At the lower limit of influence costs ($\kappa \downarrow 1/\alpha$), only the network-wide consensus survives, and equilibrium actions collapse to $\bar{\theta}\mathbf{1}$.

Figure 1: Scaling Factor ($L(\lambda)$) Mapping Eigenvalues (λ) to Geopolitical Actions ($\boldsymbol{a}$)



Note: the scaling factor ($L(\lambda) \equiv \frac{(1-\lambda)(\kappa\alpha-1)}{\kappa(1-\lambda)^2-\alpha}$) that maps the eigenvalues of a geopolitical cleavage (λ) to geopolitical actions ($\boldsymbol{a}$) for a network row sum of $\alpha = 0.2$ and three values of influence costs: $\kappa = 5.01$, $\kappa = 10$, and $\kappa = \infty$.

2.4.2 Strategic Influence and Domestic Interests

Equilibrium geopolitical actions ($\boldsymbol{a}$) in the previous subsection are shaped by the strategic influence ($\boldsymbol{\Omega}(\boldsymbol{\theta})$) induced by domestic interests ($\boldsymbol{\theta}$) given the geopolitical network ($\boldsymbol{G}$). To systematically characterize the role of strategic influence, we introduce a *contestability index*, which measures the degree of misalignment in the foreign influence directed at each country. Formally, we define the contestability index ($\chi_i(\boldsymbol{\theta}) \in [0,1]$) for each country i as one minus the absolute cosine similarity between the foreign influence that i receives and a constant vector of ones ($\mathbf{1}$):

$$\chi_i(\boldsymbol{\theta}) \equiv 1 - \frac{\left|\sum_{m \neq i} \omega_{im}(\boldsymbol{\theta})\right|}{\sqrt{(N-1)\sum_{m\neq i} \omega_{im}(\boldsymbol{\theta})^2}},$$

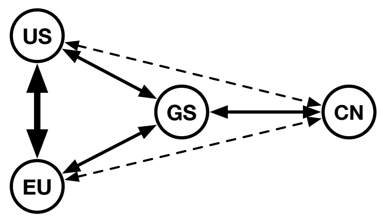
where we leave out self influence to focus on contestability from foreign influence. The cosine similarity with $\mathbf{1}$ measures how close foreign influence is to being uniform, and hence one minus its absolute value captures the degree of misalignment in foreign influence.

At the lower bound of the contestability index ($\chi_i = 0$), foreign influence is perfectly monolithic: every foreign nation pushes country i's policy by the same amount in a unified direction. Therefore, a low value for the contestability index (χ_i) indicates a strong consensus of foreign pressure, which pushes country i's action (a_i) in that unified direction. In contrast, at the upper bound of the contestability index ($\chi_i = 1$), the net influence on country i sums exactly to zero despite active foreign influence. Hence, a high value for the contestability index (χ_i) reflects a geopolitical tug-of-war, in which country i is a highly-contested swing state or a battleground state for foreign influence. In this case, foreign actors exert competing, contradictory pressures, which partially cancel each other out, and hence have less effect on country i's action (a_i).

2.4.3 Example Network

To illustrate these analytical results and provide further intuition, we consider an example network ($\boldsymbol{G}$) of four countries, as illustrated in the top panel of Figure 2. Countries 1 and 2 (we refer to them as the US and the EU respectively) are strongly connected; country 3 is equally connected to everyone (the Global South); and country 4 (China) is only weakly connected to countries 1 and 2.

Figure 2: Example of a Geopolitical Network



$$\boldsymbol{G} = \begin{bmatrix} 0 & .5 & .3 & .1 \\ .5 & 0 & .3 & .1 \\ .3 & .3 & 0 & .3 \\ .1 & .1 & .3 & .4 \end{bmatrix} \qquad \boldsymbol{u}_\ell \propto \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_\ell = \quad 0.9, \quad 0.3, \quad -0.3, \quad -0.5$$

Note: Countries 1 and 2 (the US and the EU, respectively) are strongly connected; country 3 is equally connected to everyone (the Global South (GS)); and country 4 (China (CN)) is only weakly connected to countries 1 and 2. Top panel shows a network graph with thick solid arrows denoting the strongest connections; medium solid arrows denoting intermediate connections; and medium dashed arrows showing the weakest connections. Lower panel shows the network matrix $\boldsymbol{G}$, with elements G_{ij} denoting the strength of network connections between countries i and j, and the implied eigenvalues (λ_ℓ) and eigenvectors ($\boldsymbol{u}_\ell$).

We report the assumed strength of the network connections (G_{ij}) and the implied eigenvalues (λ_ℓ) and eigenvectors ($\boldsymbol{u}_\ell$) in the lower panel of Figure 2. The first eigenvector $\boldsymbol{u}_1$ is a constant vector. The second eigenvector $\boldsymbol{u}_2$ captures a primary geopolitical cleavage: international issues where the US and the EU take one side; China takes the polar opposite; and the Global South occupies the middle. When domestic interests align with this axis, the Global South's contestability index reaches its upper bound ($\chi = 1$). Because the Western bloc and China attempt to pull the Global South in exactly opposite directions, their competing strategic influence cancel out, thereby rendering it a highly-contestable swing state for such international issues.

The third eigenvector $\boldsymbol{u}_3$ captures a different structural divide—one where China aligns with the US and the EU, but all three disagree with the Global South. In this scenario, the Global South's contestability index drops to its lower bound ($\chi = 0$). Rather than facing a tug-of-war, the Global South is subjected to monolithic foreign pressure, as all major global powers exert influence to shift its geopolitical action in the exact same direction.

Finally, the eigenvector associated with the smallest eigenvalue captures localized disagreements, such as a sharp policy fracture strictly between the US and the EU. Because the US and the EU exert immense pressure to pull the rest of the network in diametrically opposite directions, their efforts effectively neutralize each other abroad. As a result, both China and the Global South absorb perfectly offsetting foreign influence, pushing their contestability indices to exactly one ($\chi = 1$), and turning the rest of the globe into a battleground for an intra-Western schism.

2.5 Stochastic International Issues and Identification

We have so far focused on geopolitical actions in response to a single international issue, treating the vector of domestic interests for that international issue ($\boldsymbol{\theta}$) as fixed. In this section, we extend our theoretical analysis to a setting in which international issues arrive stochastically, and the domestic interests for each international issue ($\boldsymbol{\theta}$) are drawn from a fixed distribution of geopolitical shocks. We interpret these stochastic draws for domestic interests as capturing the shifting landscape of international crises, such as the Berlin Airlift, the Cuban Missile Crisis, the US proposal to purchase Greenland, or Chinese military exercises in the Taiwan Strait. We use this stochastic specification to provide conditions under which the unobserved geopolitical network ($\boldsymbol{G}$) can be identified from observed geopolitical actions ($\boldsymbol{a}$) and domestic interests ($\boldsymbol{\theta}$).

To provide a sharp analytical characterization of geopolitical actions with stochastic international issues, we make the following two assumptions. First, we are concerned with the cross-country variance and covariance of domestic interests across diverse international issues, which depends on relative domestic interests. Therefore, we normalize the expected value of domestic interests to zero ($\mathbb{E}[\boldsymbol{\theta}] = \mathbf{0}$), without loss of generality. Second, we assume that domestic interests ($\boldsymbol{\theta}$) and the geopolitical network ($\boldsymbol{G}$) inhabit the same underlying space of geopolitical issues, in the sense that the cross-country variance-covariance matrix of domestic interests ($\boldsymbol{\theta}$) shares the same eigenbasis as the geopolitical network ($\boldsymbol{G}$).

Assumption 1. ***(Graph Stationarity)*** *$\boldsymbol{\Sigma}_\theta$ is a spectral function of $\boldsymbol{G}$: it shares the same eigenbasis as $\boldsymbol{G}$.*

Assumption 1 implies that the same eigenvectors that describe the geometry of $\boldsymbol{G}$—the geopolitical cleavages—are the exact same axes along which domestic interests naturally covary across countries. It implies that $\widetilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_\theta \boldsymbol{U}$ is a diagonal matrix:

$$\widetilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_\theta \boldsymbol{U} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_N^2 \end{bmatrix}. \tag{20}$$

To interpret this assumption, note that the vector of domestic interests associated with each international issue can be written as a linear combination over latent cleavages:

$$\boldsymbol{\theta} = \widetilde{\theta}_1 \boldsymbol{u}_1 + \widetilde{\theta}_2 \boldsymbol{u}_2 + \cdots + \widetilde{\theta}_N \boldsymbol{u}_N. \tag{21}$$

Therefore, Assumption 1 implies that the coordinates of $\boldsymbol{\theta}$ are in the eigenbasis of $\boldsymbol{G}$, such that the weights on the eigencomponents ($\widetilde{\theta}_\ell$) are uncorrelated across eigencomponents, with variance σ_ℓ^2 for each eigencomponent ℓ. In the language of principal components analysis, Assumption 1 states that the principal components of domestic interests coincide with the latent geopolitical cleavages. Intuitively, we assume that the set of latent issues that determine the strength of geopolitical connections between countries is the same as those that generate differences in domestic interests across countries.[5] We provide evidence on the extent to which this assumption provides a good approximation to the data in empirical specification checks below.

Lemma 2 below establishes the key implication of Assumption 1 for our empirical approach: the covariance matrices of domestic interests ($\boldsymbol{\Sigma}_\theta$) and geopolitical actions ($\boldsymbol{\Sigma}_a$) share the same principal components, because the equilibrium mapping rescales each geopolitical cleavage without rotating it (Proposition 4).

Lemma 2. *Under Assumption 1, the cross-country covariance matrix of geopolitical actions, $\boldsymbol{\Sigma}_a$, is also a spectral function of $\boldsymbol{G}$ and shares its eigenbasis.*

Proof. Recall that $\boldsymbol{a} = \mathcal{L}\left(\boldsymbol{G}\right)\boldsymbol{\theta}$. By Proposition 2, $\mathbb{E}\left[\boldsymbol{\theta}\right] = \mathbf{0}$ implies that $\mathbb{E}\left[\boldsymbol{a}\right] = \mathbf{0}$. The covariance matrix of geopolitical actions is thus

$$\boldsymbol{\Sigma}_a \equiv Cov\left[\boldsymbol{a}\right] = \mathbb{E}\left[\boldsymbol{a}\boldsymbol{a}^\top\right] = \mathcal{L}\left(\boldsymbol{G}\right)\mathbb{E}\left[\boldsymbol{\theta}\boldsymbol{\theta}^\top\right]\mathcal{L}\left(\boldsymbol{G}\right)^\top = \mathcal{L}\left(\boldsymbol{G}\right)\boldsymbol{\Sigma}_\theta\mathcal{L}\left(\boldsymbol{G}\right)^\top.$$

The result follows immediately from Assumption 1, as $\mathcal{L}\left(\boldsymbol{G}\right)$ and $\boldsymbol{\Sigma}_\theta$ are both strictly diagonalizable by the shared eigenbasis $\boldsymbol{U}$ of $\boldsymbol{G}$. □

In other words, the equilibrium passes each principal component ℓ of domestic interests through to geopolitical actions, rescaling its variance by $L\left(\lambda_\ell\right)^2$.

Remark 1. Within our model, graph stationarity is not only sufficient but also, generically, necessary for this shared structure: $\boldsymbol{\Sigma}_\theta$ and $\boldsymbol{\Sigma}_a$ share a common set of principal components if and only if Assumption 1 holds (see Online Appendix F for the proof). Hence graph stationarity imposes no restriction on the observed covariance matrices beyond the shared principal components property, which we assess directly in the data below.

We now use these properties that the covariance matrices of domestic interests ($\boldsymbol{\Sigma}_\theta$) and geopolitical actions ($\boldsymbol{\Sigma}_a$) have the same eigenbasis as the geopolitical network ($\boldsymbol{G}$) to identify the eigenvectors ($\boldsymbol{u}_\ell$) and eigenvalues (λ_ℓ) of this network, which correspond to underlying geopolitical cleavages, and from which we can recover the entire geopolitical network.

[5] An analogous assumption is common in network research in signal processing (e.g., Marques et al. 2017).

Proposition 5. ***(Identification)*** *Let $\widetilde{\boldsymbol{\Sigma}}_a \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_a \boldsymbol{U}$, $\widetilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_\theta \boldsymbol{U}$ be the projection of $\boldsymbol{\Sigma}_a$ and $\boldsymbol{\Sigma}_\theta$ onto the eigenbasis $\boldsymbol{U}$. The eigenvalues (λ_ℓ) and eigenvectors ($\boldsymbol{u}_\ell$) of $\boldsymbol{G}$ are identified if the ℓ-th diagonal element of $\widetilde{\boldsymbol{\Sigma}}_\theta$ is non-zero, with*

$$\lambda_\ell = L^{-1}\left(\sqrt{\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell} \Big/ \sigma_\ell^2}\right), \tag{22}$$

where $L(\cdot)$ is defined in Proposition 4; σ_ℓ^2 is the ℓ-th diagonal entry of $\widetilde{\boldsymbol{\Sigma}}_\theta$; and we recover $\boldsymbol{u}_\ell$ from equation (20) using this solution for σ_ℓ^2.

Proof. See Online Appendix E. □

In this identification result, we use the property from Proposition 4 that the scaling function ($L(\lambda_\ell)$) mapping domestic interests to geopolitical actions is strictly monotonic in the eigenvalues (λ_ℓ), which implies that it can be inverted. Given the observed covariance matrices of geopolitical actions ($\boldsymbol{\Sigma}_a$) and domestic interests ($\boldsymbol{\Sigma}_\theta$), we can use Proposition 4 to identify the eigenvectors ($\boldsymbol{u}_\ell$) and eigenvalues (λ_ℓ) for each geopolitical cleavage ℓ, as long as there is variation in domestic interests along that geopolitical cleavage ($\sigma_\ell^2 > 0$). If instead $\sigma_\ell^2 = 0$, the global network never experiences a shock for which domestic interests lie along that specific cleavage, rendering the data uninformative about network behavior in that direction. Consequently, $\boldsymbol{G}$ is only identified on the subspace spanned by the eigenvectors for which domestic interests actually vary in the data. On the orthogonal complement, λ_ℓ remains unrestricted by $\boldsymbol{\Sigma}_\theta$ and $\boldsymbol{\Sigma}_a$.

Intuitively, graph stationarity in Assumption 1 and spectral decoupling in Proposition 4 imply that both domestic interests and geopolitical actions can be additively decomposed along the underlying geopolitical cleavages of the network. We can recover the eigenvectors corresponding to each of these cleavages from a spectral decomposition of the cross-country covariance matrices of domestic interests and geopolitical actions. The relative importance of each cleavage for domestic interests and geopolitical actions is controlled by the corresponding eigenvalue. Therefore, we can recover these eigenvalues from the relative responsiveness of domestic interests and domestic actions for each cleavage, as long as there is variation in domestic interests along that cleavage in the data.

Having identified these eigenvectors and eigenvalues, we can reconstruct the implied unobserved geopolitical network:

$$\boldsymbol{G} = \sum_\ell \lambda_\ell \boldsymbol{u}_\ell \boldsymbol{u}_\ell^\top,$$

where we only sum over the geopolitical cleavages that can be identified from the observed variation in the data. As a result of the property of spectral decoupling, the unobserved dimensions of $\boldsymbol{G}$ represent latent geopolitical fault lines that are never activated by historical shocks, and hence never captured in the observed variation in domestic interests and geopolitical actions. Because these silent dimensions do not interact with or spill over into the observable dimensions, it is without loss of generality to omit them from our subsequent empirical analysis.

2.6 *Ex Ante* Welfare

With stochastic international issues, we measure each country's *ex ante* welfare as its expected payoff ($\mathbb{E}_{\boldsymbol{\theta}}[U_i]$) over the distribution of the draws of domestic interests (θ). We now show that the *ex ante* welfare of each country depends on the network structure as well as the set of international issues that arise. We express each country i's *ex ante* welfare as a function of the eigenvectors and eigenvalues of $\boldsymbol{G}$, as well as of the matrix $\boldsymbol{\Sigma}_\theta$ capturing the covariance of domestic interests.

Proposition 6. ***(Ex Ante Welfare)*** *Under Assumption 1, ex ante welfare for country i can be written as:*

$$\mathbb{E}_\theta[U_i] = \sum_\ell \sigma_\ell^2 \times \Gamma_{i\ell}(\{u_k\}, \{\lambda_k\}),$$

where $\Gamma_{i\ell}(\{u_k\}, \{\lambda_k\})$ is country i's payoff along geopolitical cleavage ℓ, defined as:

$$\begin{aligned}\Gamma_{i\ell} \quad &= \quad (L(\lambda_\ell) - 1)\left[2\frac{\alpha^2 - (1-\lambda_\ell)^2}{\kappa(1-\lambda_\ell)^2 - \alpha} - \alpha(L(\lambda_\ell) - 1)\right] u_{i\ell}^2 \\ &\quad -L(\lambda_\ell)^2 \sum_q C_{\ell\ell q}(1 - \alpha - 2\lambda_\ell + \lambda_q) u_{iq} \\ &\quad -\frac{1}{\kappa}\sum_p \left(\sum_q C_{\ell p q}\left[\frac{L(\lambda_\ell)(1-\lambda_q)}{1-\lambda_p} - 1\right] u_{iq}\right)^2,\end{aligned}$$

where $L(\cdot)$ is as defined in Proposition 4, and $C_{\ell pq} \equiv \sum_{k=1}^N u_{k\ell} u_{kp} u_{kq} = \langle \boldsymbol{u}_\ell \circ \boldsymbol{u}_p, \boldsymbol{u}_q \rangle$ is the spectral interaction tensor that captures how eigenmodes interact through the nonlinearity of the country-level products of the eigenvectors $u_{k\ell} u_{kp} u_{kq}$.

Proof. See Online Appendix G. □

Proposition 6 shows that the *ex ante* welfare of country i depends on the network structure (as encoded in the eigenvectors $\{u_k\}$ and eigenvalues $\{\lambda_k\}$ of $\boldsymbol{G}$), the equilibrium interactions between countries through coordination and influence (as captured by the function $\Gamma_{i\ell}(\cdot)$), and the distribution of international issues (as captured by the variance σ_ℓ^2 with which each latent cleavage is drawn).

Ex ante welfare is additive across each of the geopolitical cleavages, because of spectral decoupling. Each of these geopolitical cleavages ℓ captures a latent axis of disagreement among countries. The function $\Gamma_{i\ell}$ measures country i's equilibrium payoff exposure to cleavage ℓ—how much country i stands to gain or lose when an international issue loads on that cleavage. This exposure depends on i's position in the network: a country can be highly exposed to one cleavage but insulated from another. For instance, the United States may be centrally positioned along the axis of US-Soviet rivalry, but peripheral to a localized conflict between India and Pakistan. How influential a country is for other nations' policy choices, how contested its policy choices are through strategic influence, and how much policy coordination it engages in are all contingent on the specific cleavage.

The variance σ_ℓ^2 determines how prominently geopolitical cleavage ℓ features in the distribution of international issues. A large variance σ_ℓ^2 implies that international issues are more likely to load on cleavage ℓ, increasing its weight in every country's *ex ante* welfare. The additive structure for *ex ante* welfare implies that one can evaluate the welfare consequences of each cleavage independently, and then aggregate across the full spectrum of cleavages.

2.7 Introducing Network Asymmetry

For expositional simplicity, we have so far assumed that the geopolitical network is symmetric. We now relax this assumption to allow the geopolitical network to be directed ($G_{ij} \neq G_{ji}$), such that the effect of country i on policy in country j can differ from that of country j on policy in country i. We thus incorporate asymmetries in the geopolitical network, reflecting differences in economic dependence, military capacity, or diplomatic reach.

We incorporate these network asymmetries by allowing for a geopolitical shifter for each country (π_i) that captures its structural importance in the network. Formally, let $\boldsymbol{H}$ be a symmetric, nonnegative matrix of network connections between countries, which plays the same role as the symmetric network in our baseline model except that its row sums may differ across countries. We define country i's structural importance as the total intensity of its network connections, $\pi_i \equiv \sum_{j=1}^{N} H_{ij}$, and let $\boldsymbol{\Pi} = \operatorname{Diag}(\boldsymbol{\pi})$; we normalize the scale of $\boldsymbol{H}$ such that $\sum_{i=1}^{N} \pi_i = 1$, which is without loss of generality. We construct the directed network as:

$$\boldsymbol{G} = (1-\alpha)\, \boldsymbol{\Pi}^{-1} \boldsymbol{H}, \tag{23}$$

so that $G_{ij} = (1-\alpha) H_{ij}/\pi_i$. Hence $G_{ij}/G_{ji} = \pi_j/\pi_i$: for a given connection H_{ij}, the country with greater structural importance responds less to the relationship than its partner does. In other words, countries with greater structural importance have a disproportionate impact on their neighbors, while being less sensitive to any single neighbor themselves.

By construction, $\boldsymbol{G}$ is nonnegative with constant row sums: $\boldsymbol{G}\mathbf{1} = (1-\alpha)\, \boldsymbol{\Pi}^{-1} \boldsymbol{H} \mathbf{1} = (1-\alpha)\, \boldsymbol{\Pi}^{-1} \boldsymbol{\pi} = (1-\alpha)\, \mathbf{1}$. Moreover, $\boldsymbol{\Pi}\boldsymbol{G}$ is symmetric, since $\pi_i G_{ij} = (1-\alpha) H_{ij} = \pi_j G_{ji}$. It follows that $\boldsymbol{G}$ is diagonalizable with real eigenvalues $\{\lambda_m\}$, bounded by $1-\alpha$ in absolute value, and that its right eigenvectors $\{\boldsymbol{u}_m^R\}$ can be normalized to be orthonormal in the $\boldsymbol{\Pi}$-inner product: $\left(\boldsymbol{u}_m^R\right)^\top \boldsymbol{\Pi} \boldsymbol{u}_p^R = \mathbf{1}_{m=p}$. The left eigenvectors are then $\boldsymbol{u}_m^L = \boldsymbol{\Pi} \boldsymbol{u}_m^R$, which satisfy biorthonormality:

$\left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{u}_p^R = \mathbf{1}_{m=p}$. The Perron eigenvalue is $1-\alpha$, with $\boldsymbol{u}_1^R = \mathbf{1}$ and $\boldsymbol{u}_1^L = \boldsymbol{\pi}$: the leading eigenvector of $\boldsymbol{G}$ in the space of actions remains the consensus direction, and each country's weight in measuring it is the country's structural importance (π_i).

We scale the costs of strategic influence by structural importance, such that the cost of country i exerting influence on country k is given by $\kappa\left(\pi_k/\pi_i\right)\omega_{ki}^2$. We thus capture the idea that influencing a country with greater structural importance (large π_k) is expensive, because its actions are anchored by dense network connections. Additionally, projecting influence from a peripheral country (small π_i) is costly, reflecting its limited diplomatic and economic reach.

Combining these assumptions, country i's payoff is now given by:

$$U_i\left(\boldsymbol{a},\boldsymbol{\Omega}\right) = -\left[\alpha\left(a_i-\theta_i\right)^2 + \sum_j G_{ij}\left(a_i-a_j\right)^2\right] - \kappa\sum_k \frac{\pi_k}{\pi_i}\omega_{ki}^2 + 2\sum_m \omega_{im}\left(a_i-\theta_i\right),$$

where a symmetric network is the special case in which $\boldsymbol{H}$ has constant row sums, so that all countries have the same structural importance and $\pi_i = \pi_k$ for all i,k.

We now show that our main analytical results for the properties of the policy coordination and strategic influence game extend to this case of asymmetric networks.

Proposition 7. *Suppose $\boldsymbol{G} = (1-\alpha)\,\boldsymbol{\Pi}^{-1}\boldsymbol{H}$ with influence costs $\kappa\left(\pi_k/\pi_i\right)\omega_{ki}^2$. Then:*

1. *Propositions 1 and 2 continue to hold. In particular:*

   (a) *Equilibrium net influence and geopolitical actions satisfy:*

   $$\boldsymbol{\Omega}\mathbf{1} = -\left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^2 - \alpha\boldsymbol{I}\right)^{-1}\left(\left(\boldsymbol{I}-\boldsymbol{G}\right)^2 - \alpha^2\boldsymbol{I}\right)\boldsymbol{\theta},$$

   $$\boldsymbol{a} = \left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^2 - \alpha\boldsymbol{I}\right)^{-1}\left(\boldsymbol{I}-\boldsymbol{G}\right)\left(\kappa\alpha-1\right)\boldsymbol{\theta}.$$

   (b) *The equilibrium inherits uniqueness, the convex combination property, $\boldsymbol{\Pi}$-weighted conservativeness ($\sum_i \pi_i a_i = \sum_i \pi_i\theta_i$ and $\sum_i \pi_i \sum_m \omega_{im} = 0$), and the $\boldsymbol{\Pi}$-weighted analogues of the two limiting benchmarks ($\kappa\to\infty$ and $\kappa\downarrow 1/\alpha$) from the symmetric baseline.*

2. *The equilibrium continues to admit a spectral representation:*

   $$a_i = \sum_m L\left(\lambda_m\right)\left[\boldsymbol{u}_m^R\right]_i\left(\boldsymbol{u}_m^L\right)^\top\boldsymbol{\theta}, \tag{24}$$

   where the scaling function $L\left(\lambda\right) \equiv \left(1-\lambda\right)\left(\kappa\alpha-1\right)/\left(\kappa\left(1-\lambda\right)^2-\alpha\right)$ is identical to that under a symmetric network.

Proof. See Online Appendix H. ☐

Proposition 7 establishes that the equilibrium vectors of net influence and geopolitical actions satisfy the same relationships as for a symmetric network in Proposition 1. The properties of the equilibrium are also similar to those with a symmetric network in Proposition 2. Finally, the spectral decomposition of geopolitical actions in terms of domestic interests in Proposition 4 extends to the asymmetric network by working in $\mathbf{\Pi}$-weighted coordinates. Specifically, we define adjusted domestic interests and geopolitical actions as $\tilde{\theta}_m \equiv \left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{\theta}$ and $\tilde{a}_m \equiv \left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{a}$. These adjusted variables with a tilde are projections of domestic interests and equilibrium actions onto the eigenbasis of $\boldsymbol{G}$: $\boldsymbol{\theta} = \sum_m \tilde{\theta}_m \boldsymbol{u}_m^R$ and $\boldsymbol{a} = \sum_m \tilde{a}_m \boldsymbol{u}_m^R$.

The equilibrium using these projected coordinates is structurally identical to that for a symmetric network. The closed-form expressions, the net influence equation, and the spectral scaling function $L(\lambda)$ all carry over unchanged. For each eigenmode, we have:

$$\tilde{a}_m = L(\lambda_m)\,\tilde{\theta}_m, \qquad L(\lambda) \equiv \frac{(1-\lambda)(\kappa\alpha - 1)}{\kappa(1-\lambda)^2 - \alpha}. \tag{25}$$

In the original coordinates, the equilibrium action of country i is:

$$a_i = \sum_m L(\lambda_m)\left[\boldsymbol{u}_m^R\right]_i \left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{\theta}. \tag{26}$$

Note that the scaling function $L(\cdot)$ is identical to the symmetric network case. Therefore, all the properties of the scaling function (non-negativity, boundedness, monotonicity in λ and κ, the two limiting benchmarks) carry over to the asymmetric network case. What changes is the geometry of geopolitical cleavages. In scalar form, the equilibrium action of country i can be written as:

$$a_i = \sum_m L(\lambda_m)\left[\boldsymbol{u}_m^R\right]_i \sum_j \pi_j \left[\boldsymbol{u}_m^R\right]_j \theta_j.$$

The loading of country i's action on geopolitical cleavage m is $\left[\boldsymbol{u}_m^R\right]_i$, and the importance of country j's domestic interest in cleavage m is $\pi_j\left[\boldsymbol{u}_m^R\right]_j$: relative to the responsiveness of its own action, the weight of a country's domestic interests in shaping the actions of others is proportional to its structural importance. With a symmetric network, all countries have the same structural importance, and the two coincide up to a common constant. In contrast, with the asymmetric network, structurally-important countries (large π_i) have magnified interest weights relative to their action loadings along every geopolitical cleavage. For these structurally-important countries, their own actions are anchored by dense network connections, while their domestic interests carry disproportionate weight in shaping other countries' policies. Peripheral countries display the reverse pattern—their interests carry little weight relative to their responsiveness—making them highly responsive to each geopolitical issue yet less consequential in determining others' actions.

We next extend our network identification result to this case of an asymmetric network, showing that we can recover the spectral properties of the network from the observed covariance matrices for domestic interests ($\boldsymbol{\Sigma_\theta}$) and geopolitical actions ($\boldsymbol{\Sigma_a}$). We again assume graph stationarity, such that the set of latent issues that determine the strength of geopolitical connections between countries is the same as those that generate differences in domestic interests across countries.

Assumption 2. ***(Graph Stationarity for Directed Networks)*** *The covariance matrix of domestic interests shares the eigenvectors of the network:* $\boldsymbol{\Sigma}_\theta = \sum_m \sigma_m^2 \boldsymbol{u}_m^R \left(\boldsymbol{u}_m^R\right)^\top$.

In the special case in which the network is symmetric ($\boldsymbol{H}$ has constant row sums, so that $\boldsymbol{\Pi} \propto \boldsymbol{I}$ and $\boldsymbol{G} \propto \boldsymbol{H}$), Assumption 2 reduces to Assumption 1. More generally, when the network is asymmetric, the $\boldsymbol{\Pi}$-weighting captures the fact that structurally-important countries can exhibit systematically larger variation in domestic interests, reflecting their dense connections to the geopolitical network. In terms of principal components, Assumption 2 states that domestic interests load on the eigenvectors of the network with mutually uncorrelated weights. Under Assumption 2, the cross-country covariance matrices for domestic interests ($\boldsymbol{\Sigma_\theta}$) and geopolitical actions ($\boldsymbol{\Sigma_a}$) in their original coordinates are given by:

$$\boldsymbol{\Sigma}_\theta = \sum_m \sigma_m^2\, \boldsymbol{u}_m^R (\boldsymbol{u}_m^R)^\top, \qquad \boldsymbol{\Sigma}_a = \sum_m L(\lambda_m)^2 \sigma_m^2\, \boldsymbol{u}_m^R (\boldsymbol{u}_m^R)^\top. \tag{27}$$

Combining Assumption 2 with the mapping from domestic interests to geopolitical actions from Proposition 7, we are again able to identify the eigenvalues (λ_m) and eigenvectors ($\boldsymbol{u}_m^R$) of the network, and hence recover the entire geopolitical network.

Proposition 8. ***(Identification)*** *Under Assumption 2, the eigenvalue* λ_m *and eigenvector* $\mathbf{u}_m^R$ *are identified whenever* $\sigma_m^2 > 0$*, via:*

$$\lambda_m = L^{-1}\left(\sqrt{\frac{[\tilde{\Sigma}_a]_{mm}}{[\tilde{\Sigma}_\theta]_{mm}}}\right),$$

where $\tilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{\Pi}^{1/2}\boldsymbol{\Sigma}_\theta\boldsymbol{\Pi}^{1/2}$ *and* $\tilde{\boldsymbol{\Sigma}}_a \equiv \boldsymbol{\Pi}^{1/2}\boldsymbol{\Sigma}_a\boldsymbol{\Pi}^{1/2}$ *are the* $\boldsymbol{\Pi}$*-weighted covariance matrices, and the diagonal entries are taken in their shared eigenbasis.*

Proof. See Online Appendix I. □

Proposition 8 identifies the eigenvalues λ_m and eigenvectors $\boldsymbol{u}_m^R$ (and hence recovers the entire geopolitical network) given the vector of structural importance ($\boldsymbol{\pi}$). For the special case of a symmetric network, this eigenbasis is pinned down by graph stationarity alone. More generally, when the network is asymmetric, there is an additional unknown—the vector of structural importance $\boldsymbol{\pi}$—since the eigenvectors of the network are only revealed after $\boldsymbol{\Pi}$-weighting the covariance matrices. If the structural importance of a country (π_i) can be directly measured empirically (e.g., using GDP shares), the vector $\boldsymbol{\pi}$ can be directly constructed using these empirical weights. Alternatively, since $\boldsymbol{\pi}$ is, by construction, the vector of row sums of $\boldsymbol{H}$, estimation can proceed via an iterative indirect inference procedure. First, given an initial guess for $\boldsymbol{\pi}$, one can implement the $\boldsymbol{\Pi}$-weighted diagonalization, compute the eigenvalues (λ_ℓ) and eigenvectors $(\boldsymbol{u}_\ell^R)$, and recover the geopolitical network $(\boldsymbol{H})$. Second, given empirical moments for the properties of the geopolitical network $(\boldsymbol{H})$, one can compare the value of these moments in the model and data, and update the initial guess for $\boldsymbol{\pi}$ to target the empirical values of the moments in the model.

Therefore, the methods that we developed in previous subsections for a symmetric network carry over to the more general case of an asymmetric network considered in this subsection. The asymmetric network extension introduces a new object, $\boldsymbol{\pi}$, which governs the heterogeneous structural importance of countries in the network. This heterogeneity in structural importance governs asymmetries in countries' incentives to coordinate policies with others and engage in strategic influence towards others. The construction $\boldsymbol{G} = (1-\alpha)\,\boldsymbol{\Pi}^{-1}\boldsymbol{H}$ preserves the full analytical tractability of the symmetric network case: the properties of the equilibrium, spectral decoupling, the scaling function $L(\lambda)$, and the identification of the eigenvalues and eigenvectors of the network all carry over unchanged in $\boldsymbol{\Pi}$-weighted coordinates. What changes is the geometry of geopolitical cleavages: the geopolitical actions of structurally-important countries respond less to disagreement with their neighbors, whereas their interests carry disproportionate weight in shaping the geopolitical actions of those neighbors. Peripheral countries display the opposite pattern—their geopolitical actions are highly responsive to each international issue, but their interests are less consequential in determining the geopolitical actions of other countries.

3 Empirical Implementation

3.1 Data

To estimate the structure of the geopolitical network and test the predictions of our theoretical model, we use two datasets to measure geopolitical actions $(\boldsymbol{a})$ and domestic interests $(\boldsymbol{\theta})$, the model's two core variables.

3.1.1 Measuring Geopolitical Actions (a): UN General Assembly Voting

We measure countries' geopolitical actions $(\boldsymbol{a})$ using United Nations General Assembly (UNGA) voting data. To construct the action covariance matrix, we take the votes in each of the six topics categorized by Bailey et al. (2017) as our empirical measure of actions.[6] We take the six topic-level actions (each topic denoted p) and compute the cross-country covariance matrix of actions for each decade

[6] The six categories of issues are: human rights, nuclear weapons/materials, Middle East, disarmament, colonialism, and economic issues. The literature refers to such measures of aggregated actions (either across all topics or within a given topic) adjusted for issue composition and the baseline tendency for agreement on specific issues as "ideal points."

d as:

$$(\Sigma_{a,d})_{ij} = \sum_{t \in d} \sum_{p=1}^{6} (a_{ipt} - \overline{a}_{pt})(a_{jpt} - \overline{a}_{pt}) \tag{28}$$

where a_{ipt} denotes country i's ideal point on topic p and $\overline{a}_{pt}$ denotes the Π-weighted average position on topic p: $\overline{a}_{pt} \equiv \sum_k \frac{\pi_{kd}}{\sum_i \pi_{id}} a_{kpt}$. The $\mathbf{\Pi}$-weighted average is each topic's loading on the consensus mode; subtracting it removes the component of each topic that reflects the arbitrary level of UN coding rather than any geopolitical cleavage; and it is precisely the demeaning under which the $\mathbf{\Pi}$-weighted covariance matrices are consistent with Assumption 2. We thus capture both cross-country and cross-issue variation, as the ij-th entry of $\Sigma_{a,d}$ will be larger when countries i and j both deviate from the average action on each issue similarly.

3.1.2 Measuring Domestic Interests ($\boldsymbol{\theta}$): Varieties of Democracy (V-Dem)

We measure countries' domestic interests ($\boldsymbol{\theta}$) using orientation towards democracy or autocracy, as a fundamental institutional characteristic of countries that affects their international positions. Specifically, we use the Varieties of Democracy (V-Dem) dataset, which aggregates expert evaluations to score countries across hundreds of disaggregated domestic political and social dimensions, such as freedom of the press, gender equality, civil liberties, and judicial independence. These metrics reflect the internal political architecture and ideological priorities of a state independently of foreign lobbying and alignment. As such, they correspond closely to the notion of domestic interests in the model, which are defined separately from strategic influence and coordination.

The V-Dem dataset contains over one thousand variables that record features of different dimensions of political institutions. These variables are aggregated into 48 separate index variables, each of which summarizes a country's political institutions relating to a given topic, such as freedom of expression or electoral democracy. However, it is a priori unclear which of these 48 indices are relevant to geopolitical actions taken on the international stage. In order to address this, we use LASSO to select which differences in these indices predict differences in countries' UNGA voting actions.[7] This model selection procedure results in 33 variables being selected, the names of which are listed in Table 1 below.

[7]To be more precise, we run a penalized regression of the form $|a_{it} - a_{jt}| = \sum_{k=1}^{48} \beta_k |v_{ikt} - v_{jkt}| + \epsilon_{ijt}$. The variable a_{it} denotes the UNGA ideal point of country i in year t, and so the outcome is the absolute difference in countries i and j's UN voting behavior in year t. We relate this outcome variable to the difference in country i and j in year t of the 48 indices of political institutions that V-Dem provides (denoted $|v_{ikt} - v_{jkt}|$); LASSO selects the dimensions k which are predictive of the outcome.

Table 1: Varieties of Democracy Variables used for Domestic Interests

Variable	Description	Variable	Description
v2x_EDcomp_thick	Electoral component index	v2x_ex_party	Ruling party dimension index
v2x_api	Additive polyarchy index	v2x_feduni	Division of power index
v2x_civlib	Civil liberties index	v2x_frassoc_thick	Freedom of association thick index
v2x_clphy	Physical violence index	v2x_gencl	Women civil liberties index
v2x_clpol	Political civil liberties index	v2x_gender	Women political empowerment index
v2x_clpriv	Private civil liberties index	v2x_genpp	Women political participation index
v2x_corr	Political corruption index	v2x_hosabort	Presidential election aborted
v2x_delibdem	Deliberative democracy index	v2x_hosinter	Chief executive no longer elected
v2x_diagacc	Diagonal accountability index	v2x_jucon	Judicial constraints on the executive index
v2x_divparctrl	Divided party control index	v2x_libdem	Liberal democracy index
v2x_egaldem	Egalitarian democracy index	v2x_liberal	Liberal component index
v2x_elecoff	Elected officials index	v2x_partipdem	Participatory democracy index
v2x_elecreg	Electoral regime index	v2x_pubcorr	Public sector corruption index
v2x_ex_confidence	Confidence dimension index	v2x_suffr	Share of population with suffrage
v2x_ex_direlect	Direct election dimension index	v2x_regime	Regimes of the world—the RoW measure
v2x_ex_hereditary	Hereditary dimension index	v2x_regime_amb	Regimes of the world—the RoW measure
v2x_ex_military	Military dimension index		(with ambiguous case categories)

Note: 33 variables from the Varieties of Democracy (V-Dem) dataset selected by LASSO as predicting differences in the UNGA actions as estimated by Bailey et al. (2017).

Let x_{imt} denote country i's V-Dem score on domestic issue m in year t. We construct the cross-country covariance matrix of domestic interests ($\mathbf{\Sigma}_{\theta,d}$) by computing the covariance between the interests vectors of countries i and j across 33 distinct domestic issues across years t within each decade d as follows:

$$(\Sigma_{\theta,d})_{ij} = \sum_{t\in d}\sum_{m=1}^{33} (x_{imt} - \overline{x}_{mt})(x_{jmt} - \overline{x}_{mt}).$$

We again capture cross-sectional variation, since the ij-th entry of $\Sigma_{\theta,d}$ will be larger if the domestic interests of countries i and j deviate from the cross-country mean for a given dimension m similarly within that decade. As for $\Sigma_{a,d}$, we demean by the $\mathbf{\Pi}$-weighted average value $\overline{x}_{mt} \equiv \sum_k \frac{\pi_{kd}}{\sum_i \pi_{id}} x_{kmt}$.

Remark. (Normalization of units.) Both geopolitical actions (UNGA voting, $\boldsymbol{a}$) and domestic interests (V-Dem indices, $\boldsymbol{\theta}$) are measured in different units. Only their cross-country dispersion carries economic meaning. Along each identified cleavage ℓ, the data pin down the measured gain $\hat{L}_\ell = \sigma_{a,\ell}/\sigma_{\theta,\ell}$—the ratio of the action to interest dispersion. The model interprets this ratio as $L(\lambda_\ell) = \hat{L}_\ell$; since $L(\cdot)$ is strictly monotone (given α, κ), inversion recovers the eigenvalue λ_ℓ. These eigenvalues are identified only up to a choice of units. To see this, fix the units of $\boldsymbol{\theta}$ and suppose that actions are rescaled by $s > 0$, so that estimation uses $s \cdot \boldsymbol{a}$ in place of $\boldsymbol{a}$. The joint diagonalization (the eigenbasis) is unaffected, while every recovered eigenvalue of $\boldsymbol{G}$ shifts: $\lambda_\ell^{(s)} = L^{-1}\big(s\,\hat{L}_\ell\big)$. This choice of units is immaterial for our counterfactuals, because we report counterfactual actions in observed data units. Expressed in those units, the equilibrium mapping from interests to actions is unaffected by the rescaling, because the two appearances of s cancel out:

$$\frac{1}{s}\,L\big(\lambda_\ell^{(s)}\big) \;=\; \frac{1}{s}\,L\Big(L^{-1}\big(s\,\hat{L}_\ell\big)\Big) \;=\; \frac{1}{s}\cdot s\,\hat{L}_\ell \;=\; \hat{L}_\ell,$$

which leaves every counterfactual action a function of the data alone. We choose the scale for the eigenvalues using the following convenient normalization. We set the largest non-consensus eigenvalue of $\boldsymbol{G}$ to be just below that of the consensus mode, whose eigenvalue the model pins at $1-\alpha$. Strict dominance of the consensus mode—the Perron–Frobenius property of a connected network—requires every cleavage eigenvalue to lie strictly inside this bound. This normalization pins down the scale of the eigenvalues of $\boldsymbol{G}$ but affects none of the results on counterfactual actions.

3.1.3 Other Data Sources and Coverage

In empirical validation checks of our inferred geopolitical network ($\boldsymbol{G_d}$), we use a range of empirical proxies for bilateral geopolitical connections between countries. We use variables from the CEPII Gravity dataset (bilateral trade, distance and indicators for cultural and historical linkages from Conte et al. 2022), the Polity Project (the difference in polity scores between countries). In addition to the measure of overall foreign influence from the Foreign Bilateral Influence Capacity (FBIC) dataset discussed above, we also use disaggregated measures of economic and security dependence from the same dataset. Finally, we also use a separate measure of strategic influence from the Country and Organization Leader Travel (COLT) dataset's record of state visits between the heads of state/government of different nations.

The number of countries used to recover the network $\boldsymbol{G_d}$ varies across decades, because of the emergence of new states and missing data for some years. As plotted in Appendix Figure A2, the coverage of V-Dem and UN voting data grows over time, ultimately converging to roughly 175 countries with non-missing data on both domestic interests and geopolitical actions. Appendix Figure A3 reports country coverage for the variables used in empirical validation checks of our inferred geopolitical network ($\boldsymbol{G_d}$). In general, we have high coverage throughout our sample period, with the exception that we currently only use bilateral trade flow data from 1990 onwards.

3.2 Estimating Geopolitical Cleavages

We implement our empirical analysis separately by decade, recognizing that $\boldsymbol{G}$ is slow moving and stable within a decade. To estimate the eigenvectors and eigenvalues of the geopolitical network, we begin by calibrating the parameters of the scaling function ($L(\lambda)$). In our baseline specification, we set $\alpha = 0.4$ and $\kappa = 5$, which ensures that the parameter restriction for the costs of strategic influence to be sufficiently convex ($\kappa > 1/\alpha$) is satisfied. In robustness checks, we examine the sensitivity of our empirical results to alternative values of these parameters.

We estimate our asymmetric network specification by weighting the covariance matrices of geopolitical actions ($\boldsymbol{\Sigma_{a,d}}$) and domestic interests ($\boldsymbol{\Sigma_{\theta,d}}$) using the structural importance matrix ($\boldsymbol{\Pi_d}$; we discuss the estimation procedure in the next section), which yields $\tilde{\boldsymbol{\Sigma}}_{\theta,d} = \boldsymbol{\Pi_d^{1/2}\Sigma_{\theta,d}\Pi_d^{1/2}}$ and $\tilde{\boldsymbol{\Sigma}}_{a,d} = \boldsymbol{\Pi_d^{1/2}\Sigma_{a,d}\Pi_d^{1/2}}$. As discussed in Section 2.5, under graph stationarity domestic interests and geopolitical actions share the same principal components, and the model maps the variance of domestic interests along each cleavage into the variance of geopolitical actions through the scaling factor $L(\lambda_m)^2$. We therefore estimate the latent cleavages as joint principal components: a single set of orthogonal directions that simultaneously accounts for the covariance structure of both domestic interests and geopolitical actions (common principal components analysis, as discussed in Flury 1984). Even if the true weighted covariance matrices share exact common principal components, their empirical counterparts need not, because of sampling variation and measurement error. Formally, for each decade d, we search for the orthonormal basis $\boldsymbol{U}_d$ that comes as close as possible to diagonalizing both weighted covariance matrices, minimizing the sum of their squared off-diagonal entries (an approximate joint diagonalization):

$$\min_{\boldsymbol{U}_d} \sum_i \sum_{j\neq i} \left\{ \left[\boldsymbol{U}_d'\tilde{\boldsymbol{\Sigma}}_{\theta,d}\boldsymbol{U}_d\right]_{ij}^2 + \left[\boldsymbol{U}_d'\tilde{\boldsymbol{\Sigma}}_{a,d}\boldsymbol{U}_d\right]_{ij}^2 \right\}.$$

When the two matrices coincide, this procedure reduces to ordinary principal components analysis; requiring a common basis for domestic interests and geopolitical actions is what disciplines the decomposition, and the relative variances along each joint principal component then identify the eigenvalues of the network through Proposition 8. We denote by $\boldsymbol{v}_{m,d}$ the columns of $\boldsymbol{U}_d$ (the estimated joint principal components); the right eigenvectors of the network are then recovered as $\boldsymbol{u}_{m,d}^R = \boldsymbol{\Pi}_d^{-1/2}\boldsymbol{v}_{m,d}$.

We examine the empirical success of this approximate joint-diagonalization procedure using the analog of an R^2: the share of total variance in the empirical second moments explained by the matrices that are reconstructed using the approximate joint basis.[8]

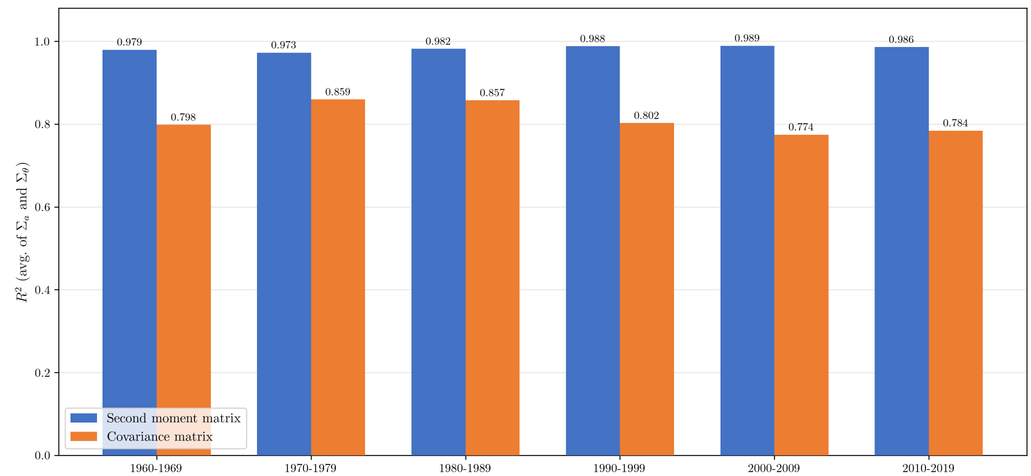
Figure 3 shows that the reconstruction quality is high. Across issues and decades, the top-5 principal components from the approximate joint-diagonalization (out of the total number of components equal to the number of countries in our sample for each decade) account for, on average across actions and domestic interests, over 95 percent of the variation in the uncentered second-moments and over 77 percent of the cross-country covariances.[9]

[8] This statistic is closely related to the least squares criterion (Frobenius Norm), which equals the sum of the squares of all the off-diagonals of $\boldsymbol{U}_d'\boldsymbol{\Sigma}_{\theta,d}\boldsymbol{U}_d$ and $\boldsymbol{U}_d'\boldsymbol{\Sigma}_{a,d}\boldsymbol{U}_d$. We obtain similar results using other commonly-used metrics for the empirical success of the joint diagonalization, such as the Amari Index or the Kullback-Leibler Divergence.

[9] Specifically, we project empirical covariance matrices ($\boldsymbol{\Sigma}_{\theta,d}$ and $\boldsymbol{\Sigma}_{a,d}$) and the corresponding uncentered second-moment matrices (covariance with the consensus retained) onto the orthonormal basis $\boldsymbol{U}_d$. We use the eigenvalues of the geopolitical action matrix to rank the components and keep the top five. We then project back to the original space and examine how much of the variation in each original matrix can be explained by these joint-diagonalized

Mechanically, adding more eigenvectors necessarily improves the fit of the joint-diagonalization. But we find that the top-five eigenvectors explain the vast majority of the observed cross-country covariance in political actions ($\boldsymbol{\Sigma}_{\boldsymbol{a},\boldsymbol{d}}$) and domestic interests ($\boldsymbol{\Sigma}_{\boldsymbol{\theta},\boldsymbol{d}}$). This pattern reflects a core feature of geopolitical networks: they are dominated by a few leading geopolitical cleavages, as captured by the top few eigenvectors, such that most other potential geopolitical cleavages do not matter in practice. The discarded geopolitical cleavages not only are characterized by low eigenvalues—meaning the network naturally neutralizes divergence along them—but they also represent obscure geopolitical fault lines that rarely manifest in actual international shocks.

Figure 3: Reconstruction Quality for the Joint Diagonalization of $\Sigma_{\theta,d}$ and $\Sigma_{a,d}$



Note: Reconstruction quality for the joint diagonalization, measured by the R^2: the share of the total variance in each empirical matrix explained by its reconstruction from the top-five principal components. Each entry is the average across the domestic-interests and geopolitical-actions matrices, reported separately for the covariance and uncentered second-moment matrices.

3.3 Estimating Structural Importance

We next turn to the estimation of the vector of structural importance ($\boldsymbol{\pi}_{\boldsymbol{d}}$) that controls the cost of influence of each country. We assume a constant elasticity relationship between structural importance and a country's size as measured by its GDP: $\pi_{id} = \text{GDP}_{id}^{\beta}$, where β is a parameter that we calibrate to match an empirical moment in the data. Intuitively, β captures the asymmetry of the network; larger β exaggerates differences in country size, whereas $\beta \to 0$ returns a symmetric network. Since this parameter reflects the asymmetry of the network, and since asymmetry implies it is cheaper for a bigger country i to influence others and more expensive for others to influence i, a natural moment to match involves the relationship between outward influence relative to inward influence and GDP.

reconstructions $\boldsymbol{U_d}\,\text{Diag}\,(\boldsymbol{U}_d'\boldsymbol{\Sigma}_{\theta,d}\boldsymbol{U}_d)_{top5}\,\boldsymbol{U}_d'$ and $\boldsymbol{U_d}\,\text{Diag}\,(\boldsymbol{U}_d'\boldsymbol{\Sigma}_{a,d}\boldsymbol{U}_d)_{top5}\,\boldsymbol{U}_d'$ (and analogously for the uncentered second-moment matrices), as measured by the R^2.

In particular, we use the estimated coefficient (b) from the following OLS regression:

$$\ln \left(\frac{\text{influence exerted}}{\text{influence received}} \right)_{id} = b \ln \text{GDP}_{id} + \epsilon_{id}, \tag{29}$$

where influence exerted is the average influence of a sender country on all receiver countries within each decade; influence received is the average influence of a receiver country from all sender countries within each decade; GDP_{id} is average GDP within each decade; ϵ_{id} is a stochastic error; and we estimate this regression pooling all decades.

We first estimate this regression using actual data on bilateral foreign influence. The empirical measure of influence that we use is from the Formal Bilateral Influence Capacity (FBIC) dataset, which measures state-sanctioned or state-sponsored and publicly acknowledged bilateral dependence between countries. Bilateral dependence is measured in the data based on diplomatic exchange, arms transfers, and goods trade. Critically, bilateral dependence is asymmetric – in a given pair, one country can depend more on the other. Consistent with the predictions of our model, in which more important countries send more outward influence to other countries relative to the inward influence they receive from other countries, we find a positive and statistically significant estimate of $\hat{b} = 0.701$.

We next estimate the same regression on model-predicted foreign influence. To obtain our model's predictions for foreign influence, we start with an assumed value for β, and generate the $\boldsymbol{\pi_d}$ vector using $\pi_{id} = \text{GDP}_{id}^{\beta}$. Using the resulting $\boldsymbol{\Pi_d}$ matrix, we can compute adjusted domestic interests and geopolitical actions using the procedure from Section 2.7, and recover the asymmetric geopolitical network ($\boldsymbol{G_d}$) for our assumed value of β. Given this asymmetric geopolitical network ($\boldsymbol{G_d}$) and the observed actions ($\boldsymbol{a_d}$) from UN votes, we can invert our model's first-order conditions (14) and (15) to recover the implied vector of domestic interests ($\boldsymbol{\theta_d}$) and matrix of influence ($\boldsymbol{\Omega_d}$). Using this matrix of bilateral influence ($\boldsymbol{\Omega_d}$), we can compute the average influence exerted by a sender country on all receiver countries, and the average influence experienced by a receiver country from all sender countries. Given these model predictions for inward and outward influence for our assumed value of β, we estimate the coefficient b in the regression (29). Finally, we update our assumed value of β to minimize the sum of squared deviations between the estimated coefficients $\widehat{b}$ using actual and model-generated foreign influence.

We find that the exponent on GDP for which the model matches the empirical relationship between relative inward and outward influence and GDP in the data is $\beta = 0.78$. Given this parameter value, we set $\pi_{id} = \text{GDP}_{id}^{0.78}$ for each decade, and use the resulting $\boldsymbol{\Pi_d}$ matrix to recover the asymmetric $\boldsymbol{G_d}$ network for each decade.

3.4 Validation of the Inferred Geopolitical Network

We use the top-five eigenvectors ($\boldsymbol{v}_{m,d}$) and eigenvalues ($\lambda_{m,d}$) from our joint diagonalization in the previous subsection to construct the unobserved geopolitical network in each decade ($\boldsymbol{G_d} = \boldsymbol{\Pi}_d^{-1/2}\boldsymbol{U_d}\boldsymbol{\Lambda_d}\boldsymbol{U_d}^\top\boldsymbol{\Pi}_d^{1/2}$). We next present evidence that our recovered decade-specific asymmetric geopolitical network $\boldsymbol{G_d}$ reflects a range of empirical proxies for geopolitical connections between countries used in the existing economics and political science literature. Specifically, we estimate the following bilateral regression of our geopolitical connection for receiver i and sender j in decade d (G_{ijd}) on each of these empirical proxies (x_{ijd}):

$$G_{ijd} = bx_{ijd} + \alpha_{id} + \gamma_{jd} + \epsilon_{ijd}, \tag{30}$$

where our empirical proxies include bilateral trade, geographical distance and indicators of common religion, language and historical colonizers; α_{id} are receiver-decade fixed effects; γ_{jd} are sender-decade fixed effects; and ϵ_{ijd} is a stochastic error.

We standardize all variables such that their mean is zero and their standard deviation is one. We estimate (30) using the linear fixed effects estimator. Therefore, the estimated coefficients have the interpretation that a one standard deviation change in the right-hand side variable (x_{ijd}) is associated with a change in the left-hand side variable (G_{ijd}) of b standard deviations. The inclusion of the receiver-decade and sender-decade fixed effects ensures that the coefficient b is identified from the bilateral variation in network connections. We cluster the standard errors at the (undirected) country pair level to allow for correlation in the error term between the directed links for each undirected country pair and over time.

We find an intuitive pattern of estimation results, as reported in Table 2. Countries with more similar political institutions (column 1) and more cultural/historical ties (columns 4-6) have stronger geopolitical connections in our recovered network, whereas countries that are more politically dissimilar (column 1) or more geographically distant (column 3) have weaker geopolitical connections in our recovered network. Therefore, our inferred geopolitical network is systematically related in the expected way to these empirical proxies for network connections between countries.

This pattern of results is far from mechanical. It could be natural to expect the cross-country covariance matrices of geopolitical actions and domestic interests to be related to bilateral trade or geographical distance. However, our estimates here are for the edges in our geopolitical network $\boldsymbol{G_d}$, which are given by $G_{ijd} = \sqrt{\frac{\pi_{jd}}{\pi_{id}}}\sum_{m=1}^{5}\lambda_{md}[\boldsymbol{v}_{md}]_i[\boldsymbol{v}_{md}]_j = \sqrt{\frac{\pi_{jd}}{\pi_{id}}}\sum_{m=1}^{5}L^{-1}\left(\sqrt{\frac{[\tilde{\Sigma}_{a,d}]_{mm}}{[\tilde{\Sigma}_{\theta,d}]_{mm}}}\right)[\boldsymbol{v}_{md}]_i[\boldsymbol{v}_{md}]_j$. Therefore, these edges involve a linear combination of (i) the structural importance weights (π_{id}), (ii) the eigenvalues of the geopolitical network (λ_{md}), and (iii) entries of the basis vectors (v_{id}). The eigenvalues in turn depend on the *relative* responsiveness of geopolitical actions and domestic interests for each geopolitical cleavage. It is far from obvious that this complex transformation of the data implied by our model should be related in the expected way to empirical proxies for geopolitical connections between countries. Therefore, these results provide validation for our approach.

The empirical proxies in the first three columns — polity score distance, trade flow, and geographical distance — are all standardized on the decade level, whereas those in the final three columns are indicator variables. Of the continuous covariates, polity score distance has the largest (in magnitude) association with the network edges, whereas two countries' populations sharing a religion is the largest predictor of network edges for the indicator variables. Overall, the results in Table 2 provide evidence that our recovered geopolitical network exhibits sensible cross-sectional variation that qualitatively matches economic and cultural interactions between countries. Given the robustness of these exercises, we next provide descriptive evidence regarding features of the estimated network itself, rather than its relation to external measures.

Table 2: Geopolitical Network Edges and Measures of Countries' Ties

	Outcome: G_{ijd}					
	$\lvert \text{Polity}_{id} - \text{Polity}_{jd} \rvert$	Trade flow$_{ijd}$	Distance$_{ij}$	Common religion$_{ij}$	Common language$_{ij}$	Common colonizer$_{ij}$
	(1)	(2)	(3)	(4)	(5)	(6)
x_{ijd}	-0.0935***	0.135**	-0.0800***	0.164***	0.0970***	0.0575***
	(0.00759)	(0.0557)	(0.00725)	(0.0178)	(0.0199)	(0.00947)
Observations	113270	60535	113456	111936	113456	113456
Sender × decade FE:	✓	✓	✓	✓	✓	✓
Receiver × decade FE:	✓	✓	✓	✓	✓	✓

Note: This table regresses the ij-th entry of the G network for decade d on decade-level averages of dyadic variables. Column 1's covariate is the distance between country i and country j's average polity scores in each decade d. Column 2's covariate is directed exports from country i to country j in decade d (CEPII BACI, origin-to-destination), not total bilateral trade. Column 3's covariate is the geographical distance between country i and country j. Columns 4-6 have binary covariates denoting whether or not countries i and j share a common religion, common language, or common colonizer. CEPII trade flow data (Column 2) goes back to 1990, whereas polity data (Column 1) covers the entire panel. Fixed effects are separate sender ($j\times$ decade) and receiver ($i\times$ decade) effects. Standard errors are clustered on the unordered pair level, with significance levels denoted * $p < 0.1$, ** $p < .05$, *** $p < 0.01$.

In order to further demonstrate that the recovered networks $\boldsymbol{G}_d$ reflect geopolitical ties, we carry out two placebo exercises that illustrate the robustness of our estimates which can be found in Appendix Figure A4. The first exercise (Panel A) is a permutation placebo: we randomly relabel the countries in the action covariance matrix (and, in separate variants, both the action and preference covariances) before recovering $\boldsymbol{G}_d$. This severs the true correspondence between a country's actions and its preferences, so that any structure that survived is mechanical rather than substantive. This exercise demonstrates that the regression results disappear when they should. In contrast, the second check (Panel B) is a residual bootstrap exercise which shows that the results are robust to measurement error. Because our estimation procedure finds an approximate joint basis for the two covariance matrices, imperfect fit in the joint basis means that the original covariance matrices have some portion that is explained by the top five modes of the joint basis, and some part that is a residual. To ensure that this residual is not large enough to alter our regression results, we repeatedly replace the discarded residual variation with random noise of the same magnitude, re-estimate $\boldsymbol{G}_d$, and re-run our validation regressions. These two placebo exercises behave exactly as they should if $\boldsymbol{G}_d$ captures real geopolitical ties: the validation coefficients are not statistically distinguishable from zero when the inputs are scrambled, but the coefficients remain stable in sign and magnitude when we bootstrap the residual. Together these two exercises indicate that the estimated $\boldsymbol{G}_d$ networks do not simply reflect some mechanical consequences of our estimation procedure, but instead recover genuine geopolitical structure whose relation to external measures like distance or trade behaves as expected.

3.5 Inferred Strategic Influence and State Visits

As a further validation check, we show that the patterns of strategic influence implied by our model are robustly associated with independent data on strategic influence not used in the quantification of our model. We use the Country and Organization Leader Travel (COLT) dataset's records of state visits between the heads of state/government of different nations. We measure strategic influence using the number of state visits for each country pair-year, since many of the most important efforts at cross-country coordination happen in bilateral country visits or forums like the G7. These state visits are plausibly undirected, since France's president travelling to the USA versus the USA's president travelling to France does not carry a substantially different interpretation. Therefore, we aggregate state visits at the country pair-year level.

We generate model predictions for strategic influence following a similar procedure as for the estimation of the vector of structural importance ($\boldsymbol{\pi}_d$). We first take UN resolution votes in each year t as the empirical actions ($\boldsymbol{a_t}$) and back out domestic interests ($\boldsymbol{\theta_t}$) using the first-order condition for actions ($\boldsymbol{a_t} = \mathcal{L}(\boldsymbol{G_d})\boldsymbol{\theta_t}$). We then use the recovered domestic interests ($\boldsymbol{\theta_t}$) to back out the model-implied influence ($\boldsymbol{\Omega_t}$) from the first-order condition for influence $\boldsymbol{\Omega_t} = C(\boldsymbol{G_d})\boldsymbol{\theta_t}$. This first-order condition for influence from country i to country $k \neq i$ in year t is given by

$$\omega_{kit} = \frac{\pi_{id}}{\pi_{kd}} \frac{1}{\kappa} \sum_j G_{ijd}(a_{it} - a_{jt}) \left[(\boldsymbol{I} - \boldsymbol{G_d})^{-1} \right]_{jk},$$

where actions and influence vary by year t, but we assume that the geopolitical network ($\boldsymbol{G_d}$) is slow-moving, and hence only varies by decade d, as recovered from our joint diagonalization in a previous subsection.

Observing actions thus allows us to back out ω_{kit} (since $\boldsymbol{G_d}$ and $\boldsymbol{\Pi_d}$ are already known). In the model, influence ω_{kit} can be negative or positive as countries may want others to take a higher or lower action. State visits, however, measure magnitude of influence rather than the political intention/direction of influence. To make the empirical and model objects cohere, we simply record the magnitude of influence $|\omega_{kit}|$ and average across all observed actions (UN votes), to get the average magnitude of influence $\overline{|\boldsymbol{\Omega}_t|}$. To make the model-implied object match the structure of state visits, we then construct $\frac{1}{2}\left[\overline{|\boldsymbol{\Omega}_t|}+\overline{|\boldsymbol{\Omega}_t|}^T\right]$, which averages the magnitude of influence within a country pair so that it is symmetric/undirected. We denote the ij-th element of the average matrix as $\overline{\omega}_{(ij)t}$. This gives a model-implied measure of the magnitude of influence between country pairs, which we can then relate to state visits. We use a Poisson specification to allow for zero state visits in the data. We estimate the following regression between state visits in the data (State visits$_{(ij)t}$) and our model's predictions for the magnitude of influence ($\overline{\omega}_{(ij)t}$):

$$\text{State visits}_{(ij)t} = \exp\left[b\overline{\omega}_{(ij)t} + \gamma_{(i)t} + \eta_{(j)t}\right],$$

where the subscript notation emphasizes that the regression is unordered, such that our outcome variable is only defined on the unordered pair involving i and j, and not on the ordered pair $i \to j$ versus $j \to i$. Consistent with this, we denote the country-year fixed effects as $\gamma_{(i)t}$ to indicate that we have a fixed effect for whenever country i appears in a pair in year t (and similarly for country j).

Table 3: State Visits and Model-implied Influence

	Outcome: State visits$_{(ij)t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\overline{\omega}_{(ij)t}$	0.0159***	0.1073***	0.1536***	0.0228***	0.0496***	0.0474***
	(0.0043)	(0.0068)	(0.0084)	(0.0060)	(0.0108)	(0.0119)
Sample restriction: pair had visit in	year	decade	ever	year	decade	ever
Year FE	✓	✓	✓	✗	✗	✗
Country-year FE	✗	✗	✗	✓	✓	✓
Observations	35,794	122,808	295,608	35,794	122,808	295,608

Note: Poisson pseudo-maximum-likelihood regressions of leader-visit counts on the standardized model-implied bilateral influence $\overline{\omega}_{(ij)t}$, recovered from *UN resolution votes* on the ideal-point-covariance G network (Σ_a = the π-demeaned covariance of the six topic ideal points, rescaled by a per-decade constant so the largest non-Perron eigenvalue equals 0.59; the Perron eigenvalue $1-\alpha = 0.6$ is the strict top of the spectrum). The unit is the *unordered* country-pair-year (both directions collapsed to a single row by averaging). Influence is z-scored by decade. Columns 1–3 include year fixed effects; columns 4–6 include a single *shared country-year* fixed effect that loads on *both* endpoints of the pair (e.g. USA-2010 is one effect, not a separate "sender" and "receiver" effect). Each column restricts the sample to pair-years in which the pair had at least one leader visit that same year (columns 1, 4), anytime that decade (columns 2, 5), or ever in the panel (columns 3, 6). Standard errors clustered on the unordered pair in parentheses, with significance levels denoted * $p < 0.1$, ** $p < .05$, *** $p < 0.01$.

In comparing the model's predictions to the data, we take into account the extensive and intensive margins of state visits in the data. In the model, countries face a convex cost of influence and so always choose some positive magnitude of influence on all other countries. However, not all countries have state visits within a given year or even decade. We explore the roles of the extensive and intensive margin in two ways, reporting the results in Table 3. First, we use a Poisson regression (rather than OLS) so that our estimates are robust to zeros in the outcome variable. Second, we consider three different sample restrictions: (i) country-pairs that have at least one visit in a given year

(Columns (1) and (4)); (ii) country-pairs that have at least one visit in a given decade (Columns (2) and (5)); and (iii) country-pairs that have at least one visit in the entire sample (Columns (3) and (6)). Across all of these different samples, we find a strong relationship between our model's predictions for the magnitude of foreign influence and state visits in the data.

We also demonstrate the robustness of our results to the inclusion of different sets of fixed effects. Columns (1)-(3) report results with year fixed effects, such that the estimated coefficient reflects cross-sectional variation, illustrating that country-pairs with higher model-predicted influence also have more state visits. Although we intentionally construct the model to allow for asymmetries, one concern about this specification could be that large countries have more state visits for reasons other than those in the model. To address this concern, Columns (4)-(6) present results with country-year fixed effects, and show that even after controlling for country size or other sources of unobserved heterogeneity across countries, we continue to find a strong relationship between our model's predictions for the magnitude of foreign influence and state visits in the data.

4 Properties of the Geopolitical Network

Having validated our geopolitical network, we now describe its empirical properties.

4.1 Recovered Geopolitical Cleavages and Their Evolution

One way to describe the estimated geopolitical network is to examine its eigenvectors for leading geopolitical cleavages and their evolution over time. Figure 4 visualizes $\boldsymbol{u}_2$, the eigenvector associated with the second-largest eigenvalue, across our sample period. Recall that this eigenvector captures the network's deepest geopolitical cleavage—international issues where tightly connected blocs exhibit high internal consensus but stark cross-bloc polarization. In the figure, each panel represents a distinct decade, with the heatmap diverging from red to blue to denote polar opposite domestic interests along this structural axis.

The evolution of the leading nontrivial eigenvector tracks major shifts in the structure of international politics. In the 1960s, it aligns closely with the Cold War division between the Western and Soviet blocs. During the 1970s, however, the dominant cleavage shifts toward a divide between the major powers—including the United States and the Soviet Union—and the developing world, reflecting the growing political salience of the Third World (see for example Westad 2019). By the 1980s, the principal cleavage again resembles the East–West divide, before changing sharply after the collapse of the Soviet Union.

The post-1989 realignment is especially visible in the spatial distribution of this cleavage. Former Soviet-aligned states in Eastern Europe, such as Poland and the Czech Republic, move away from Russia and China and toward the Western pole. At the same time, the former Soviet bloc no longer anchors the same extreme position it occupied during the Cold War. Since the 1990s, the leading cleavage has instead been organized around a configuration in which the United States and Western Europe occupy one pole, China occupies the other, and many countries in the Global South lie between them.

This evolution also captures changes in the position of groups of countries relative to the Cold War divide. From the 1970s onward, countries in Asia, Africa and Latin America appear as a more cohesive Global South, including many former colonies of Western powers and generally positioned away from the Western bloc. Notably, China loads with the major-power bloc during this period, rather than the developing world, despite its self-identification as a leader of the Third World. Over the post-2000 decades, the heatmap reflects a broader structural reorganization of the international network, with China emerging as the central pole of a geopolitical bloc opposed to the United States and Western Europe. Thus, the estimated eigenvector recovers not only the Cold War cleavage itself, but also the subsequent realignments through which that cleavage was transformed.

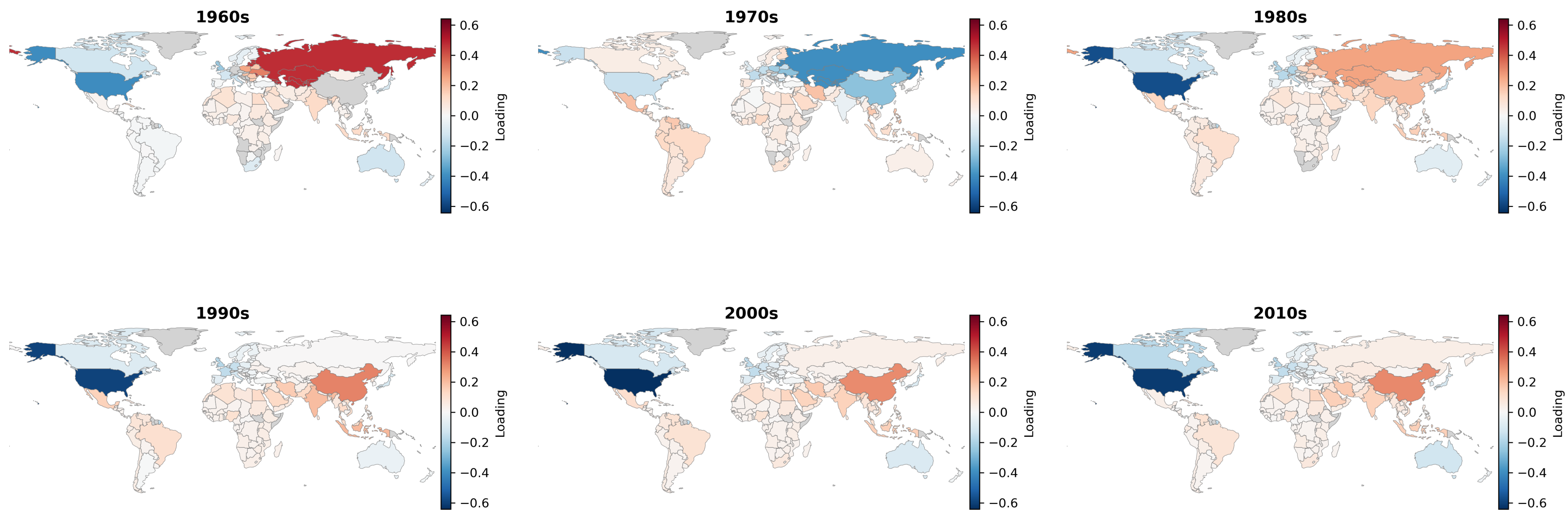
Therefore, our network-based approach recovers the leading geopolitical cleavage over our sample period from the observed data without imposing it, and accurately captures changes in the landscape of this geopolitical cleavage over time. The remaining eigenvectors capture less polarized geopolitical cleavages, each of which is orthogonal to the others, including for example divisions among more versus less ideological members of the Soviet bloc.

When we estimate the relationship between our geopolitical network connections separately with the economic and security dependence, we find that in the early decades of our sample period, security dependence dominates economic dependence, which is consistent with the heightened geopolitical tension in these peak years of the Cold War. Since the 1980s, economic dependence grew and eventually became more important relative to security dependence, which is consistent with the globalization of the world economy increasing the relative importance of economic relationships between countries. See Appendix Section J for details.

4.2 Individual Countries' Network Connections

We next examine the geopolitical network of the USA and China as the two leading powers at the end of our sample period. The top panel of Figure 5 maps the geopolitical network for the USA during 2010-2019. The top-left map displays the importance of other countries j to the USA (the USA's outward network). The top-right map shows the importance of the USA to other countries j (the USA's inward network). We find an intuitive pattern with many similarities between the outward and inward networks. For example, the USA and the UK have a mutual interest in one another. But there are other more one-sided relationships. For example, the USA cares relatively more about China than China cares about the USA, while New Zealand places a relatively higher weight on the USA than the USA places on New Zealand. Perhaps the most striking difference between the two panels is the

Figure 4: Second Dominant Eigenvector by Decade



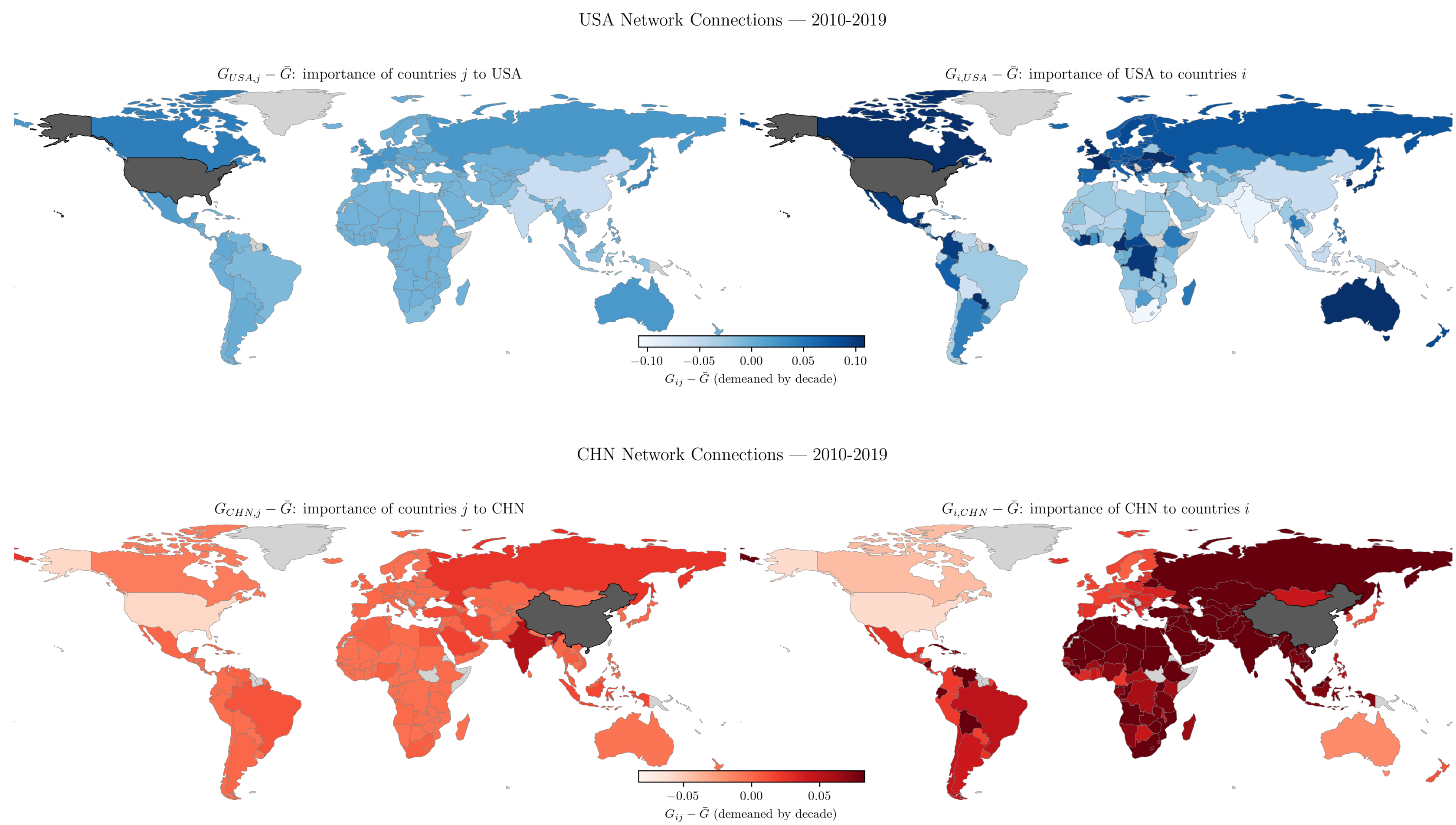
Note: Figure shows the second dominant eigenvector, which captures the network's deepest geopolitical cleavage, for which tightly connected blocs exhibit high internal consensus but stark cross-bloc polarization; second dominant eigenvector displayed by decade; heatmap from red to blue, where the pair of countries with the darkest red and the darkest blue are those with the greatest geopolitical cleavage.

upper and lower limits of the color scales. The range for other countries' weights on the USA is an order of magnitude larger than that for the USA's weights on other countries. This difference reflects the asymmetry of the geopolitical network, in which the USA is more structurally important than other countries. Therefore, other countries are more influenced by their geopolitical connections to the USA than the USA is influenced by its geopolitical connections to them.

Following a similar structure, the bottom panel of Figure 5 maps the geopolitical network for China during 2010-2019. The bottom-left map displays the importance of other countries j to China (China's outward network). The bottom-right map shows the importance of China to other countries j (China's inward network). We find that China has a very different geopolitical network than the United States, with strong connections to countries such as Russia, North Korea and India with which the USA has relatively weak links. More broadly, the US has the strongest connections to Canada and Western Europe, whereas China has some of its strongest connections in Asia. China's inward and outward networks also exhibit some similarities. For example, Russia and China have a mutual interest in one another. But there are again differences between these inward and outward networks. Besides the USA-China example discussed above, many African countries care relatively more about China than China cares about them. As before, the upper and lower limits of the color scales differ markedly between the two panels, because of the asymmetries in the geopolitical network. Given China's structural importance in the network, other countries are more influenced by their geopolitical connections to China than China is influenced by its geopolitical connections to them.

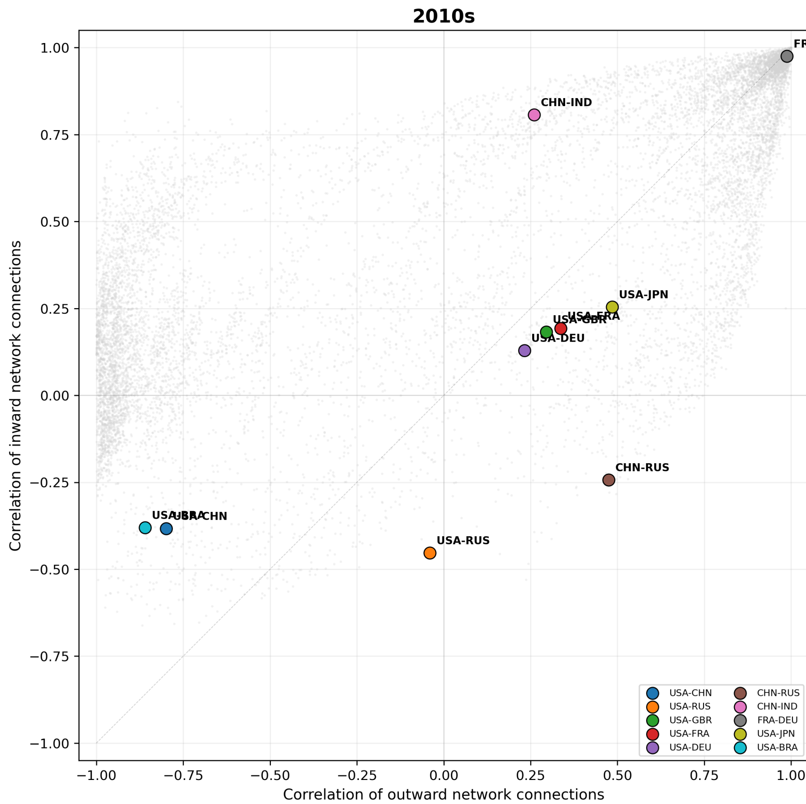
This finding that China and the USA have substantially different geopolitical networks raises the question of whether this simply reflects geopolitical rivalry between these two countries or whether these cross-country differences are a more general feature of the data. In Figure 6, we explore the correlations between countries' geopolitical networks more broadly. On the vertical axis, we display the correlation of countries' inward geopolitical networks (the similarity with which other countries care about them). On the horizontal axis, we display the correlation of countries' outward geopolitical networks (the similarity with which they care about other countries). Each dot in the figure corresponds to a bilateral pair of countries.

Figure 5: USA and China Outward and Inward Connections



Note: Top panel displays the USA's geopolitical network connections; top-left map shows the USA's outward connections (the importance of other countries to the USA); top-right map presents the USA's inward connections (the importance of the USA to other countries); bottom panel displays China's geopolitical network connections; bottom-left panel shows China's outward connections (the importance of other countries to China); bottom-right panel presents China's inward connections (the importance of China to other countries); in the top panel, darker blue denotes a more positive weight; in the bottom panel, darker red indicates a more positive weight; scales vary across all four maps.

Figure 6: Correlation of Inward and Outward Connections for Each Country Pair in the 2010s



Note: For each pair of countries in our sample (e.g., USA-CHN) for the decade 2010-19, the figure displays the correlation between the two countries' in-network connections (vertical axis) against the correlation between the two countries' out-network connections (horizontal axis); each gray dot shows a different country pair; selected country pairs highlighted using color as noted in the legend; a perfectly symmetric network would feature all country pairs along the diagonal; in-network connections (vertical axis) measure influence received from others; out-network connections (horizontal axis) measure influence exerted on others.

Perhaps the most remarkable feature of this figure is the heterogeneity in the structure of countries' geopolitical networks. For some structurally-important countries with different positions in the geopolitical network (e.g., USA-CHN), we find a low correlation of both their inward and their outward geopolitical networks. For other pairs of countries with similar structural importance and similar positions in the geopolitical network (e.g., FRA-DEU), we find a much higher degree of correlation between their outward networks than their inward networks. For yet other pairs of countries with apparently similar structural importance and positions in the geopolitical network (e.g., CHN-IND), we find that their inward networks can be positively correlated at the same time as their outward networks are negatively correlated. This heterogeneity highlights the insights from our approach of being able to recover unobserved geopolitical networks from observed domestic interests and geopolitical actions. Based on the regressions reported in Table 2 above, this heterogeneity in geopolitical networks is influenced by a rich combination of differences in economic size, geography and history, including alliance structures, language and colonial relationships.

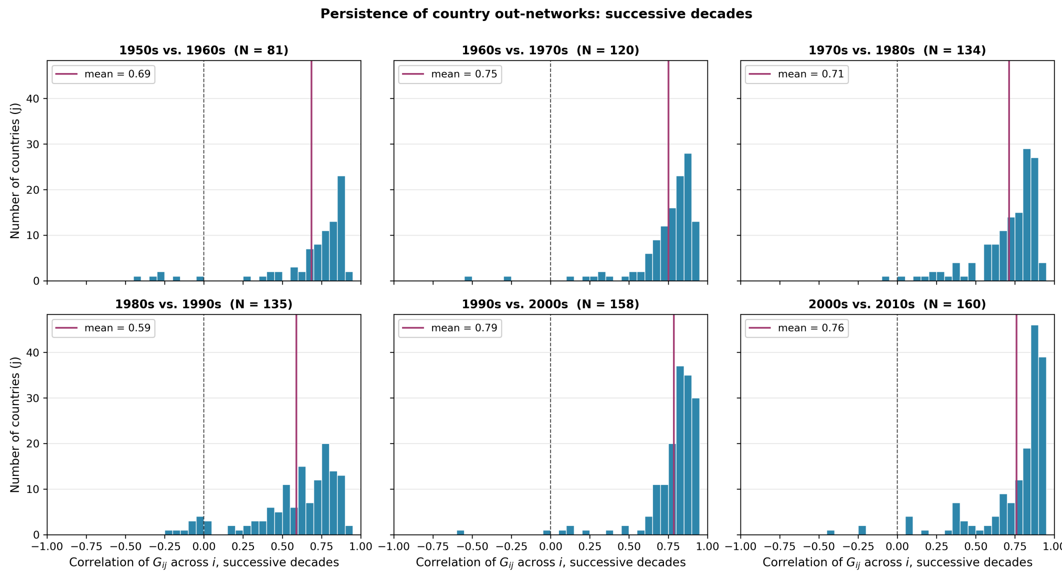
4.3 Temporal Stability of the Geopolitical Network

Our findings for the changing importance of economic and security dependence over time in Section J, and our results for the change in the Cold War cleavage over time in Subsection 4.1, raise the more general question of the stability of the geopolitical network over time. To provide evidence on this question, we correlate each country's outward network connections in each decade with those in the previous decade. Outward connections for each country capture how much it cares about other countries. Positive correlations indicate persistence of geopolitical ties over time; negative correlations represent reversals in geopolitical links over time.

In Figure 7, we display the distribution of these correlation coefficients across countries as histograms for each successive pair of decades. In most cases, we find strong persistence in geopolitical connection over time. The average correlation coefficient between decades is typically 0.69 or higher. There is a mass of countries with correlation coefficients concentrated at values of above 0.50. Few countries display negative correlations between decades. A notable exception is the comparison between the 1980s and 1990s, which reflects the collapse of the Iron Curtain. The average correlation coefficient for this pair of decades falls to 0.59. There is a reduction in the mass of countries with high correlation coefficients of above 0.50. There is an increase in the number of countries with correlation coefficients that are close to zero or even negative.

This pattern of results provides evidence that geopolitical network connections are typically stable over time, which is consistent with persistent effects of geography, history and alliance structures on international relations between countries. Nevertheless, large-scale geopolitical alignments can occasionally occur, such as the transformation of geopolitical networks that occurred in the aftermath of the fall of the Iron Curtain. More broadly, there is substantial heterogeneity across countries in the persistence of geopolitical networks. This heterogeneity is perhaps not surprising given the degree to which many nations' political institutions and roles in international politics have shifted in the last half-century. Again our approach explicitly allows for this heterogeneity by recovering the evolving geopolitical network from changes in the cross-country covariance structure of countries' geopolitical actions and domestic interests without imposing it on the data.

Figure 7: Stability of Geopolitical Networks over Time



Note: For each country, we correlate its outward network connections in each decade with those in the previous decade; outward connections for each country correspond to how much it cares about other countries; we display the distribution of these correlation coefficients across countries as histograms for each successive pair of decades.

4.4 Direct Versus Indirect Network Connections

Finally, we quantify the importance of indirect connections through the network versus direct connections between countries. We simulate alternative international issues by taking random draws for domestic interests from a normal distribution with mean zero and variance-covariance matrix equal to our estimated cross-country covariance matrix for domestic interests ($\boldsymbol{\Sigma_\theta}$) in each decade. For each vector of realizations for domestic interests ($\boldsymbol{\theta}$), we compute equilibrium influence ($\boldsymbol{\Omega}$) and geopolitical actions ($\boldsymbol{a}$) given our inferred geopolitical network ($\boldsymbol{G}$) for that decade. Finally, we use the first-order condition for influence to decompose the equilibrium influence exerted from each country i to each country k into the following contributions of direct and indirect influence:

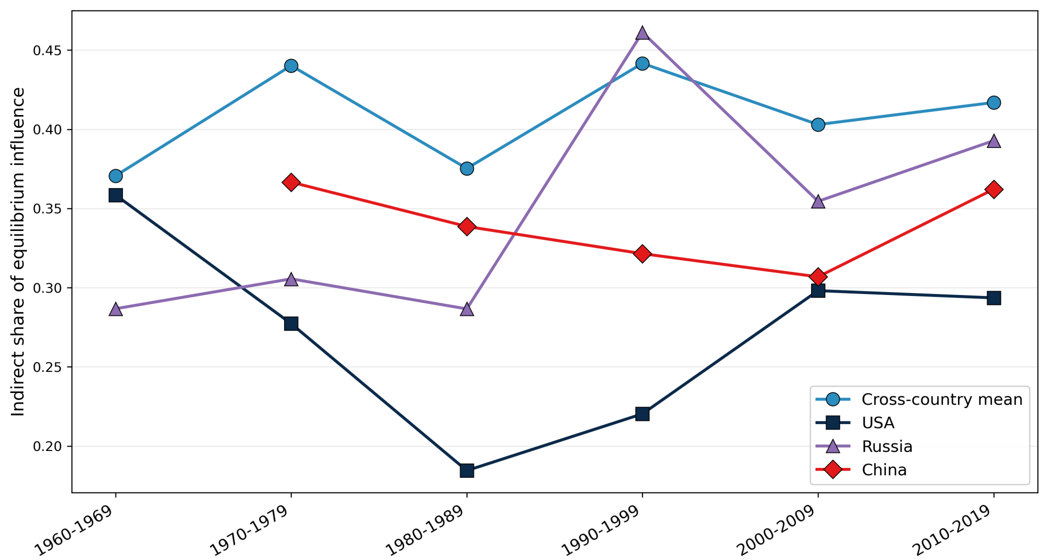
$$\kappa(\pi_k/\pi_i)\omega_{ki} = \underbrace{G_{ik}(a_i - a_k)}_{\text{direct}} + \underbrace{\sum_j G_{ij}(a_i - a_j)G_{jk} + \sum_j G_{ij}(a_i - a_j)(G^2)_{jk} + \ldots,}_{\text{indirect}} \tag{31}$$

where the first term captures the direct influence that country i has on country k; the remaining terms capture the indirect or higher-order influence that country i has on country k (through country k's immediate neighbors, from the neighbors of its immediate neighbors, and so on).

We repeat this simulation 1,000 times. We compute the average shares of direct and indirect influence in equation (31) across the 1,000 vectors of realizations for domestic interests. In Figure 8, we display these average shares of indirect influence in overall influence based on our estimated variance-covariance matrix of interests and geopolitical network for each decade. We show these averages for all countries (blue line with circles), for the United States (black line with squares), for Russia (purple line with triangles), and for China (red line with diamonds). For the world as a whole, we find an average share of indirect influence of around 45 percent, highlighting the insights from taking a network approach that incorporates indirect connections. For the most structurally-important countries of the United States and China, we find average shares of indirect influence that are smaller, but still substantial at around 35 percent. Russia starts with a similar share of indirect influence as the United States at the beginning of our sample period, but becomes more like the world as a whole by the end of our sample period. This pattern of results is consistent with the idea that dominant powers can achieve more through direct bilateral influence, whereas smaller nations are more reliant on indirect influence through the network structure.

In the majority of cases, we find an increase in the average share of indirect influence over time, suggesting an increase in the intensity of network connections over time.

Figure 8: Share of Indirect Influence in Overall Influence



Note: average shares of indirect influence in overall influence in each decade of our sample for individual countries and the world as a whole across 1,000 simulations. Shares of indirect influence computed by taking random draws for domestic interests from a normal distribution with mean zero and variance-covariance matrix equal to our estimated cross-country covariance matrix for domestic interests in each decade ($\boldsymbol{\Sigma}_{\boldsymbol{\theta},d}$). For each vector of realizations for domestic interests ($\boldsymbol{\theta}$), we compute equilibrium influence ($\boldsymbol{\Omega}$) and geopolitical actions ($\boldsymbol{a}$) given our inferred geopolitical network for that decade ($\boldsymbol{G}_d$). Shares of indirect influence computed from the first-order condition for influence (31).

5 Counterfactuals

Our network approach provides a tractable platform for undertaking counterfactuals to assess the role of domestic interests and network connections in shaping geopolitical actions. We present two counterfactual exercises that assess the geopolitical implications of shifting individual countries' domestic interests. In ongoing work, we conduct counterfactual exercises that quantify the geopolitical implications of the changing structure of the geopolitical network.

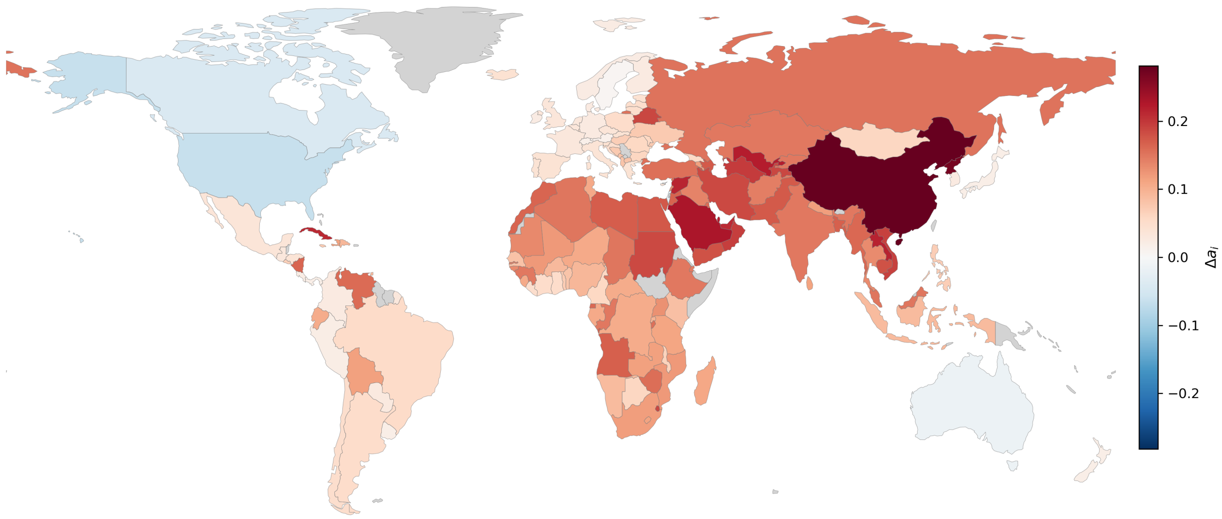
5.1 What Are the Geopolitical Implications of Converging Interests Between China and the U.S.?

In the aftermath of the fall of the Iron Curtain, Francis Fukuyama famously and perhaps prematurely speculated that the "end of history" would involve a convergence of all nations towards Western liberal democracy. In a similar vein, many Western policymakers espoused the view that China's economic growth would lead to its political convergence towards the Western Powers. In this first counterfactual exercise, we examine how different the global geopolitical landscape would be if China were indeed to converge towards the domestic interests of the USA.

We take the dominant non-Perron eigenmode as the vector of domestic interests for an international issue ($\boldsymbol{\theta}$) in which the USA and its Western allies fall positively on one side of the issue, whereas China, Russia, and their allies fall negatively on the other side of the issue. Using our inferred geopolitical network ($\boldsymbol{G}$) in the 2010s, we evaluate countries' geopolitical actions ($\boldsymbol{a}$) in response to this international issue ($\boldsymbol{\theta}$) under the status quo domestic interests in comparison with a scenario where we set China's domestic interests equal to that of the United States, holding all other countries' domestic interests unchanged.

Figure 9: Equilibrium responses to China with American preferences

(a) Map of action changes



(b) Histogram of action changes

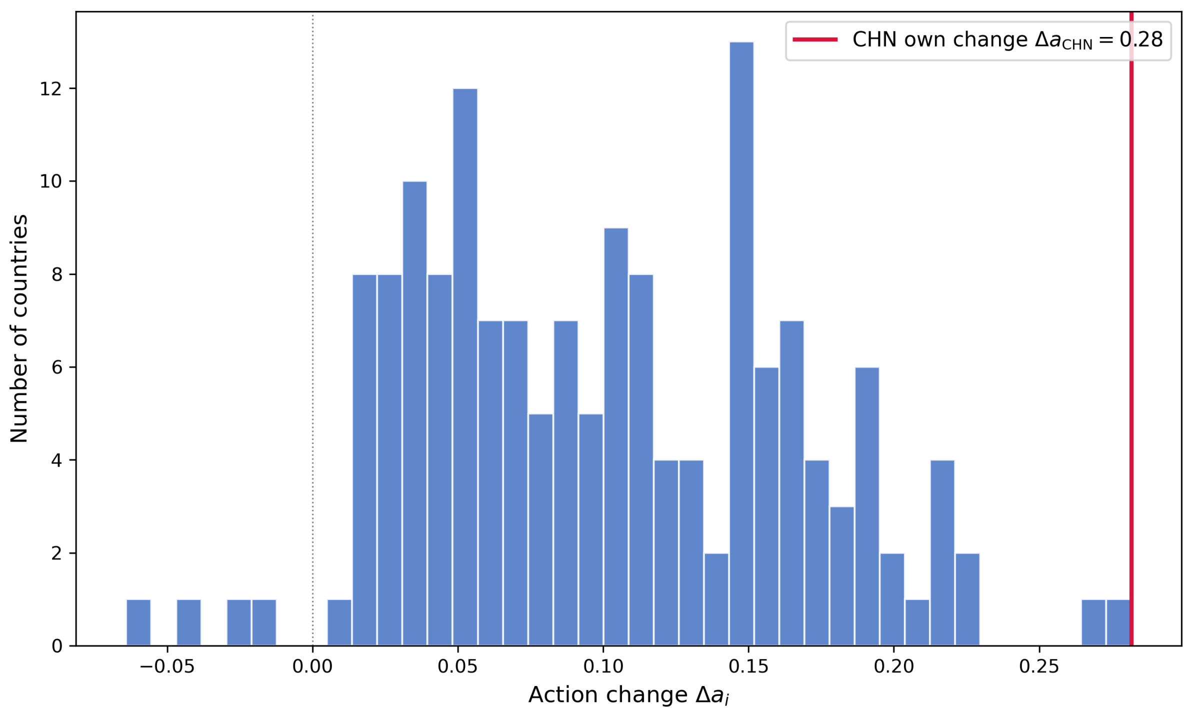


Figure 9 illustrates how the geopolitical network mediates the consequences of a shift in China's domestic interests toward those of the United States. Panel A maps the resulting changes in equilibrium geopolitical actions, while Panel B plots the distribution of these changes across countries.[10] As

[10] In Appendix Figure A6, we additionally map geopolitical actions for each country under these three different sce-

China's domestic interests converge toward those of the United States, its equilibrium geopolitical actions shift markedly in the same direction. Yet the adjustment remains incomplete: even when China shares the same domestic interests as the United States, its geopolitical actions do not fully converge to those of the United States. The reason is that China's actions are determined not only by its own domestic interests but also by incentives to coordinate with its neighbors in the geopolitical network. Because many of China's most important network partners—including Russia, India, and other regional powers—retain different domestic interests, China's equilibrium actions remain partially anchored to its existing network. This result illustrates how established geopolitical relationships constrain countries' behavior, limiting the extent to which changes in domestic preferences translate into changes in international actions.

The effects of the shift in China's domestic interests extend well beyond China itself. As China's domestic interests shift, the equilibrium actions of many neighboring countries also move toward the United States, despite no change in their own domestic interests. The largest adjustments occur in Russia, Central Asia, the Middle East, and parts of Africa, reflecting the propagation of influence through the geopolitical network.

An intriguing exception is a small group of four countries—the United States and three of its closest allies (Canada, Israel, and Australia)—whose equilibrium actions move slightly toward China as China's domestic interests converge toward those of the United States. This seemingly paradoxical result reflects a general equilibrium effect. As China and many of its geopolitical partners shift toward the United States, the United States and its closest allies face stronger incentives to coordinate with the rest of the network. Their own equilibrium actions therefore become somewhat more moderate, moving marginally toward China. Rather than simply pulling other countries toward itself, a changing China also reshapes the incentives facing the United States and its allies, reducing dispersion in global actions from both sides.

Appendix Figure A5 reports the analogous counterfactual in which the United States' domestic interests converge toward those of China. The qualitative patterns are similar: changes in the domestic interests of a major power propagate throughout the geopolitical network, affecting the equilibrium actions of many countries whose own domestic interests remain unchanged. One notable difference, however, is that the moderating effect is substantially stronger. A much larger set of countries shifts its actions, together with China, toward the United States, reflecting the broader reach of the United States' indirect connections.

Taken together, these findings illustrate three defining features of our framework. First, countries' geopolitical actions are constrained by their network relationships, so changes in domestic

narios. The top panel corresponds to the case in which the USA and China have their original preferences as given by the top non-Perron eigenmode in the 2010s network. The second panel corresponds to the case in which China lies at the midpoint between its original domestic interest and that of the United States. The third panel presents the case in which China and the USA have the same domestic interests.

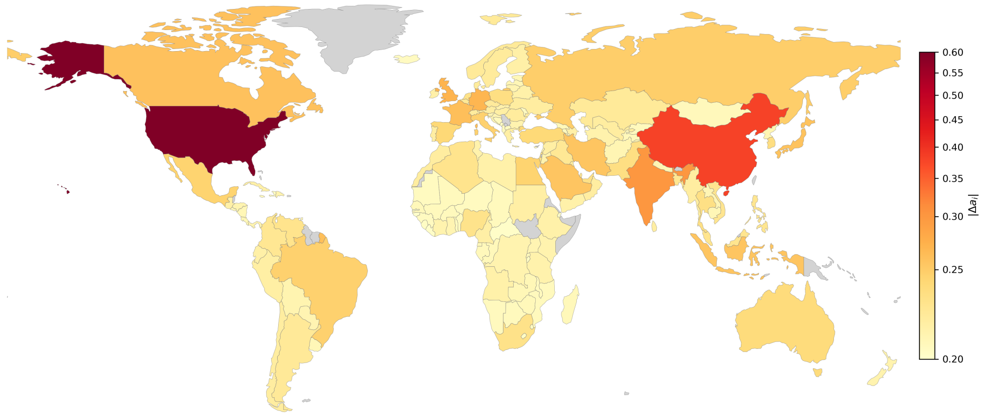
interests translate only imperfectly into changes in international behavior. Second, shocks propagate through the geopolitical network: a change in one country's domestic interests affects the equilibrium actions of many others, even when their own domestic interests remain unchanged. Third, these adjustments are inherently general equilibrium. As countries connected to China move toward the United States, they also moderate the actions of the United States and its closest allies, reducing polarization from both sides. More broadly, the results highlight China's structural influence in the geopolitical network: many countries behave less like the United States not because their own domestic interests require it, but because their geopolitical positions are shaped by coordination with China and its network of partners.

5.2 Geopolitical Power: Do Shifts in Domestic Interests Translate to Changes in Actions?

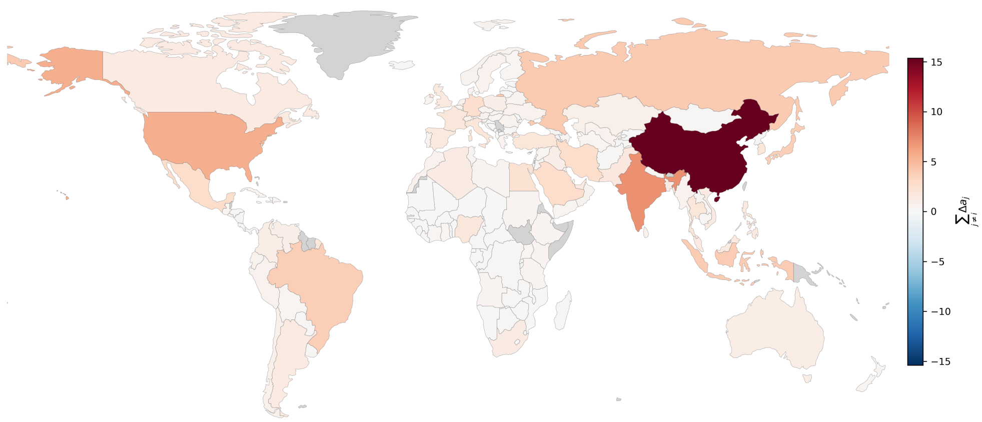
In our second counterfactual, we use our network approach to evaluate each country's international power for a given international issue, defined as its ability to get other countries to change their actions in response to its domestic interests. We use our inferred geopolitical network ($\boldsymbol{G}$) in the 2010s and set domestic interests for an international issue ($\boldsymbol{\theta}$) equal to the second dominant eigenvector of that network (its deepest geopolitical cleavage). We first compute the equilibrium geopolitical action vector ($\boldsymbol{a} = \mathcal{L}(\boldsymbol{G})\boldsymbol{\theta}$) given these domestic interests ($\boldsymbol{\theta}$). We next perturb the domestic interests of one country i ($\theta_i' = \theta_i + 1$), holding the domestic interests of all other countries unchanged ($\theta_j' = \theta_j$ for $j \neq i$). Using these new domestic interests, we compute the equilibrium geopolitical action vector for all countries ($\boldsymbol{a'} = \mathcal{L}(\boldsymbol{G})\boldsymbol{\theta'}$), and evaluate the magnitude of the change in the geopolitical actions of country i (Δa_i) and all other countries ($\sum_{j \neq i} \Delta a_j$) in response to these new domestic interests of country i. Finally, we repeat this process for each country i in turn.

Figure 10: Domestic interest shocks and equilibrium implications

(a) Change in own actions Δa_i



(b) Change in other's actions $\sum_{j \neq i} \Delta a_j$



(c) Own vs. others action changes

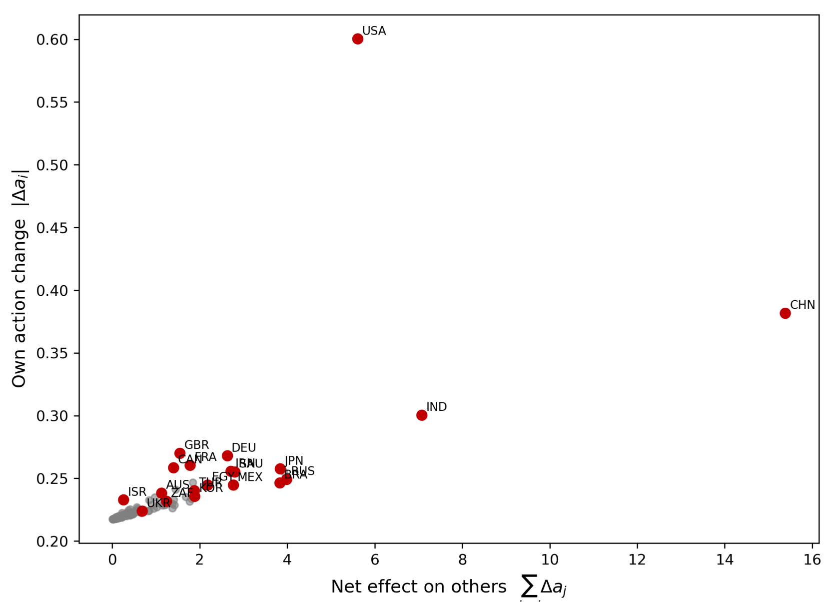


Figure 10 decomposes geopolitical power into two dimensions: a country's ability to translate changes in its domestic interests into its own geopolitical actions, and its ability to induce changes in the actions of other countries. Panel A maps the change in equilibrium action in own country where domestic interests change occur. Two countries stand out: the United States and China. Relative to all other countries, changes in their domestic interests translate much more strongly into changes in their own actions. For most countries, by contrast, equilibrium actions respond only weakly to shifts in domestic interests because they are constrained by incentives to coordinate with neighbors and by influence from other countries in the geopolitical network. In this sense, the United States and China possess substantial geopolitical autonomy: they are unusually able to act in accordance with their own domestic interests, even in a world where countries' actions are jointly determined. Nevertheless, even these two powers remain constrained by the network, as their equilibrium actions adjust substantially less than one-for-one with changes in their domestic interests.

Panel B maps the second dimension of geopolitical power: a country's ability to influence the equilibrium actions of others as a result of its domestic interest change. For each country, we aggregate the changes in all other countries' actions following a shift in its domestic interests. China emerges as the most influential country, followed by the United States and India. While large economies naturally possess greater influence, geopolitical power is not determined by size alone. It also depends on a country's position in the network and the strength of its geopolitical connections. China, for example, exerts greater influence over other countries' actions than the United States despite having a smaller economy, while Russia's influence is similarly disproportionate relative to its economic size. These results illustrate that geopolitical power is fundamentally a network property rather than simply a consequence of national capabilities.

Finally, Panel C relates these two dimensions of power. Countries that are better able to translate their domestic interests into their own actions also tend to exert greater influence over others, suggesting that autonomy and influence generally go hand in hand. At the same time, there is substantial heterogeneity around this relationship. The United States exhibits greater autonomy than influence: changes in its domestic interests are transmitted relatively strongly to its own actions but less strongly to those of other countries. China displays the opposite pattern. Its own actions remain more constrained by its geopolitical relationships, yet changes in its domestic interests generate especially large spillovers throughout the network. This contrast highlights two distinct sources of geopolitical power. Countries may derive power from the freedom to pursue their own preferred policies, or from the ability to shape the behavior of others. While the United States is comparatively stronger along the first dimension, China is stronger along the second.

6 Conclusion

The central question of this paper is how domestic interests are translated into geopolitical actions in an interconnected world. We argue that this translation depends not only on countries' own preferences but also on their positions in a geopolitical network and on the strategic influence exerted by other countries. To study this problem, we develop a tractable theoretical and empirical framework that combines network coordination, strategic influence, and spectral identification of the underlying geopolitical network.

Our framework yields several insights. Theoretically, it shows that the mapping from domestic interests to geopolitical actions admits a simple spectral representation, allowing geopolitical disagreements to be decomposed into latent network cleavages. Empirically, we recover a small number of interpretable geopolitical cleavages that closely track major historical realignments, from the Cold War to the contemporary U.S.–China rivalry. The estimated network also provides a platform for counterfactual analysis, revealing how changes in one country's domestic interests propagate through the international system and how geopolitical power depends jointly on domestic interests, network position, and indirect influence.

More broadly, our findings suggest that incorporating network structure fundamentally changes how we think about international politics. Power is not simply a function of national capabilities or bilateral leverage, but also of where countries are positioned within the network and how influence propagates through it. Likewise, geopolitical outcomes are not determined solely by domestic preferences, but emerge from equilibrium interactions among interconnected countries. This perspective provides a richer way to think about power, coalition formation, polarity, and the sources of international polarization.

Finally, we view our framework as a foundation rather than a complete model of geopolitical networks. Many economic and political environments—from geoeconomics and technology competition to production networks, financial systems, and institutional diffusion—involve actors whose decisions are jointly shaped by their own objectives, their network relationships, and their ability to influence others. By combining a tractable model of strategic interaction with an empirical approach to recovering latent network structure, our framework provides a general template for studying how network connections shape strategic behavior and aggregate outcomes in a broad class of interconnected systems.

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Online Appendix for "A Network Approach to Geopolitics" (Not for Publication)*

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Introduction

This appendix reports the technical derivations of the theoretical results reported in the paper (including the proofs of propositions) and presents supplementary empirical results that are discussed in the paper. Section A provides the proof of Proposition 1 in the paper. Section B provides the proof of Proposition 2 in the paper. Section C provides the proof of Proposition 3 in the paper. Section D provides the proof of Proposition 4 in the paper. Section E provides the proof of Proposition 5 in the paper. Section F provides the proof of Remark 1 in the paper. Section G provides the proof of Proposition 6 in the paper. Section H provides the Proof of Proposition 7 in the paper. Section I provides the proof of Proposition 8 in the paper. Section J reports additional empirical evidence on the relative importance of economic and security dependence for geopolitical network connections over time. Section K includes additional figures and tables discussed in the paper.

A Proof of Proposition 1

Proposition. *(Proposition 1 in the paper) The equilibrium vectors of net influence and geopolitical actions satisfy:*

$$\boldsymbol{\Omega}\mathbf{1} = -\left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^2 - \alpha\boldsymbol{I}\right)^{-1}\left(\left(\boldsymbol{I}-\boldsymbol{G}\right)^2 - \alpha^2\boldsymbol{I}\right)\boldsymbol{\theta}$$
$$\boldsymbol{a} = \left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^2 - \alpha\boldsymbol{I}\right)^{-1}\left(\boldsymbol{I}-\boldsymbol{G}\right)\left(\kappa\alpha - 1\right)\boldsymbol{\theta}$$

Moreover, as $\kappa \to \infty$, $\boldsymbol{\Omega}\mathbf{1} \to \mathbf{0}$ and $\boldsymbol{a} \to \alpha\left(\boldsymbol{I}-\boldsymbol{G}\right)^{-1}\boldsymbol{\theta}$; and as $\kappa \downarrow 1/\alpha$,

$$\boldsymbol{a} \to \bar{\theta}\,\mathbf{1}, \qquad \boldsymbol{\Omega}\mathbf{1} \to \alpha\left(\bar{\theta}\,\mathbf{1} - \boldsymbol{\theta}\right),$$

where $\bar{\theta} \equiv \frac{1}{N}\sum_i \theta_i$.

Proof. The proof proceeds in four steps: (i) solve the coordination stage for $\boldsymbol{a}$ given $\boldsymbol{\Omega}$; (ii) solve the influence stage for $\boldsymbol{\Omega}$ given the coordination-stage response; (iii) combine the two fixed-point conditions into the closed-form solutions; and (iv) take the stated limits in κ.

Step 1: Coordination stage. Taking $\boldsymbol{\Omega}$ as given, country i chooses a_i to maximize (5). The first-order condition is

$$0 = \frac{\partial U_i}{\partial a_i} = -2\alpha\left(a_i - \theta_i\right) - 2\sum_j G_{ij}\left(a_i - a_j\right) + 2\sum_m \omega_{im},$$

i.e., $a_i = \alpha\theta_i + \sum_j G_{ij}a_j + \sum_m \omega_{im}$, or in vector form, using $[\boldsymbol{\Omega}\mathbf{1}]_i = \sum_m \omega_{im}$,

$$\boldsymbol{a} = \alpha\boldsymbol{\theta} + \boldsymbol{G}\boldsymbol{a} + \boldsymbol{\Omega}\mathbf{1}.$$

Since $\alpha \in (0,1)$ and $\boldsymbol{G}$ has spectral radius $1-\alpha < 1$ (by Perron–Frobenius, as $\boldsymbol{G}$ is nonnegative with constant row sums $1-\alpha$), $(\boldsymbol{I}-\boldsymbol{G})$ is invertible, and the coordination-stage best responses admit the unique fixed point

$$\boldsymbol{a}=(\boldsymbol{I}-\boldsymbol{G})^{-1}(\alpha\boldsymbol{\theta}+\boldsymbol{\Omega}\mathbf{1})\,. \tag{32}$$

Step 2: Influence stage. Country i chooses $\{\omega_{ki}\}_k$ to maximize $U_i(\boldsymbol{a}(\boldsymbol{\Omega}),\boldsymbol{\Omega})$, anticipating the coordination-stage response of all actions to $\boldsymbol{\Omega}$. By the chain rule, the total derivative with respect to ω_{ki} decomposes as

$$0=\frac{\mathrm{d}U_i}{\mathrm{d}\omega_{ki}}=\underbrace{\frac{\partial U_i}{\partial a_i}\frac{\partial a_i}{\partial \omega_{ki}}}_{=0\text{ (envelope)}}+\sum_{j\neq i}\frac{\partial U_i}{\partial a_j}\frac{\partial a_j}{\partial \omega_{ki}}+\left.\frac{\partial U_i}{\partial \omega_{ki}}\right|_{\boldsymbol{a}}.$$

The first term vanishes because a_i is chosen optimally in Step 1, so $\partial U_i/\partial a_i=0$ at the equilibrium action (envelope theorem). The remaining two terms are computed directly from (5) and (32):

$$\frac{\partial U_i}{\partial a_j}=2G_{ij}(a_i-a_j)\quad(j\neq i)\,,\qquad \frac{\partial a_j}{\partial \omega_{ki}}=\left[(\boldsymbol{I}-\boldsymbol{G})^{-1}\right]_{jk},$$

where the second equality follows because, from (32), $\partial\boldsymbol{a}/\partial\omega_{ki}=(\boldsymbol{I}-\boldsymbol{G})^{-1}\boldsymbol{e}_k$ (only the k-th entry of $\boldsymbol{\Omega}\mathbf{1}$ responds to ω_{ki}, with unit coefficient). Since $G_{ii}(a_i-a_i)=0$, the sum over $j\neq i$ can be extended to all j. For the direct effect, only the terms $-\kappa\omega_{ki}^2$ and, when $k=i$, $2\omega_{ii}(a_i-\theta_i)$ in (5) depend on ω_{ki}, so

$$\left.\frac{\partial U_i}{\partial \omega_{ki}}\right|_{\boldsymbol{a}}=-2\kappa\omega_{ki}+2\cdot\mathbf{1}_{k=i}(a_i-\theta_i)\,.$$

Collecting terms and dividing by 2, the first-order condition for ω_{ki} is

$$\kappa\,\omega_{ki}=\sum_j G_{ij}(a_i-a_j)\left[(\boldsymbol{I}-\boldsymbol{G})^{-1}\right]_{jk}+\mathbf{1}_{k=i}(a_i-\theta_i)\,. \tag{33}$$

Summing (33) over $i=1,\ldots,N$ (i.e., over senders) and using $\sum_i\mathbf{1}_{k=i}(a_i-\theta_i)=a_k-\theta_k$:

$$\kappa\sum_i\omega_{ki}=\sum_i\sum_j G_{ij}(a_i-a_j)\left[(\boldsymbol{I}-\boldsymbol{G})^{-1}\right]_{jk}+a_k-\theta_k.$$

Because $\boldsymbol{G}$ is symmetric with constant row sums $1-\alpha$, $\sum_i G_{ij}(a_i-a_j)=[\boldsymbol{G}\boldsymbol{a}]_j-(1-\alpha)a_j$, so in vector form

$$\kappa\boldsymbol{\Omega}\mathbf{1}=(\boldsymbol{I}-\boldsymbol{G})^{-1}\left[\boldsymbol{G}\boldsymbol{a}-(1-\alpha)\boldsymbol{a}\right]+\boldsymbol{a}-\boldsymbol{\theta}=(\boldsymbol{I}-\boldsymbol{G})^{-1}\left[-(\boldsymbol{I}-\boldsymbol{G})a+\alpha a\right]+\boldsymbol{a}-\boldsymbol{\theta},$$

which simplifies to

$$\kappa\boldsymbol{\Omega}\mathbf{1}=\alpha(\boldsymbol{I}-\boldsymbol{G})^{-1}\boldsymbol{a}-\boldsymbol{\theta}. \tag{34}$$

Step 3: Closed-form solution. Equations (32) and (34) are two linear equations in the unknowns $\boldsymbol{a}$ and $\boldsymbol{\Omega}\mathbf{1}$. Substituting $\boldsymbol{\Omega}\mathbf{1}=\frac{1}{\kappa}\left[\alpha(\boldsymbol{I}-\boldsymbol{G})^{-1}\boldsymbol{a}-\boldsymbol{\theta}\right]$ from (34) into (32) gives

$$\boldsymbol{a}=\alpha(\boldsymbol{I}-\boldsymbol{G})^{-1}\boldsymbol{\theta}+\frac{\alpha}{\kappa}(\boldsymbol{I}-\boldsymbol{G})^{-2}\boldsymbol{a}-\frac{1}{\kappa}(\boldsymbol{I}-\boldsymbol{G})^{-1}\boldsymbol{\theta}.$$

Since $\boldsymbol{G}$ is symmetric, all matrices appearing here are polynomials or rational functions of $\boldsymbol{G}$ and therefore commute. Left-multiplying both sides by $\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}$ and collecting the $\boldsymbol{a}$ terms on the left and the $\boldsymbol{\theta}$ terms on the right:

$$\left[\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha\boldsymbol{I}\right]\boldsymbol{a}=\left(\kappa\alpha-1\right)\left(\boldsymbol{I}-\boldsymbol{G}\right)\boldsymbol{\theta},$$

so that

$$\boldsymbol{a}=\left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha\boldsymbol{I}\right)^{-1}\left(\boldsymbol{I}-\boldsymbol{G}\right)\left(\kappa\alpha-1\right)\boldsymbol{\theta},\tag{35}$$

which is the second expression in the proposition. (The matrix $\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha\boldsymbol{I}$ is invertible because $\kappa>1/\alpha$ and $\boldsymbol{G}$'s eigenvalues lie in $[-\left(1-\alpha\right),1-\alpha]$, so every eigenvalue of $\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha\boldsymbol{I}$ is bounded below by $\kappa\alpha^{2}-\alpha=\alpha\left(\kappa\alpha-1\right)>0$.) Substituting (35) back into (34), and using commutativity to write $\left(\boldsymbol{I}-\boldsymbol{G}\right)^{-1}\left(\boldsymbol{I}-\boldsymbol{G}\right)=\boldsymbol{I}$ inside the resolvent:

$$\begin{aligned}\kappa\boldsymbol{\Omega}\boldsymbol{1}&=\alpha\left(\kappa\alpha-1\right)\left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha\boldsymbol{I}\right)^{-1}\boldsymbol{\theta}-\boldsymbol{\theta}\\&=\left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha\boldsymbol{I}\right)^{-1}\left[\alpha\left(\kappa\alpha-1\right)\boldsymbol{I}-\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}+\alpha\boldsymbol{I}\right]\boldsymbol{\theta}\\&=\left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha\boldsymbol{I}\right)^{-1}\kappa\left[\alpha^{2}\boldsymbol{I}-\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}\right]\boldsymbol{\theta},\end{aligned}$$

using $\alpha\left(\kappa\alpha-1\right)+\alpha=\kappa\alpha^{2}$ in the last line. Dividing by κ:

$$\boldsymbol{\Omega}\boldsymbol{1}=-\left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha\boldsymbol{I}\right)^{-1}\left(\left(\boldsymbol{I}-\boldsymbol{G}\right)^{2}-\alpha^{2}\boldsymbol{I}\right)\boldsymbol{\theta},\tag{36}$$

which is the first expression in the proposition.

Step 4: Limits in κ. Because $\boldsymbol{G}$ is real and symmetric, write its spectral decomposition $\boldsymbol{G}=\sum_{m}\lambda_{m}\boldsymbol{u}_{m}\boldsymbol{u}_{m}^{\top}$ with orthonormal eigenvectors $\{\boldsymbol{u}_{m}\}$. Expanding $\boldsymbol{\theta}=\sum_{m}\tilde{\theta}_{m}\boldsymbol{u}_{m}$ with $\tilde{\theta}_{m}=\boldsymbol{u}_{m}^{\top}\boldsymbol{\theta}$, equations (35) and (36) act mode by mode:

$$\boldsymbol{a}=\sum_{m}L\left(\lambda_{m}\right)\tilde{\theta}_{m}\boldsymbol{u}_{m},\qquad\boldsymbol{\Omega}\boldsymbol{1}=\sum_{m}\Phi\left(\lambda_{m}\right)\tilde{\theta}_{m}\boldsymbol{u}_{m},$$

$$L\left(\lambda\right)\equiv\frac{\left(1-\lambda\right)\left(\kappa\alpha-1\right)}{\kappa\left(1-\lambda\right)^{2}-\alpha},\qquad\Phi\left(\lambda\right)\equiv-\frac{\left(1-\lambda\right)^{2}-\alpha^{2}}{\kappa\left(1-\lambda\right)^{2}-\alpha}.$$

Limit $\kappa\to\infty$. For every eigenvalue λ, $L\left(\lambda\right)\to\alpha\left(1-\lambda\right)^{-1}$ and $\Phi\left(\lambda\right)\to0$ as $\kappa\to\infty$ (divide numerator and denominator by κ). Hence $\boldsymbol{\Omega}\boldsymbol{1}\to\boldsymbol{0}$ and $\boldsymbol{a}\to\sum_{m}\alpha\left(1-\lambda_{m}\right)^{-1}\tilde{\theta}_{m}\boldsymbol{u}_{m}=\alpha\left(\boldsymbol{I}-\boldsymbol{G}\right)^{-1}\boldsymbol{\theta}$.

Limit $\kappa\downarrow1/\alpha$. Since $\boldsymbol{G}$ has nonnegative entries and constant row sums $1-\alpha$, $\boldsymbol{G}\boldsymbol{1}=\left(1-\alpha\right)\boldsymbol{1}$, so $\boldsymbol{u}_{1}\equiv\boldsymbol{1}/\sqrt{N}$ is a unit eigenvector with eigenvalue $\lambda_{1}=1-\alpha$; by Perron–Frobenius, λ_{1} is the (simple) eigenvalue of largest modulus whenever the network is connected, so $|1-\lambda_{m}|>\alpha$ for every $m\neq1$ while $1-\lambda_{1}=\alpha$. Evaluating L at λ_{1}:

$$L\left(\lambda_{1}\right)=\frac{\alpha\left(\kappa\alpha-1\right)}{\kappa\alpha^{2}-\alpha}=\frac{\alpha\left(\kappa\alpha-1\right)}{\alpha\left(\kappa\alpha-1\right)}=1\quad\text{for all }\kappa>1/\alpha,$$

so trivially $L(\lambda_1) \to 1$ as $\kappa \downarrow 1/\alpha$. For $m \neq 1$, the denominator $\kappa(1-\lambda_m)^2 - \alpha \to \alpha^{-1}(1-\lambda_m)^2 - \alpha > 0$ (strictly, since $(1-\lambda_m)^2 > \alpha^2$) remains bounded away from zero as $\kappa \to 1/\alpha$, while the numerator $(1-\lambda_m)(\kappa\alpha - 1) \to 0$; hence $L(\lambda_m) \to 0$ for $m \neq 1$. Therefore

$$\boldsymbol{a} \to L(\lambda_1)\,\tilde{\theta}_1 \boldsymbol{u}_1 = \tilde{\theta}_1 \boldsymbol{u}_1 = \left(\boldsymbol{u}_1^\top \boldsymbol{\theta}\right)\boldsymbol{u}_1 = \left(\frac{\mathbf{1}^\top \boldsymbol{\theta}}{N}\right)\mathbf{1} = \bar{\theta}\,\mathbf{1}.$$

For the influence term, $\Phi(\lambda) = (1-\lambda)L(\lambda) - \alpha$ (a direct algebraic identity, since $(1-\lambda)^2 - \alpha^2 = (1-\lambda)(1-\lambda) - \alpha^2$ and $(1-\lambda)L(\lambda) - \alpha = \frac{(1-\lambda)^2(\kappa\alpha-1) - \alpha\left[\kappa(1-\lambda)^2 - \alpha\right]}{\kappa(1-\lambda)^2 - \alpha} = \frac{-(1-\lambda)^2 + \alpha^2}{\kappa(1-\lambda)^2 - \alpha} = \Phi(\lambda)$). As $\kappa \downarrow 1/\alpha$, $\Phi(\lambda_1) \to (1-\lambda_1)\cdot 1 - \alpha = \alpha - \alpha = 0$, while for $m \neq 1$, $\Phi(\lambda_m) \to (1-\lambda_m)\cdot 0 - \alpha = -\alpha$. Hence

$$\boldsymbol{\Omega}\mathbf{1} \to \sum_{m \neq 1} (-\alpha)\,\tilde{\theta}_m \boldsymbol{u}_m = -\alpha\left(\boldsymbol{\theta} - \tilde{\theta}_1 \boldsymbol{u}_1\right) = -\alpha\left(\boldsymbol{\theta} - \bar{\theta}\,\mathbf{1}\right) = \alpha\left(\bar{\theta}\,\mathbf{1} - \boldsymbol{\theta}\right).$$

This establishes both limits and completes the proof. □

B Proof of Proposition 2

Proposition. ***(Proposition 2 in the paper)*** *The equilibrium has the following properties.*

1. *The equilibrium is unique.*
2. *Each country's equilibrium action is a convex combination over the domestic interests of all countries.*
3. *The equilibrium is translation invariant to $\boldsymbol{\theta}$: adding a constant c to all θ_i's results in a new equilibrium in which all actions are also shifted by c, without any change in payoffs.*
4. *The equilibrium is conservative: the average geopolitical action and the average domestic interest across countries coincide: $\mathbf{1}'\boldsymbol{a} = \mathbf{1}'\boldsymbol{\theta}$.*
5. *The equilibrium strategic influence nets out to zero: $\mathbf{1}'\boldsymbol{\Omega}\mathbf{1} = 0$.*

Proof. As established in Proposition 1, the equilibrium satisfies

$$\boldsymbol{a} = \boldsymbol{M}\boldsymbol{\theta}, \qquad \boldsymbol{M} \equiv \left(\kappa(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}\right)^{-1}(\boldsymbol{I}-\boldsymbol{G})(\kappa\alpha - 1), \tag{37}$$

$$\boldsymbol{\Omega}\mathbf{1} = \boldsymbol{N}\boldsymbol{\theta}, \qquad \boldsymbol{N} \equiv -\left(\kappa(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}\right)^{-1}\left((\boldsymbol{I}-\boldsymbol{G})^2 - \alpha^2\boldsymbol{I}\right). \tag{38}$$

Two elementary facts about $\boldsymbol{M}$ and $\boldsymbol{N}$ will be used repeatedly.

Fact A (symmetry). $\boldsymbol{M}$ and $\boldsymbol{N}$ are symmetric. Indeed, $\boldsymbol{G} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^\top$ for an orthogonal $\boldsymbol{U}$ and diagonal $\boldsymbol{\Lambda}$, so every matrix built from $\boldsymbol{I}, \boldsymbol{G}$, sums, products, and inverses thereof (as $\boldsymbol{M}$ and $\boldsymbol{N}$ are) can be written as $\boldsymbol{U} f(\boldsymbol{\Lambda})\,\boldsymbol{U}^\top$ for a diagonal matrix $f(\boldsymbol{\Lambda})$; any such matrix is symmetric.

Fact B (action on $\mathbf{1}$). Since $\boldsymbol{G}$ has constant row sums $1-\alpha$, $\boldsymbol{G}\mathbf{1} = (1-\alpha)\,\mathbf{1}$, so $(\boldsymbol{I}-\boldsymbol{G})\,\mathbf{1} = \alpha\mathbf{1}$ and $(\boldsymbol{I}-\boldsymbol{G})^2\,\mathbf{1} = \alpha^2\mathbf{1}$. Hence

$$\left(\kappa\,(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}\right)\mathbf{1} = \left(\kappa\alpha^2 - \alpha\right)\mathbf{1} = \alpha\,(\kappa\alpha - 1)\,\mathbf{1},$$

so $\mathbf{1}$ is an eigenvector of $\left(\kappa\,(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}\right)^{-1}$ with eigenvalue $[\alpha\,(\kappa\alpha-1)]^{-1}$, and therefore

$$\boldsymbol{M}\mathbf{1} = (\kappa\alpha-1)\,[\alpha\,(\kappa\alpha-1)]^{-1}\,\alpha\mathbf{1} = \mathbf{1}, \qquad \boldsymbol{N}\mathbf{1} = -\,[\alpha\,(\kappa\alpha-1)]^{-1}\left(\alpha^2-\alpha^2\right)\mathbf{1} = \mathbf{0}. \quad (39)$$

1. Uniqueness. At the coordination stage, for any fixed $\boldsymbol{\Omega}$, U_i is strictly concave in a_i: from (5), $\partial^2 U_i/\partial a_i^2 = -2\left(\alpha + \sum_j G_{ij}\right) < 0$, since $\alpha > 0$ and $G_{ij} \geq 0$. Hence the first-order condition has a unique solution, and because $\boldsymbol{I}-\boldsymbol{G}$ is invertible ($\boldsymbol{G}$'s spectral radius is $1-\alpha < 1$ by Perron–Frobenius, as $\boldsymbol{G}$ is nonnegative with constant row sums $1-\alpha$), the coordination-stage best response is the unique linear map $\boldsymbol{a} = (\boldsymbol{I}-\boldsymbol{G})^{-1}(\alpha\boldsymbol{\theta} + \boldsymbol{\Omega}\mathbf{1})$. At the influence stage, anticipating this continuation, $U_i\left(\boldsymbol{a}\left(\boldsymbol{\Omega}\right), \boldsymbol{\Omega}\right)$ is quadratic in country i's own controls $\{\omega_{ki}\}_k$ with Hessian $-2\kappa\boldsymbol{I}$ (the terms through $\boldsymbol{a}$ enter linearly in $\boldsymbol{\Omega}$, so they contribute nothing to the Hessian), which is negative definite since $\kappa > 0$; hence country i's best response at the influence stage is also unique and linear, given by the influence-stage best response displayed below. The full system of first-order conditions across all i and k is therefore a linear system in $(\boldsymbol{a}, \boldsymbol{\Omega})$, and it has a unique solution because the matrix $\kappa\,(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}$ is invertible (every eigenvalue of $\boldsymbol{G}$ lies in $[-(1-\alpha), 1-\alpha]$, so every eigenvalue of $\kappa\,(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}$ is at least $\kappa\alpha^2 - \alpha = \alpha\,(\kappa\alpha-1) > 0$). Since each player's strategy is a strict best response to the (unique) strategies of others at every information set, this linear fixed point is the unique subgame-perfect equilibrium; existence is established by the explicit closed-form solution (37)–(38).

2. Convex combination. We show $\boldsymbol{M}$ has nonnegative entries and row sums equal to one; combined with $\boldsymbol{a} = \boldsymbol{M}\boldsymbol{\theta}$, this means $a_i = \sum_k M_{ik}\theta_k$ is a convex combination of $\{\theta_k\}$. The row-sum property is $\boldsymbol{M}\mathbf{1} = \mathbf{1}$, already established in Fact B. For nonnegativity, work in $\boldsymbol{G}$'s eigenbasis: $\boldsymbol{M}$ has eigenvalues

$$L(\lambda) = \frac{(1-\lambda)\,(\kappa\alpha-1)}{\kappa\,(1-\lambda)^2 - \alpha}$$

at each eigenvalue λ of $\boldsymbol{G}$. Write $z \equiv 1-\lambda$ and $\beta \equiv \sqrt{\alpha/\kappa}$; since $\kappa > 1/\alpha$, we have $\beta^2 = \alpha/\kappa < \alpha^2$, so $\beta < \alpha < 1$. By partial fractions,

$$L(\lambda) = \frac{\kappa\alpha-1}{\kappa}\cdot\frac{z}{z^2-\beta^2} = \frac{\kappa\alpha-1}{2\kappa}\left[\frac{1}{(1-\beta)-\lambda} + \frac{1}{(1+\beta)-\lambda}\right].$$

Because $|\lambda| \leq 1-\alpha < 1-\beta < 1+\beta$ (the first inequality by Perron–Frobenius, the second because $\beta < \alpha$), both terms admit convergent geometric expansions in λ:

$$\frac{1}{(1-\beta)-\lambda} = \sum_{n=0}^{\infty}\frac{\lambda^n}{(1-\beta)^{n+1}}, \qquad \frac{1}{(1+\beta)-\lambda} = \sum_{n=0}^{\infty}\frac{\lambda^n}{(1+\beta)^{n+1}}.$$

Hence $L(\lambda) = \sum_{n\geq 0} d_n \lambda^n$ with

$$d_n = \frac{\kappa\alpha - 1}{2\kappa}\left[(1-\beta)^{-(n+1)} + (1+\beta)^{-(n+1)}\right] > 0 \quad \text{for every } n \geq 0,$$

since $\kappa\alpha - 1 > 0$ (as $\kappa > 1/\alpha$) and $0 < 1-\beta < 1+\beta$. Applying the same expansion to the matrix $\boldsymbol{G}$ (valid since $\boldsymbol{G}$'s eigenvalues satisfy the same bound), $\boldsymbol{M} = \sum_{n\geq 0} d_n \boldsymbol{G}^n$. Each $\boldsymbol{G}^n$ has nonnegative entries, because $\boldsymbol{G}$ does and products of nonnegative matrices are nonnegative; since each $d_n > 0$, every entry of $\boldsymbol{M}$ is a convergent sum of nonnegative terms, hence $\boldsymbol{M}_{ik} \geq 0$ for all i, k. Together with $\boldsymbol{M}\mathbf{1} = \mathbf{1}$, this proves each row of $\boldsymbol{M}$ is a set of convex-combination weights.

3. Translation invariance. Let $\boldsymbol{\theta}' = \boldsymbol{\theta} + c\mathbf{1}$ for a constant c. We claim that $(\boldsymbol{a}', \boldsymbol{\Omega}') = (\boldsymbol{a} + c\mathbf{1}, \boldsymbol{\Omega})$ — the same influence matrix, with every action shifted by c — is the equilibrium associated with $\boldsymbol{\theta}'$. First, by (37) and $\boldsymbol{M}\mathbf{1} = \mathbf{1}$ (Fact B),

$$\boldsymbol{a}' = \boldsymbol{M}\boldsymbol{\theta}' = \boldsymbol{M}\boldsymbol{\theta} + c\boldsymbol{M}\mathbf{1} = \boldsymbol{a} + c\mathbf{1},$$

confirming the postulated shift in actions. Next, we verify this $(\boldsymbol{a}', \boldsymbol{\Omega}')$ satisfies the equilibrium conditions for $\boldsymbol{\theta}'$. The coordination-stage FOC, $a_i = \alpha\theta_i + \sum_j G_{ij} a_j + \sum_m \omega_{im}$, holds at $(\boldsymbol{a}', \boldsymbol{\theta}', \boldsymbol{\Omega}' = \boldsymbol{\Omega})$ because both sides increase by exactly $\alpha c + (1-\alpha)c = c$ relative to the original equilibrium (using $\sum_j G_{ij} = 1 - \alpha$), matching $a_i' = a_i + c$. The influence-stage best response,

$$\kappa\,\omega_{ki} = \sum_j G_{ij}(a_i - a_j)\left[(\boldsymbol{I} - \boldsymbol{G})^{-1}\right]_{jk} + \mathbf{1}_{k=i}(a_i - \theta_i),$$

depends on $(\boldsymbol{a}, \boldsymbol{\theta})$ only through the differences $a_i - a_j$ and $a_i - \theta_i$, both of which are unchanged when $\boldsymbol{a} \to \boldsymbol{a} + c\mathbf{1}$ and $\boldsymbol{\theta} \to \boldsymbol{\theta} + c\mathbf{1}$. Hence $\boldsymbol{\Omega}' = \boldsymbol{\Omega}$ continues to satisfy every bilateral first-order condition unchanged, verifying the claim. Finally, payoffs in (5) depend on $(\boldsymbol{a}, \boldsymbol{\theta}, \boldsymbol{\Omega})$ only through $a_i - \theta_i$, $a_i - a_j$, and $\boldsymbol{\Omega}$ itself, all of which are identical at $(\boldsymbol{a}', \boldsymbol{\theta}', \boldsymbol{\Omega}')$ and $(\boldsymbol{a}, \boldsymbol{\theta}, \boldsymbol{\Omega})$; hence U_i is unchanged for every i.

4. Conservation. By Fact A, $\boldsymbol{M}$ is symmetric, so $\boldsymbol{M}^\top = \boldsymbol{M}$. By Fact B, $\boldsymbol{M}\mathbf{1} = \mathbf{1}$, so

$$\mathbf{1}^\top \boldsymbol{M} = \left(\boldsymbol{M}^\top \mathbf{1}\right)^\top = (\boldsymbol{M}\mathbf{1})^\top = \mathbf{1}^\top.$$

Therefore $\mathbf{1}^\top \boldsymbol{a} = \mathbf{1}^\top \boldsymbol{M}\boldsymbol{\theta} = \mathbf{1}^\top \boldsymbol{\theta}$ for every $\boldsymbol{\theta}$.

5. Zero net influence. By Fact A, $\boldsymbol{N}$ is symmetric, so $\boldsymbol{N}^\top = \boldsymbol{N}$. By Fact B, $\boldsymbol{N}\mathbf{1} = \mathbf{0}$, so

$$\mathbf{1}^\top \boldsymbol{N} = \left(\boldsymbol{N}^\top \mathbf{1}\right)^\top = (\boldsymbol{N}\mathbf{1})^\top = \mathbf{0}^\top.$$

Therefore $\mathbf{1}^\top \boldsymbol{\Omega}\mathbf{1} = \mathbf{1}^\top \boldsymbol{N}\boldsymbol{\theta} = 0$ for every $\boldsymbol{\theta}$. $\square$

C Proof of Proposition 3

Proposition. ***(Proposition 3 in the paper)*** *The network disagreement of an eigenvector $\boldsymbol{u}_k$ is $\mathcal{S}(\boldsymbol{u}_k) = 1 - \alpha - \lambda_k$. Moreover,*

$$\boldsymbol{u}_1 = \arg\min_{\|\boldsymbol{\theta}\|=1} \mathcal{S}(\boldsymbol{\theta}) \propto \mathbf{1}, \qquad \lambda_1 = 1 - \alpha.$$

$$\boldsymbol{u}_k = \arg\min_{\boldsymbol{\theta} \perp \{\boldsymbol{u}_j\}_{j<k}, \|\boldsymbol{\theta}\|=1} \mathcal{S}(\boldsymbol{\theta}) \text{ for all } k > 1.$$

Proof. **Part 1: $\mathcal{S}(\boldsymbol{u}_k) = 1 - \alpha - \lambda_k$.** Expand the square in the definition of $\mathcal{S}$:

$$\sum_{i,j} G_{ij} (\theta_i - \theta_j)^2 = \sum_{i,j} G_{ij} \theta_i^2 - 2 \sum_{i,j} G_{ij} \theta_i \theta_j + \sum_{i,j} G_{ij} \theta_j^2.$$

Since every row sum of $\boldsymbol{G}$ equals $1 - \alpha$, $\sum_j G_{ij} = 1 - \alpha$ for every i, so the first term is $\sum_i \theta_i^2 (1 - \alpha) = (1 - \alpha) \|\boldsymbol{\theta}\|^2$; by symmetry of $\boldsymbol{G}$, column sums equal row sums, so the third term is likewise $(1 - \alpha) \|\boldsymbol{\theta}\|^2$. The middle term is $2\boldsymbol{\theta}^\top \boldsymbol{G} \boldsymbol{\theta}$. Hence

$$\mathcal{S}(\boldsymbol{\theta}) = \frac{1}{2} \left[2 (1 - \alpha) \|\boldsymbol{\theta}\|^2 - 2\boldsymbol{\theta}^\top \boldsymbol{G} \boldsymbol{\theta} \right] = (1 - \alpha) \|\boldsymbol{\theta}\|^2 - \boldsymbol{\theta}^\top \boldsymbol{G} \boldsymbol{\theta}. \tag{40}$$

This identifies $\mathcal{S}(\boldsymbol{\theta}) = \boldsymbol{\theta}^\top \boldsymbol{L} \boldsymbol{\theta}$ with $\boldsymbol{L} \equiv (1 - \alpha) \boldsymbol{I} - \boldsymbol{G}$, the (weighted) graph Laplacian of $\boldsymbol{G}$, sharing $\boldsymbol{G}$'s eigenbasis with eigenvalues $1 - \alpha - \lambda_k$. In particular, for a unit eigenvector $\boldsymbol{\theta} = \boldsymbol{u}_k$, $\boldsymbol{u}_k^\top \boldsymbol{G} \boldsymbol{u}_k = \lambda_k$, so

$$\mathcal{S}(\boldsymbol{u}_k) = (1 - \alpha) - \lambda_k.$$

Part 2: $\boldsymbol{u}_1 \propto \mathbf{1}$ and $\lambda_1 = 1 - \alpha$. Since $\boldsymbol{G}\mathbf{1} = (1 - \alpha) \mathbf{1}$ (constant row sums), $\mathbf{1}$ is an eigenvector of $\boldsymbol{G}$ with eigenvalue $1 - \alpha$. For any eigenvalue λ of $\boldsymbol{G}$ with eigenvector $\boldsymbol{v}$, letting i index the largest entry of $|\boldsymbol{v}|$, the i-th row of $\boldsymbol{G}\boldsymbol{v} = \lambda \boldsymbol{v}$ gives $|\lambda| |v_i| = \left| \sum_j G_{ij} v_j \right| \leq \sum_j G_{ij} |v_j| \leq \left(\sum_j G_{ij} \right) |v_i| = (1 - \alpha) |v_i|$, so $|\lambda| \leq 1 - \alpha$ for every eigenvalue. Hence $1 - \alpha$ is the largest eigenvalue in absolute value, and since it is attained (by $\mathbf{1}$) it is also the algebraically largest eigenvalue: $\lambda_1 = 1 - \alpha$. Because the network is connected, $\boldsymbol{G}$ is an irreducible nonnegative matrix, so by the Perron–Frobenius theorem the eigenvalue of largest modulus is simple and its eigenspace is spanned by a strictly positive vector; since $\mathbf{1}$ is strictly positive and already an eigenvector for $\lambda_1 = 1 - \alpha$, it must (up to normalization) be this Perron eigenvector, i.e., $\boldsymbol{u}_1 = \mathbf{1}/\sqrt{N} \propto \mathbf{1}$.

Part 3: sequential variational characterization. By (40), for any unit vector $\boldsymbol{\theta}$, $\mathcal{S}(\boldsymbol{\theta}) = (1 - \alpha) - \boldsymbol{\theta}^\top \boldsymbol{G} \boldsymbol{\theta}$, an affine, strictly decreasing function of the Rayleigh quotient $\boldsymbol{\theta}^\top \boldsymbol{G} \boldsymbol{\theta}$. Hence minimizing $\mathcal{S}(\boldsymbol{\theta})$ over any constraint set is equivalent to maximizing $\boldsymbol{\theta}^\top \boldsymbol{G} \boldsymbol{\theta}$ over the same set, and it suffices to establish the following standard sequential characterization of the eigenbasis of a symmetric matrix.

Claim. For each $k = 1, \ldots, N$,

$$\boldsymbol{u}_k \in \underset{\boldsymbol{\theta} \perp \{\boldsymbol{u}_j\}_{j<k},\ \|\boldsymbol{\theta}\|=1}{\arg\max} \boldsymbol{\theta}^\top \boldsymbol{G} \boldsymbol{\theta}, \qquad \text{with maximal value } \lambda_k.$$

(For $k = 1$ the orthogonality constraint is vacuous.)

Proof of Claim. Fix k and let $\boldsymbol{\theta}$ be any unit vector orthogonal to $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{k-1}$. Expand $\boldsymbol{\theta} = \sum_{j \geq k} c_j \boldsymbol{u}_j$ in the orthonormal eigenbasis; orthogonality to $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{k-1}$ forces the coefficients on those directions to vanish, and $\|\boldsymbol{\theta}\| = 1$ gives $\sum_{j \geq k} c_j^2 = 1$ by Parseval's identity. Then

$$\boldsymbol{\theta}^\top \boldsymbol{G} \boldsymbol{\theta} = \sum_{j \geq k} \lambda_j c_j^2 \leq \lambda_k \sum_{j \geq k} c_j^2 = \lambda_k,$$

using $\lambda_j \leq \lambda_k$ for every $j \geq k$ (eigenvalues listed in decreasing algebraic order). The bound is attained at $\boldsymbol{\theta} = \boldsymbol{u}_k$ (taking $c_k = 1$ and $c_j = 0$ for $j > k$), proving the claim; equality throughout requires $c_j = 0$ whenever $\lambda_j < \lambda_k$, so the maximizer is unique up to sign unless λ_k is a repeated eigenvalue, in which case any unit vector in its eigenspace (orthogonal to $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{k-1}$) attains the same maximal value.

Applying the Claim with $k = 1$ and using the strictly decreasing affine relationship between $\mathcal{S}$ and the Rayleigh quotient gives $\boldsymbol{u}_1 = \arg\min_{\|\boldsymbol{\theta}\|=1} \mathcal{S}(\boldsymbol{\theta})$, already identified as $\mathbf{1}/\sqrt{N}$ in Part 2. Applying the Claim for general $k > 1$ gives $\boldsymbol{u}_k = \arg\min_{\boldsymbol{\theta} \perp \{\boldsymbol{u}_j\}_{j<k}, \|\boldsymbol{\theta}\|=1} \mathcal{S}(\boldsymbol{\theta})$, completing the proof. $\square$

D Proof of Proposition 4

Proposition. ***(Proposition 4 in the paper)*** *The operator $\mathcal{L}(\boldsymbol{G})$, mapping from domestic interests to geopolitical actions, is a spectral function of $\boldsymbol{G}$: it shares the same eigenbasis as $\boldsymbol{G}$. This implies spectral decoupling: when domestic interests are decomposed along $\boldsymbol{G}$'s eigenvectors, the equilibrium mapping scales each dimension of cleavages independently without spilling over into others:*

$$\boldsymbol{a} = \sum_{\ell} L(\lambda_\ell) \widetilde{\theta}_\ell \boldsymbol{u}_\ell, \qquad L(\lambda) \equiv \frac{(1-\lambda)(\kappa\alpha - 1)}{\kappa(1-\lambda)^2 - \alpha};$$

or, equivalently, projecting the equilibrium action profile onto $\boldsymbol{G}$'s eigenbasis $\widetilde{\boldsymbol{a}} \equiv \boldsymbol{U}^\top \boldsymbol{a}$, we have:

$$\widetilde{a}_\ell = L(\lambda_\ell) \widetilde{\theta}_\ell.$$

The scaling factor $L(\cdot)$ is non-negative, bounded above by one ($0 \leq L(\cdot) \leq 1$), and is strictly increasing in the eigenvalue λ_ℓ ($L'(\cdot) > 0$), implying that eigenvectors with larger eigenvalues translate into larger differences in geopolitical actions. Finally, the scaling factor $L(\cdot)$ is increasing in κ.

Proof. As established in Proposition 1, equilibrium geopolitical actions satisfy $\boldsymbol{a} = \mathcal{L}(\boldsymbol{G})\,\boldsymbol{\theta}$, where

$$\mathcal{L}(\boldsymbol{G}) \equiv \left(\kappa(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}\right)^{-1}(\boldsymbol{I}-\boldsymbol{G})(\kappa\alpha-1), \qquad \kappa > 1/\alpha. \tag{41}$$

Domestic interests project onto $\boldsymbol{G}$'s eigenbasis as $\boldsymbol{\theta} = \sum_\ell \widetilde{\theta}_\ell \boldsymbol{u}_\ell$, $\widetilde{\theta}_\ell = \boldsymbol{u}_\ell^\top \boldsymbol{\theta}$.

Spectral decoupling. Because $\boldsymbol{G} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^\top$, every matrix built from $\boldsymbol{I}$ and $\boldsymbol{G}$ by addition, multiplication, and inversion is diagonalized by the same orthogonal matrix $\boldsymbol{U}$: for instance, $(\boldsymbol{I}-\boldsymbol{G})^n = \boldsymbol{U}(\boldsymbol{I}-\boldsymbol{\Lambda})^n\boldsymbol{U}^\top$ for any n, and $\left(\kappa(\boldsymbol{I}-\boldsymbol{G})^2-\alpha\boldsymbol{I}\right)^{-1} = \boldsymbol{U}\left(\kappa(\boldsymbol{I}-\boldsymbol{\Lambda})^2-\alpha\boldsymbol{I}\right)^{-1}\boldsymbol{U}^\top$, since $\boldsymbol{U}$ is orthogonal ($\boldsymbol{U}\boldsymbol{U}^\top = \boldsymbol{I}$) so that sums, products, and inverses of diagonal matrices sandwiched between $\boldsymbol{U}$ and $\boldsymbol{U}^\top$ combine correctly (e.g., $\boldsymbol{U}D_1\boldsymbol{U}^\top \cdot \boldsymbol{U}D_2\boldsymbol{U}^\top = \boldsymbol{U}D_1D_2\boldsymbol{U}^\top$). Substituting into (41),

$$\mathcal{L}(\boldsymbol{G}) = \boldsymbol{U}\left(\kappa(\boldsymbol{I}-\boldsymbol{\Lambda})^2-\alpha\boldsymbol{I}\right)^{-1}(\boldsymbol{I}-\boldsymbol{\Lambda})(\kappa\alpha-1)\boldsymbol{U}^\top = \boldsymbol{U}\ \mathrm{Diag}\left(L(\lambda_1),\ldots,L(\lambda_N)\right)\boldsymbol{U}^\top,$$

where $L(\lambda) \equiv (1-\lambda)(\kappa\alpha-1)/\left(\kappa(1-\lambda)^2-\alpha\right)$ is precisely the scalar function obtained by applying the same rational expression entrywise to each diagonal eigenvalue. Hence $\mathcal{L}(\boldsymbol{G})$ has eigenvectors $\boldsymbol{u}_\ell$ with eigenvalues $L(\lambda_\ell)$: it is a spectral function of $\boldsymbol{G}$, sharing $\boldsymbol{G}$'s eigenbasis. Applying $\mathcal{L}(\boldsymbol{G})$ to $\boldsymbol{\theta} = \sum_\ell \widetilde{\theta}_\ell\boldsymbol{u}_\ell$ term by term gives $\boldsymbol{a} = \mathcal{L}(\boldsymbol{G})\,\boldsymbol{\theta} = \sum_\ell L(\lambda_\ell)\,\widetilde{\theta}_\ell\boldsymbol{u}_\ell$, and projecting onto $\boldsymbol{u}_\ell$ (using orthonormality) gives $\widetilde{a}_\ell = \boldsymbol{u}_\ell^\top\boldsymbol{a} = L(\lambda_\ell)\,\widetilde{\theta}_\ell$. This is the claimed spectral decoupling: each cleavage ℓ is scaled only by its own eigenvalue, with no mixing across cleavages.

For the remaining properties of $L(\cdot)$, substitute $z \equiv 1-\lambda$, so that $\boldsymbol{G}$'s eigenvalues $\lambda \in [-(1-\alpha), 1-\alpha]$ (a bound established in the proof of Proposition 3 via Perron–Frobenius) correspond to $z \in [\alpha, 2-\alpha]$, and write

$$L(z) = \frac{(\kappa\alpha-1)\,z}{\kappa z^2-\alpha}.$$

Sign and denominator positivity. Since $\kappa > 1/\alpha$, $\kappa\alpha - 1 > 0$. For $z \geq \alpha > 0$, $\kappa z^2 - \alpha \geq \kappa\alpha^2 - \alpha = \alpha(\kappa\alpha-1) > 0$ (the map $z \mapsto \kappa z^2 - \alpha$ is increasing for $z > 0$, so its minimum over $z \geq \alpha$ is attained at $z = \alpha$). Hence both numerator and denominator of $L(z)$ are strictly positive throughout the relevant range, so $L(\lambda) > 0$ for every eigenvalue λ of $\boldsymbol{G}$; in particular $L(\cdot) \geq 0$.

Upper bound $L(\cdot) \leq 1$. We must show $(\kappa\alpha-1)\,z \leq \kappa z^2 - \alpha$ for $z \geq \alpha$, i.e., $\kappa z^2 - (\kappa\alpha-1)\,z - \alpha \geq 0$. Factor:

$$\kappa z^2 - (\kappa\alpha-1)\,z - \alpha = \kappa z(z-\alpha) + (z-\alpha) = (z-\alpha)(\kappa z+1).$$

Since $z \geq \alpha$, $z - \alpha \geq 0$, and $\kappa z + 1 > 0$; hence the product is nonnegative, proving $L(z) \leq 1$ for all $z \geq \alpha$, with equality if and only if $z = \alpha$, i.e., $\lambda = 1-\alpha$ (the top eigenvalue, associated with $\boldsymbol{u}_1 \propto \mathbf{1}$). Combined with the previous step, $0 \leq L(\cdot) \leq 1$.

Monotonicity in λ. Differentiate L with respect to z using the quotient rule:

$$\frac{\mathrm{d}L}{\mathrm{d}z} = \frac{(\kappa\alpha - 1)\left(\kappa z^2 - \alpha\right) - (\kappa\alpha - 1)\, z\, (2\kappa z)}{\left(\kappa z^2 - \alpha\right)^2} = \frac{-\left(\kappa\alpha - 1\right)\left(\kappa z^2 + \alpha\right)}{\left(\kappa z^2 - \alpha\right)^2} < 0,$$

since $\kappa\alpha - 1 > 0$ and $\kappa z^2 + \alpha > 0$. Because $z = 1 - \lambda$, the chain rule gives $\mathrm{d}L/\mathrm{d}\lambda = -\mathrm{d}L/\mathrm{d}z > 0$: $L(\cdot)$ is strictly increasing in λ, so cleavages with larger eigenvalues are scaled more strongly into equilibrium actions.

Monotonicity in κ. Differentiate L with respect to κ at fixed z, using the quotient rule:

$$\begin{aligned}\frac{\partial L}{\partial \kappa} &= \frac{\alpha z\left(\kappa z^2 - \alpha\right) - (\kappa\alpha - 1)\, z \cdot z^2}{\left(\kappa z^2 - \alpha\right)^2} = \frac{z\left[\alpha\kappa z^2 - \alpha^2 - \kappa\alpha z^2 + z^2\right]}{\left(\kappa z^2 - \alpha\right)^2} \\ &= \frac{z\left(z^2 - \alpha^2\right)}{\left(\kappa z^2 - \alpha\right)^2} = \frac{z\,(z - \alpha)\,(z + \alpha)}{\left(\kappa z^2 - \alpha\right)^2}.\end{aligned}$$

Since $z \geq \alpha > 0$, every factor in the numerator is nonnegative, so $\partial L/\partial\kappa \geq 0$ for every eigenvalue $\lambda \leq 1 - \alpha$, with equality only at $z = \alpha$ (i.e., $\lambda = 1 - \alpha$, where $L \equiv 1$ regardless of κ, consistent with the equality case above). For every other eigenvalue, $L(\cdot)$ is strictly increasing in κ: as strategic influence becomes cheaper (lower κ), the scaling factor on every cleavage other than the consensus direction $\mathbf{1}$ falls, pulling equilibrium actions toward the network consensus. $\square$

E Proof of Proposition 5

Proposition. ***(Proposition 5 from the paper)*** *Let $\widetilde{\boldsymbol{\Sigma}}_a \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_a \boldsymbol{U}$, $\widetilde{\boldsymbol{\Sigma}}_\theta \equiv \boldsymbol{U}^\top \boldsymbol{\Sigma}_\theta \boldsymbol{U}$ be the projection of $\boldsymbol{\Sigma}_a$ and $\boldsymbol{\Sigma}_\theta$ onto the eigenbasis $\boldsymbol{U}$. The eigenvalues (λ_ℓ) and eigenvectors ($\boldsymbol{u}_\ell$) of $\boldsymbol{G}$ are identified if the ℓ-th diagonal element of $\widetilde{\boldsymbol{\Sigma}}_\theta$ is non-zero, with*

$$\lambda_\ell = L^{-1}\left(\sqrt{\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell} \Big/ \sigma_\ell^2}\right), \tag{42}$$

where $L(\cdot)$ is defined in Proposition 4; σ_ℓ^2 is the ℓ-th diagonal entry of $\widetilde{\boldsymbol{\Sigma}}_\theta$; and we recover $\boldsymbol{u}_\ell$ from equation (20) using this solution for σ_ℓ^2.

Proof. The identification argument has two parts: recovering the eigenvectors $\boldsymbol{u}_\ell$ from $\boldsymbol{\Sigma}_\theta$ alone, and then recovering the associated eigenvalues λ_ℓ by combining $\boldsymbol{\Sigma}_\theta$ with $\boldsymbol{\Sigma}_a$.

Recovering $\boldsymbol{u}_\ell$. By Assumption 1, $\boldsymbol{\Sigma}_\theta = \boldsymbol{U}\widetilde{\boldsymbol{\Sigma}}_\theta\boldsymbol{U}^\top$ with $\widetilde{\boldsymbol{\Sigma}}_\theta$ diagonal: this is exactly the spectral decomposition of the (observable) matrix $\boldsymbol{\Sigma}_\theta$, with $\boldsymbol{u}_\ell$ the eigenvector associated with eigenvalue σ_ℓ^2. By the spectral theorem, this decomposition is recoverable directly from data on $\boldsymbol{\Sigma}_\theta$: whenever the σ_ℓ^2 are pairwise distinct, the eigenvectors $\{\boldsymbol{u}_\ell\}$ are uniquely determined up to sign, and since $\boldsymbol{G} = \sum_\ell \lambda_\ell \boldsymbol{u}_\ell \boldsymbol{u}_\ell^\top$ depends on each $\boldsymbol{u}_\ell$ only through the sign-invariant outer product $\boldsymbol{u}_\ell\boldsymbol{u}_\ell^\top$, this sign ambiguity is immaterial for recovering $\boldsymbol{G}$. (If some σ_ℓ^2's coincide, the corresponding eigenvectors are only determined up to an orthogonal rotation within their shared eigenspace; this is the standard identification caveat for any eigenvector-based decomposition and does not affect the argument below.)

Recovering λ_ℓ. By Lemma 2,

$$\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell} = L\left(\lambda_\ell\right)^2 \sigma_\ell^2.$$

If $\sigma_\ell^2 \neq 0$, divide both sides by σ_ℓ^2 and take the (nonnegative) square root:

$$\left|L\left(\lambda_\ell\right)\right| = \sqrt{\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell} \Big/ \sigma_\ell^2}.$$

By Proposition 4, $L\left(\lambda\right) \geq 0$ for every eigenvalue λ of $\boldsymbol{G}$ in the relevant range $\left[-\left(1-\alpha\right), 1-\alpha\right]$, so $\left|L\left(\lambda_\ell\right)\right| = L\left(\lambda_\ell\right)$ and hence

$$L\left(\lambda_\ell\right) = \sqrt{\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell} \Big/ \sigma_\ell^2}.$$

By Proposition 4, $L\left(\cdot\right)$ is strictly increasing on this range ($L'\left(\cdot\right) > 0$), hence injective, and therefore invertible on its image. Applying L^{-1} to both sides:

$$\lambda_\ell = L^{-1}\left(\sqrt{\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell} \Big/ \sigma_\ell^2}\right),$$

which is uniquely determined by the observable quantities $\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell}$ and σ_ℓ^2. Together with the recovered eigenvector $\boldsymbol{u}_\ell$, this identifies the pair $\left(\lambda_\ell, \boldsymbol{u}_\ell\right)$, and hence the term $\lambda_\ell \boldsymbol{u}_\ell \boldsymbol{u}_\ell^\top$ of $\boldsymbol{G} = \sum_\ell \lambda_\ell \boldsymbol{u}_\ell \boldsymbol{u}_\ell^\top$, whenever $\sigma_\ell^2 \neq 0$.

Failure of identification when $\sigma_\ell^2 = 0$. If instead $\sigma_\ell^2 = 0$, then by Lemma 2, $\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell} = L\left(\lambda_\ell\right)^2 \cdot 0 = 0$ identically, for *every* value of λ_ℓ: the observed moment $\left(\widetilde{\boldsymbol{\Sigma}}_a\right)_{\ell\ell}$ carries no information about λ_ℓ, since domestic interests never vary along the cleavage $\boldsymbol{u}_\ell$ and hence this direction of $\boldsymbol{G}$ is never excited by any observed issue. Consequently, $\boldsymbol{G}$ is identified only on the subspace spanned by $\{\boldsymbol{u}_\ell : \sigma_\ell^2 > 0\}$; on the orthogonal complement, λ_ℓ remains unrestricted by $\left(\boldsymbol{\Sigma}_a, \boldsymbol{\Sigma}_\theta\right)$. $\square$

F Proof of Remark 1

(**Remark 1 in the paper**) Suppose that the eigenvalues of $\boldsymbol{G}$ are pairwise distinct and that $\boldsymbol{\Sigma}_\theta$ is nonsingular. Then $\boldsymbol{\Sigma}_\theta$ and $\boldsymbol{\Sigma}_a$ share a common set of principal components if and only if Assumption 1 holds.

Proof. Two symmetric matrices share a common orthonormal eigenbasis if and only if they commute; we therefore characterize when $\boldsymbol{\Sigma}_\theta \boldsymbol{\Sigma}_a = \boldsymbol{\Sigma}_a \boldsymbol{\Sigma}_\theta$. If Assumption 1 holds, then by Lemma 2 both $\boldsymbol{\Sigma}_\theta$ and $\boldsymbol{\Sigma}_a$ are spectral functions of $\boldsymbol{G}$, so they commute. For the converse, work in the eigenbasis of $\boldsymbol{G}$: write $\boldsymbol{G} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^\top$, $\boldsymbol{S} \equiv \boldsymbol{U}^\top\boldsymbol{\Sigma}_\theta\boldsymbol{U}$, and $\boldsymbol{D} \equiv \mathrm{Diag}\left(L\left(\lambda_1\right), \ldots, L\left(\lambda_N\right)\right)$, so that $\boldsymbol{U}^\top\boldsymbol{\Sigma}_a\boldsymbol{U} = \boldsymbol{D}\boldsymbol{S}\boldsymbol{D}$ by Proposition 4. Direct computation gives

$$\left[\boldsymbol{U}^\top\left(\boldsymbol{\Sigma}_\theta\boldsymbol{\Sigma}_a - \boldsymbol{\Sigma}_a\boldsymbol{\Sigma}_\theta\right)\boldsymbol{U}\right]_{mp} = \left(L\left(\lambda_p\right) - L\left(\lambda_m\right)\right)\left[\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}\right]_{mp}.$$

Because $L\left(\cdot\right)$ is strictly increasing (Proposition 4) and the eigenvalues are pairwise distinct, $L\left(\lambda_m\right) \neq L\left(\lambda_p\right)$ for $m \neq p$, so commutation requires $\left[\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}\right]_{mp} = 0$ for all $m \neq p$: the matrix $\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}$ is diagonal. Now define $\boldsymbol{B} \equiv \boldsymbol{D}^{1/2}\boldsymbol{S}\boldsymbol{D}^{1/2}$, which is symmetric and positive definite, since $\boldsymbol{S}$ is positive definite ($\boldsymbol{\Sigma}_\theta$ is a nonsingular covariance matrix) and $\boldsymbol{D}$ is positive definite ($L\left(\lambda\right) > 0$ on $\left[-\left(1-\alpha\right), 1-\alpha\right]$, by Proposition 4). Then $\boldsymbol{B}^2 = \boldsymbol{D}^{1/2}\left(\boldsymbol{S}\boldsymbol{D}\boldsymbol{S}\right)\boldsymbol{D}^{1/2}$ is diagonal with strictly positive entries. Since a positive definite matrix has a unique positive definite square root, and the diagonal matrix $\left(\boldsymbol{B}^2\right)^{1/2}$ is one such root, $\boldsymbol{B}$ is diagonal; hence $\boldsymbol{S} = \boldsymbol{D}^{-1/2}\boldsymbol{B}\boldsymbol{D}^{-1/2}$ is diagonal, which is precisely Assumption 1. If some eigenvalues of $\boldsymbol{G}$ coincide, the same argument forces $\boldsymbol{S}$ to be block-diagonal across distinct eigenvalues, so that Assumption 1 holds up to rotations within repeated eigenspaces, which leave $\boldsymbol{G}$ unchanged. The same argument in the $\boldsymbol{\Pi}$-weighted coordinates of Section 2.7 delivers the analogous characterization of Assumption 2. $\square$

G Proof of Proposition 6

Proposition. ***(Proposition 6 in the paper)*** *Under Assumption 1, ex ante welfare for country i can be written as:*

$$\mathbb{E}_\theta\left[U_i\right] = \sum_\ell \sigma_\ell^2 \times \Gamma_{i\ell}\left(\left\{u_k\right\}, \left\{\lambda_k\right\}\right),$$

where $\Gamma_{i\ell}\left(\left\{u_k\right\}, \left\{\lambda_k\right\}\right)$ is country i's payoff along geopolitical cleavage ℓ, defined as:

$$\begin{aligned}\Gamma_{i\ell} &= \left(L\left(\lambda_\ell\right) - 1\right)\left[2\frac{\alpha^2 - \left(1-\lambda_\ell\right)^2}{\kappa\left(1-\lambda_\ell\right)^2 - \alpha} - \alpha\left(L\left(\lambda_\ell\right) - 1\right)\right]u_{i\ell}^2 \\ &\quad - L\left(\lambda_\ell\right)^2\sum_q C_{\ell\ell q}\left(1 - \alpha - 2\lambda_\ell + \lambda_q\right)u_{iq} \\ &\quad - \frac{1}{\kappa}\sum_p\left(\sum_q C_{\ell pq}\left[\frac{L\left(\lambda_\ell\right)\left(1-\lambda_q\right)}{1-\lambda_p} - 1\right]u_{iq}\right)^2,\end{aligned}$$

where $L\left(\cdot\right)$ is as defined in Proposition 4, and $C_{\ell pq} \equiv \sum_{k=1}^N u_{k\ell}u_{kp}u_{kq} = \langle \boldsymbol{u}_\ell \circ \boldsymbol{u}_p, \boldsymbol{u}_q\rangle$ is the spectral interaction tensor that captures how eigenmodes interact through the nonlinearity of the country-level products of the eigenvectors $u_{k\ell}u_{kp}u_{kq}$.

Proof. We derive the expected payoff $\mathbb{E}\left[U_i\right]$ when $\boldsymbol{\theta}$ is drawn from a distribution whose covariance

$\boldsymbol{\Sigma}_\theta$ is diagonal in $\boldsymbol{G}$'s eigenbasis. The payoff of country i is

$$U_i = \underbrace{-\left[\alpha\left(a_i-\theta_i\right)^2+\sum_j G_{ij}\left(a_i-a_j\right)^2\right]}_{\text{First term (beauty contest)}} + \underbrace{2\left(\sum_j \omega_{ij}\right)\left(a_i-\theta_i\right)}_{\text{Second term (influence received)}} - \underbrace{\kappa\sum_k \omega_{ki}^2}_{\text{Third term (influence exerted)}}. \tag{43}$$

Preliminaries. Let $\boldsymbol{G} = \sum_m \lambda_m \boldsymbol{u}_m\boldsymbol{u}_m^\top$ with $\{\boldsymbol{u}_m\}$ orthonormal. Define $\tilde{\theta}_m = \boldsymbol{u}_m^\top\boldsymbol{\theta}$ and $\widetilde{\boldsymbol{\Sigma}}_\theta = \mathrm{Diag}\left(\sigma_1^2,\ldots,\sigma_N^2\right)$, so that $\mathbb{E}\left[\tilde{\theta}_m\tilde{\theta}_p\right] = \sigma_m^2\,\mathbf{1}_{m=p}$. The equilibrium action in mode m is $\tilde{a}_m = L_m\,\tilde{\theta}_m$ where

$$L\left(\lambda\right) \equiv \frac{\left(\kappa\alpha-1\right)\left(1-\lambda\right)}{\kappa\left(1-\lambda\right)^2-\alpha}. \tag{44}$$

The equilibrium net influence received, $\sum_j \omega_{ij}$, has mode-m projection $\Phi_m\,\tilde{\theta}_m$ where

$$\Phi\left(\lambda\right) \equiv \frac{\alpha^2-\left(1-\lambda\right)^2}{\kappa\left(1-\lambda\right)^2-\alpha} = \left(1-\lambda\right)L\left(\lambda\right)-\alpha, \tag{45}$$

the second equality following from $\sum_j \omega_{ij} = \left[\left(\boldsymbol{I}-\boldsymbol{G}\right)\boldsymbol{a}-\alpha\boldsymbol{\theta}\right]_i$. In country coordinates:

$$a_i-\theta_i = \sum_m \left(L_m-1\right)\tilde{\theta}_m\,u_{im}, \tag{46}$$

$$\sum_j \omega_{ij} = \sum_m \Phi_m\,\tilde{\theta}_m\,u_{im}. \tag{47}$$

Overlap tensor. Define

$$C_{mpq} \equiv \sum_{k=1}^N u_{km}\,u_{kp}\,u_{kq}. \tag{48}$$

The expansion identity $u_{im}\,u_{ip} = \sum_q C_{mpq}\,u_{iq}$ and the $\left(\boldsymbol{I}-\boldsymbol{G}\right)$-identity

$$u_{im}\,u_{ip} - \sum_j G_{ij}\,u_{jm}\,u_{jp} = \sum_q C_{mpq}\left(1-\lambda_q\right)u_{iq} \tag{49}$$

will be used throughout.

First term (beauty contest). From (46):

$$\mathbb{E}\left[\alpha\left(a_i-\theta_i\right)^2\right] = \alpha\sum_m \sigma_m^2\left(L_m-1\right)^2 u_{im}^2.$$

For the coordination loss, $a_i-a_j = \sum_m L_m\tilde{\theta}_m\left(u_{im}-u_{jm}\right)$, so

$$\mathbb{E}\left[\sum_j G_{ij}\left(a_i-a_j\right)^2\right] = \sum_m \sigma_m^2 L_m^2 \sum_j G_{ij}\left(u_{im}-u_{jm}\right)^2.$$

Expanding $\sum_j G_{ij}\left(u_{im}-u_{jm}\right)^2$ and applying $\sum_j G_{ij}u_{jm}^2=\sum_q C_{mmq}\lambda_q u_{iq}$:

$$\sum_j G_{ij}\left(u_{im}-u_{jm}\right)^2=\left(1-\alpha-2\lambda_m\right)u_{im}^2+\sum_q C_{mmq}\lambda_q\,u_{iq}. \tag{50}$$

Second term (influence received). From (46) and (47), using $\mathbb{E}\left[\tilde{\theta}_m\tilde{\theta}_p\right]=\sigma_m^2\,\mathbf{1}_{m=p}$:

$$\mathbb{E}\left[2\left(\sum_j \omega_{ij}\right)\left(a_i-\theta_i\right)\right]=2\sum_m \sigma_m^2\,\Phi_m\left(L_m-1\right)u_{im}^2. \tag{51}$$

Third term (influence exerted). The bilateral FOC for the optimal influence ω_{ki} exerted by i on k is:

$$\kappa\omega_{ki}=\sum_j G_{ij}\left(a_i-a_j\right)\left[\left(\boldsymbol{I}-\boldsymbol{G}\right)^{-1}\right]_{jk}+\mathbf{1}_{k=i}\left(a_i-\theta_i\right).$$

Define $B_{ip}\equiv\kappa\boldsymbol{\omega}_{i\cdot}^{\top}\boldsymbol{u}_p$. By Parseval, $\kappa^2\sum_k\omega_{ki}^2=\sum_p B_{ip}^2$. Projecting the FOC onto $\boldsymbol{u}_p$, substituting the spectral expansions, and applying (49):

$$B_{ip}=\sum_m\tilde{\theta}_m\sum_q C_{mpq}\left[\frac{L_m\left(1-\lambda_q\right)}{1-\lambda_p}-1\right]u_{iq}. \tag{52}$$

Taking expectations:

$$\mathbb{E}\left[\kappa\sum_k\omega_{ki}^2\right]=\frac{1}{\kappa}\sum_m\sigma_m^2\sum_p\left(\sum_q C_{mpq}\left[\frac{L_m\left(1-\lambda_q\right)}{1-\lambda_p}-1\right]u_{iq}\right)^2. \tag{53}$$

Combining. $\mathbb{E}\left[U_i\right]=\sum_m\sigma_m^2\,\Gamma_{im}$ where

$$\boxed{\begin{aligned}\Gamma_{im}=\;&\underbrace{-\alpha\left(L_m-1\right)^2u_{im}^2-L_m^2\left[\left(1-\alpha-2\lambda_m\right)u_{im}^2+\sum_q C_{mmq}\lambda_q\,u_{iq}\right]}_{\text{First term (beauty contest)}}\\&+\underbrace{2\,\Phi_m\left(L_m-1\right)u_{im}^2}_{\text{Second term (influence received)}}\\&\underbrace{-\frac{1}{\kappa}\sum_p\left(\sum_q C_{mpq}\left[\frac{L_m\left(1-\lambda_q\right)}{1-\lambda_p}-1\right]u_{iq}\right)^2}_{\text{Third term (influence exerted)}}.\end{aligned}} \tag{54}$$

Interpretation. The first term captures the beauty-contest payoff: both the cost of deviating from national interest ($\alpha\left(L_m-1\right)^2$) and the coordination loss with neighbors. The coordination loss couples eigenmodes through the overlap tensor C_{mmq}, reflecting how country i's squared loading on cleavage m relates to its neighbors' loadings on other cleavages. The second term captures the benefit of influence received: since $\Phi_m \leq 0$ and $L_m \leq 1$, this term is nonnegative, so influence receipt partially offsets the beauty-contest distortion. The third term is the cost of influence exerted, which couples all eigenmodes through the products C_{mpq}—even though equilibrium actions decouple mode by mode, the quadratic cost of bilateral influence introduces cross-mode interactions. □

H Proof of Proposition 7

Proposition. ***(Proposition 7 in the paper)*** *Suppose $\boldsymbol{G} = (1-\alpha)\,\boldsymbol{\Pi}^{-1}\boldsymbol{H}$ with influence costs $\kappa\,(\pi_k/\pi_i)\,\omega_{ki}^2$. Then:*

1. *Propositions 1 and 2 continue to hold. In particular:*

   (a) *Equilibrium net influence and geopolitical actions satisfy:*

$$\boldsymbol{\Omega}\mathbf{1} = -\left(\kappa\,(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}\right)^{-1}\left((\boldsymbol{I}-\boldsymbol{G})^2 - \alpha^2\boldsymbol{I}\right)\boldsymbol{\theta},$$

$$\boldsymbol{a} = \left(\kappa\,(\boldsymbol{I}-\boldsymbol{G})^2 - \alpha\boldsymbol{I}\right)^{-1}(\boldsymbol{I}-\boldsymbol{G})\,(\kappa\alpha-1)\,\boldsymbol{\theta}.$$

   (b) *The equilibrium inherits uniqueness, the convex combination property, $\boldsymbol{\Pi}$-weighted conservativeness ($\sum_i \pi_i a_i = \sum_i \pi_i\theta_i$ and $\sum_i \pi_i \sum_m \omega_{im} = 0$), and the $\boldsymbol{\Pi}$-weighted analogues of the two limiting benchmarks ($\kappa \to \infty$ and $\kappa \downarrow 1/\alpha$) from the symmetric baseline.*

2. *The equilibrium continues to admit a spectral representation:*

$$a_i = \sum_m L(\lambda_m)\left[\boldsymbol{u}_m^R\right]_i \left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{\theta}, \tag{55}$$

   where the scaling function $L(\lambda) \equiv (1-\lambda)(\kappa\alpha-1)\,/\,\left(\kappa\,(1-\lambda)^2 - \alpha\right)$ is identical to that under a symmetric network.

Proof. Throughout this proof, write $\tilde{\boldsymbol{H}} \equiv (1-\alpha)\,\boldsymbol{\Pi}^{-1/2}\boldsymbol{H}\boldsymbol{\Pi}^{-1/2}$ for the symmetrized network and $\sqrt{\boldsymbol{\pi}}$ for the vector with entries $\sqrt{\pi_i}$. By construction, $\boldsymbol{G} = \boldsymbol{\Pi}^{-1/2}\tilde{\boldsymbol{H}}\boldsymbol{\Pi}^{1/2}$, the matrix $\tilde{\boldsymbol{H}}$ is symmetric and nonnegative, $\tilde{\boldsymbol{H}}\sqrt{\boldsymbol{\pi}} = (1-\alpha)\sqrt{\boldsymbol{\pi}}$ (since $\boldsymbol{H}\mathbf{1} = \boldsymbol{\pi}$), and $\left\|\sqrt{\boldsymbol{\pi}}\right\| = 1$ (since $\sum_i \pi_i = 1$). The influence cost of i exerting ω_{ki} on k is $\kappa\,(\pi_k/\pi_i)\,\omega_{ki}^2$, and country i's payoff is:

$$U_i(\boldsymbol{a},\boldsymbol{\Omega}) = -\left[\alpha\,(a_i-\theta_i)^2 + \sum_j G_{ij}\,(a_i-a_j)^2\right] - \kappa\sum_k \frac{\pi_k}{\pi_i}\omega_{ki}^2 + 2\sum_m \omega_{im}\,(a_i-\theta_i)\,. \tag{56}$$

We first collect four structural facts about the pair $\left(\boldsymbol{G},\tilde{\boldsymbol{H}}\right)$ that drive every part of the proof.

Fact 1 (G is nonnegative with constant row sums). $G_{ij} = \left(\sqrt{\pi_j}/\sqrt{\pi_i}\right)\tilde{H}_{ij} \geq 0$ since $\tilde{H}_{ij} \geq 0$ and $\sqrt{\pi_i}, \sqrt{\pi_j} > 0$. Since $\tilde{\boldsymbol{H}}\sqrt{\boldsymbol{\pi}} = (1-\alpha)\sqrt{\boldsymbol{\pi}}$, $\boldsymbol{G}\mathbf{1} = \boldsymbol{\Pi}^{-1/2}\tilde{\boldsymbol{H}}\boldsymbol{\Pi}^{1/2}\mathbf{1} = \boldsymbol{\Pi}^{-1/2}\tilde{\boldsymbol{H}}\sqrt{\boldsymbol{\pi}} = (1-\alpha)\boldsymbol{\Pi}^{-1/2}\sqrt{\boldsymbol{\pi}} = (1-\alpha)\mathbf{1}$.

Fact 2 (similarity transfer). Because $\boldsymbol{G} = \boldsymbol{\Pi}^{-1/2}\tilde{\boldsymbol{H}}\boldsymbol{\Pi}^{1/2}$, induction gives $\boldsymbol{G}^n = \boldsymbol{\Pi}^{-1/2}\tilde{\boldsymbol{H}}^n\boldsymbol{\Pi}^{1/2}$ for every $n \geq 0$, so for any polynomial (or, where the relevant inverse exists, rational function) $p(\cdot)$, $p(\boldsymbol{G}) = \boldsymbol{\Pi}^{-1/2}p\left(\tilde{\boldsymbol{H}}\right)\boldsymbol{\Pi}^{1/2}$. In particular $\boldsymbol{G}$ and $\tilde{\boldsymbol{H}}$ are similar and share the same eigenvalues $\{\lambda_m\}$, all real (as $\tilde{\boldsymbol{H}}$ is symmetric) and satisfying $|\lambda_m| \leq 1-\alpha$ by Perron–Frobenius applied to $\tilde{\boldsymbol{H}}$. Substituting the spectral form $\tilde{\boldsymbol{H}} = \sum_m \lambda_m \boldsymbol{v}_m\boldsymbol{v}_m^\top$,

$$p(\boldsymbol{G}) = \boldsymbol{\Pi}^{-1/2}\left(\sum_m p(\lambda_m)\boldsymbol{v}_m\boldsymbol{v}_m^\top\right)\boldsymbol{\Pi}^{1/2} = \sum_m p(\lambda_m)\left(\boldsymbol{\Pi}^{-1/2}\boldsymbol{v}_m\right)\left(\boldsymbol{\Pi}^{1/2}\boldsymbol{v}_m\right)^\top = \sum_m p(\lambda_m)\boldsymbol{u}_m^R\left(\boldsymbol{u}_m^L\right)^\top, \tag{57}$$

with $\boldsymbol{u}_m^R \equiv \boldsymbol{\Pi}^{-1/2}\boldsymbol{v}_m$, $\boldsymbol{u}_m^L \equiv \boldsymbol{\Pi}^{1/2}\boldsymbol{v}_m$ satisfying biorthonormality $\left(\boldsymbol{u}_m^L\right)^\top\boldsymbol{u}_p^R = \boldsymbol{v}_m^\top\boldsymbol{\Pi}^{1/2}\boldsymbol{\Pi}^{-1/2}\boldsymbol{v}_p = \boldsymbol{v}_m^\top\boldsymbol{v}_p = \mathbf{1}_{m=p}$; since $\{\boldsymbol{v}_m\}$ is a basis and $\boldsymbol{\Pi}^{1/2}$ is invertible, $\left\{\boldsymbol{u}_m^R\right\}$ is also a basis, with $\left\{\boldsymbol{u}_m^L\right\}$ its (unique) dual basis, so every vector $\boldsymbol{\theta}$ has the expansion $\boldsymbol{\theta} = \sum_m \tau_m\boldsymbol{u}_m^R$, $\tau_m \equiv \left(\boldsymbol{u}_m^L\right)^\top\boldsymbol{\theta}$. Note $\boldsymbol{u}_1^R = \boldsymbol{\Pi}^{-1/2}\boldsymbol{v}_1 = \boldsymbol{\Pi}^{-1/2}\sqrt{\boldsymbol{\pi}} = \mathbf{1}$ (since $\boldsymbol{v}_1 = \sqrt{\boldsymbol{\pi}}$) and $\boldsymbol{u}_1^L = \boldsymbol{\Pi}^{1/2}\boldsymbol{v}_1 = \boldsymbol{\Pi}^{1/2}\sqrt{\boldsymbol{\pi}}$, i.e., $\left[\boldsymbol{u}_1^L\right]_i = \pi_i$.

Fact 3 ($\boldsymbol{\Pi}\boldsymbol{G}$ is symmetric). Define $\boldsymbol{W} \equiv \boldsymbol{\Pi}\boldsymbol{G} = \boldsymbol{\Pi}^{1/2}\tilde{\boldsymbol{H}}\boldsymbol{\Pi}^{1/2}$ (using $\boldsymbol{G} = \boldsymbol{\Pi}^{-1/2}\tilde{\boldsymbol{H}}\boldsymbol{\Pi}^{1/2}$). Since $\boldsymbol{\Pi}^{1/2}$ and $\tilde{\boldsymbol{H}}$ are both symmetric, $\boldsymbol{W}^\top = \boldsymbol{\Pi}^{1/2}\tilde{\boldsymbol{H}}^\top\boldsymbol{\Pi}^{1/2} = \boldsymbol{\Pi}^{1/2}\tilde{\boldsymbol{H}}\boldsymbol{\Pi}^{1/2} = \boldsymbol{W}$. Equivalently, $\pi_i G_{ij} = \sqrt{\pi_i}\sqrt{\pi_j}\tilde{H}_{ij} = \sqrt{\pi_j}\sqrt{\pi_i}\tilde{H}_{ji} = \pi_j G_{ji}$ for every i, j.

Fact 4 (row/column sums of $\boldsymbol{W}$). By Fact 1, $\boldsymbol{W}\mathbf{1} = \boldsymbol{\Pi}\boldsymbol{G}\mathbf{1} = (1-\alpha)\boldsymbol{\Pi}\mathbf{1}$, i.e., row j of $\boldsymbol{W}$ sums to $(1-\alpha)\pi_j$; by symmetry (Fact 3), column j sums to the same value: $\sum_i W_{ij} = (1-\alpha)\pi_j$.

Part 1(a): the closed-form solutions. *Coordination stage.* The coordination-game component of (56) is unchanged from the symmetric model, so the first-order condition for a_i and the resulting fixed point are identical in form:

$$\boldsymbol{a} = (\boldsymbol{I} - \boldsymbol{G})^{-1}(\alpha\boldsymbol{\theta} + \boldsymbol{\Omega}\mathbf{1}), \tag{58}$$

using invertibility of $\boldsymbol{I} - \boldsymbol{G}$, which holds because $|\lambda_m| \leq 1 - \alpha < 1$ for every eigenvalue of $\boldsymbol{G}$ (Fact 2).

Influence stage. The envelope-theorem argument is unchanged except for the direct effect, which now involves the asymmetric cost: $\partial U_i/\partial\omega_{ki}\big|_{\boldsymbol{a}} = -2\kappa(\pi_k/\pi_i)\omega_{ki} + 2\cdot\mathbf{1}_{k=i}(a_i - \theta_i)$, while $\partial U_i/\partial a_j = 2G_{ij}(a_i - a_j)$ and $\partial a_j/\partial\omega_{ki} = \left[(\boldsymbol{I} - \boldsymbol{G})^{-1}\right]_{jk}$ are exactly as before (the latter because only the k-th entry of $\boldsymbol{\Omega}\mathbf{1} = \sum_i \omega_{ki}$ responds to ω_{ki}, with unit coefficient, regardless of the cost function). The first-order condition is thus

$$\kappa\left(\frac{\pi_k}{\pi_i}\right)\omega_{ki} = \sum_j G_{ij}(a_i - a_j)\left[(\boldsymbol{I} - \boldsymbol{G})^{-1}\right]_{jk} + \mathbf{1}_{k=i}(a_i - \theta_i). \tag{59}$$

Multiplying by π_i/π_k and summing over senders $i = 1, \ldots, N$:

$$\kappa \sum_i \omega_{ki} = \frac{1}{\pi_k} \sum_j \left[(\boldsymbol{I} - \boldsymbol{G})^{-1} \right]_{jk} \left(\sum_i \pi_i G_{ij} \left(a_i - a_j \right) \right) + \left(a_k - \theta_k \right),$$

where the second term uses $\sum_i \pi_i \mathbf{1}_{k=i} \left(a_i - \theta_i \right) / \pi_k = a_k - \theta_k$. For the inner sum, use $W_{ij} = \pi_i G_{ij} = \pi_j G_{ji}$ (Fact 3) together with $\sum_i W_{ij} = (1 - \alpha) \pi_j$ (Fact 4):

$$\sum_i \pi_i G_{ij} \left(a_i - a_j \right) = \sum_i W_{ij} a_i - a_j \sum_i W_{ij} = \sum_i W_{ji} a_i - (1 - \alpha) \pi_j a_j = \pi_j \left[\boldsymbol{G a} \right]_j - (1 - \alpha) \pi_j a_j = \pi_j c_j,$$

using symmetry $W_{ij} = W_{ji}$ so that $\sum_i W_{ji} a_i = [\boldsymbol{W a}]_j = \pi_j [\boldsymbol{G a}]_j$ (as $\boldsymbol{W} = \boldsymbol{\Pi G}$), and writing $\boldsymbol{c} \equiv \boldsymbol{G a} - (1 - \alpha) \boldsymbol{a} = -(\boldsymbol{I} - \boldsymbol{G}) \boldsymbol{a} + \alpha \boldsymbol{a}$. Substituting, and noting that the sum over j contracts the *first* index of $(\boldsymbol{I} - \boldsymbol{G})^{-1}$ (so this is genuinely a transpose contraction, since $\boldsymbol{G}$ need not be symmetric):

$$\kappa \boldsymbol{\Omega 1} = \boldsymbol{\Pi}^{-1} (\boldsymbol{I} - \boldsymbol{G})^{-\top} \boldsymbol{\Pi c} + \boldsymbol{a} - \boldsymbol{\theta}.$$

Since $\boldsymbol{G}^\top = \boldsymbol{\Pi}^{1/2} \tilde{\boldsymbol{H}} \boldsymbol{\Pi}^{-1/2}$ (using $\boldsymbol{G} = \boldsymbol{\Pi}^{-1/2} \tilde{\boldsymbol{H}} \boldsymbol{\Pi}^{1/2}$ and $\tilde{\boldsymbol{H}}^\top = \tilde{\boldsymbol{H}}$), $\boldsymbol{I} - \boldsymbol{G}^\top = \boldsymbol{\Pi}^{1/2} \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right) \boldsymbol{\Pi}^{-1/2}$, so $(\boldsymbol{I} - \boldsymbol{G})^{-\top} = \boldsymbol{\Pi}^{1/2} \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1} \boldsymbol{\Pi}^{-1/2}$ and, writing $\tilde{\boldsymbol{a}} \equiv \boldsymbol{\Pi}^{1/2} \boldsymbol{a}$,

$$\boldsymbol{\Pi}^{-1} (\boldsymbol{I} - \boldsymbol{G})^{-\top} \boldsymbol{\Pi c} = \boldsymbol{\Pi}^{-1/2} \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1} \boldsymbol{\Pi}^{1/2} \boldsymbol{c}, \qquad \boldsymbol{\Pi}^{1/2} \boldsymbol{c} = -\left(\tilde{\boldsymbol{a}} - \tilde{\boldsymbol{H}} \tilde{\boldsymbol{a}} \right) + \alpha \tilde{\boldsymbol{a}} = \left(\tilde{\boldsymbol{H}} - (1 - \alpha) \boldsymbol{I} \right) \tilde{\boldsymbol{a}},$$

using $\boldsymbol{\Pi}^{1/2} (\boldsymbol{I} - \boldsymbol{G}) \boldsymbol{a} = \boldsymbol{\Pi}^{1/2} \boldsymbol{a} - \tilde{\boldsymbol{H}} \boldsymbol{\Pi}^{1/2} \boldsymbol{a} = \tilde{\boldsymbol{a}} - \tilde{\boldsymbol{H}} \tilde{\boldsymbol{a}}$. Since $\left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1} \left(\tilde{\boldsymbol{H}} - (1 - \alpha) \boldsymbol{I} \right) = \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1} \left[-\left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right) + \alpha \boldsymbol{I} \right] = -\boldsymbol{I} + \alpha \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1}$,

$$\boldsymbol{\Pi}^{-1/2} \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1} \boldsymbol{\Pi}^{1/2} \boldsymbol{c} = \boldsymbol{\Pi}^{-1/2} \left[-\boldsymbol{I} + \alpha \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1} \right] \tilde{\boldsymbol{a}} = -\boldsymbol{a} + \alpha \boldsymbol{\Pi}^{-1/2} \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1} \boldsymbol{\Pi}^{1/2} \boldsymbol{a} = -\boldsymbol{a} + \alpha (\boldsymbol{I} -$$

using $(\boldsymbol{I} - \boldsymbol{G})^{-1} = \boldsymbol{\Pi}^{-1/2} \left(\boldsymbol{I} - \tilde{\boldsymbol{H}} \right)^{-1} \boldsymbol{\Pi}^{1/2}$ (Fact 2) in the last step. Substituting back,

$$\kappa \boldsymbol{\Omega 1} = \left[-\boldsymbol{a} + \alpha (\boldsymbol{I} - \boldsymbol{G})^{-1} \boldsymbol{a} \right] + \boldsymbol{a} - \boldsymbol{\theta} = \alpha (\boldsymbol{I} - \boldsymbol{G})^{-1} \boldsymbol{a} - \boldsymbol{\theta}, \tag{60}$$

identical in form to the symmetric case. Equations (58) and (60) are *identical in form* to the corresponding coordination-stage and influence-stage fixed-point equations in the proof of Proposition 1, with $\boldsymbol{G}$ now asymmetric. Since the algebra combining these two equations into closed form (Step 3 of that proof) uses only that $(\boldsymbol{I} - \boldsymbol{G})$, $(\boldsymbol{I} - \boldsymbol{G})^{-1}$, and $\left(\kappa (\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I} \right)^{-1}$ are all polynomials or rational functions of the single matrix $\boldsymbol{G}$ — and such functions always commute, whether or not $\boldsymbol{G}$ is symmetric — that algebra applies verbatim here, yielding

$$\boldsymbol{a} = \left(\kappa (\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I} \right)^{-1} (\boldsymbol{I} - \boldsymbol{G}) (\kappa \alpha - 1) \boldsymbol{\theta}, \qquad \boldsymbol{\Omega 1} = -\left(\kappa (\boldsymbol{I} - \boldsymbol{G})^2 - \alpha \boldsymbol{I} \right)^{-1} \left((\boldsymbol{I} - \boldsymbol{G})^2 - \alpha^2 \boldsymbol{I} \right) \boldsymbol{\theta},$$

invertibility of $\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^2-\alpha\boldsymbol{I}$ again following from $|\lambda_m|\leq 1-\alpha$ for every eigenvalue of $\boldsymbol{G}$ (Fact 2), exactly as in Proposition 1. Write $\boldsymbol{a}=\boldsymbol{M}\boldsymbol{\theta}, \boldsymbol{\Omega}\boldsymbol{1}=\boldsymbol{N}\boldsymbol{\theta}$ for these two matrices.

Part 1(b)(i): uniqueness. U_i is strictly concave in a_i (coefficient $-2\left(\alpha+\sum_j G_{ij}\right)<0$, unchanged) and, at the influence stage, strictly concave in each ω_{ki} (coefficient $-2\kappa\left(\pi_k/\pi_i\right)<0$). The same backward-induction argument as in the proof of Proposition 2 applies: each stage's best response is unique and linear, the resulting linear system in $(\boldsymbol{a},\boldsymbol{\Omega})$ has a unique solution because $\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^2-\alpha\boldsymbol{I}$ is invertible, and existence is given by the closed form just derived.

Part 1(b)(ii): convex combination. By Fact 2 with $p=L$, $\boldsymbol{M}=L\left(\boldsymbol{G}\right)=\sum_{n\geq 0}d_n\boldsymbol{G}^n$ using the same partial-fraction expansion as in the proof of Proposition 2, with the same coefficients $d_n>0$ (the expansion depends only on κ,α and the eigenvalue bound $|\lambda_m|\leq 1-\alpha$, both unchanged). Since $G_{ij}\geq 0$ (Fact 1, regardless of symmetry), $\boldsymbol{G}^n$ has nonnegative entries for every n, so $\boldsymbol{M}$ has nonnegative entries. Row sums equal one because $\boldsymbol{G}\boldsymbol{1}=(1-\alpha)\,\boldsymbol{1}$ (Fact 1) gives $\left(\kappa\left(\boldsymbol{I}-\boldsymbol{G}\right)^2-\alpha\boldsymbol{I}\right)\boldsymbol{1}=\alpha\left(\kappa\alpha-1\right)\boldsymbol{1}$ exactly as in Fact B of the proof of Proposition 2, so $\boldsymbol{M}\boldsymbol{1}=\boldsymbol{1}$. Hence each $a_i=\sum_k M_{ik}\theta_k$ is a convex combination of $\{\theta_k\}$.

Part 1(b)(iii): $\boldsymbol{\Pi}$-weighted conservativeness and zero net influence. By Fact 2, $\boldsymbol{M}=\boldsymbol{\Pi}^{-1/2}\boldsymbol{M}_H\boldsymbol{\Pi}^{1/2}$ and $\boldsymbol{N}=\boldsymbol{\Pi}^{-1/2}\boldsymbol{N}_H\boldsymbol{\Pi}^{1/2}$, where

$$\boldsymbol{M}_H\equiv\left(\kappa\left(\boldsymbol{I}-\tilde{\boldsymbol{H}}\right)^2-\alpha\boldsymbol{I}\right)^{-1}\left(\boldsymbol{I}-\tilde{\boldsymbol{H}}\right)\left(\kappa\alpha-1\right),\qquad \boldsymbol{N}_H\equiv-\left(\kappa\left(\boldsymbol{I}-\tilde{\boldsymbol{H}}\right)^2-\alpha\boldsymbol{I}\right)^{-1}\left(\left(\boldsymbol{I}-\tilde{\boldsymbol{H}}\right)^2-\alpha$$

are the same rational functions of $\tilde{\boldsymbol{H}}$ that $\boldsymbol{M},\boldsymbol{N}$ are of $\boldsymbol{G}$; since $\tilde{\boldsymbol{H}}$ is symmetric, $\boldsymbol{M}_H$ and $\boldsymbol{N}_H$ are symmetric (Fact A of the proof of Proposition 2, applied to $\tilde{\boldsymbol{H}}$). Because $\tilde{\boldsymbol{H}}\sqrt{\boldsymbol{\pi}}=(1-\alpha)\sqrt{\boldsymbol{\pi}}$, the same computation as Fact B of that proof (with $\sqrt{\boldsymbol{\pi}}$ in place of $\boldsymbol{1}$, and $\tilde{\boldsymbol{H}}$ in place of $\boldsymbol{G}$) gives

$$\boldsymbol{M}_H\sqrt{\boldsymbol{\pi}}=\sqrt{\boldsymbol{\pi}},\qquad \boldsymbol{N}_H\sqrt{\boldsymbol{\pi}}=\boldsymbol{0}. \tag{61}$$

Now, $\boldsymbol{\Pi}\boldsymbol{M}=\boldsymbol{\Pi}^{1/2}\boldsymbol{M}_H\boldsymbol{\Pi}^{1/2}$ (using $\boldsymbol{\Pi}\boldsymbol{\Pi}^{-1/2}=\boldsymbol{\Pi}^{1/2}$), and this is symmetric since $\boldsymbol{M}_H$ and $\boldsymbol{\Pi}^{1/2}$ are. For a symmetric matrix $\boldsymbol{S}$, $\boldsymbol{1}^\top\boldsymbol{S}=\left(\boldsymbol{S}\boldsymbol{1}\right)^\top$; applying this with $\boldsymbol{S}=\boldsymbol{\Pi}\boldsymbol{M}=\boldsymbol{\Pi}^{1/2}\boldsymbol{M}_H\boldsymbol{\Pi}^{1/2}$ and using $\boldsymbol{\Pi}^{1/2}\boldsymbol{1}=\sqrt{\boldsymbol{\pi}}$ and (61),

$$\boldsymbol{1}^\top\boldsymbol{\Pi}\boldsymbol{M}=\left(\boldsymbol{\Pi}^{1/2}\boldsymbol{M}_H\boldsymbol{\Pi}^{1/2}\boldsymbol{1}\right)^\top=\left(\boldsymbol{\Pi}^{1/2}\boldsymbol{M}_H\sqrt{\boldsymbol{\pi}}\right)^\top=\left(\boldsymbol{\Pi}^{1/2}\sqrt{\boldsymbol{\pi}}\right)^\top=\boldsymbol{1}^\top\boldsymbol{\Pi},$$

since $\boldsymbol{\Pi}^{1/2}\sqrt{\boldsymbol{\pi}}$ has i-th entry π_i, matching $\boldsymbol{\Pi}\boldsymbol{1}$. Therefore $\left(\boldsymbol{\Pi}\boldsymbol{1}\right)^\top\boldsymbol{a}=\left(\boldsymbol{\Pi}\boldsymbol{1}\right)^\top\boldsymbol{M}\boldsymbol{\theta}=\left(\boldsymbol{\Pi}\boldsymbol{1}\right)^\top\boldsymbol{\theta}$ for every $\boldsymbol{\theta}$, i.e., $\sum_i\pi_i a_i=\sum_i\pi_i\theta_i$. By the identical argument applied to $\boldsymbol{N}$, using $\boldsymbol{N}_H\sqrt{\boldsymbol{\pi}}=\boldsymbol{0}$ in place of $\boldsymbol{M}_H\sqrt{\boldsymbol{\pi}}=\sqrt{\boldsymbol{\pi}}$,

$$\boldsymbol{1}^\top\boldsymbol{\Pi}\boldsymbol{N}=\left(\boldsymbol{\Pi}^{1/2}\boldsymbol{N}_H\sqrt{\boldsymbol{\pi}}\right)^\top=\boldsymbol{0}^\top,$$

so $\sum_i\pi_i\left[\boldsymbol{\Omega}\boldsymbol{1}\right]_i=\left(\boldsymbol{\Pi}\boldsymbol{1}\right)^\top\boldsymbol{N}\boldsymbol{\theta}=0$ for every $\boldsymbol{\theta}$, i.e., $\sum_i\pi_i\sum_m\omega_{im}=0$.

Part 1(b)(iv): the two limits. By (57) applied with $p = L$ (recalling $\boldsymbol{a} = \boldsymbol{M\theta} = L(\boldsymbol{G})\,\boldsymbol{\theta}$),

$$\boldsymbol{a} = \sum_m L(\lambda_m)\,\tau_m \boldsymbol{u}_m^R, \qquad \tau_m = \left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{\theta}. \tag{62}$$

This is precisely the spectral representation claimed in Part 2 of the proposition; we establish it here since the two limits follow directly from it. As $\kappa \to \infty$, $L(\lambda_m) \to \alpha/(1-\lambda_m)$ for every m (dividing numerator and denominator by κ), so by (57) applied to $p(\lambda) = \alpha/(1-\lambda)$, $\boldsymbol{a} \to \sum_m \frac{\alpha}{1-\lambda_m}\tau_m \boldsymbol{u}_m^R = \alpha(\boldsymbol{I}-\boldsymbol{G})^{-1}\boldsymbol{\theta}$; similarly $\boldsymbol{\Omega}\mathbf{1} \to \mathbf{0}$, since the corresponding scaling function $\Phi(\lambda) \to 0$ for every eigenvalue. As $\kappa \downarrow 1/\alpha$: exactly as in the proof of Proposition 1, $L(\lambda_1) = L(1-\alpha) \equiv 1$ for every κ (the numerator and denominator both equal $\alpha(\kappa\alpha - 1)$ at $\lambda = 1-\alpha$), while $L(\lambda_m) \to 0$ for $m \neq 1$ (the denominator stays bounded away from zero as $\kappa \to 1/\alpha$ since $|1-\lambda_m| \neq \alpha$ for $m \neq 1$, while the numerator $\to 0$). Using $\boldsymbol{u}_1^R = \mathbf{1}$ and $\tau_1 = \left(\boldsymbol{u}_1^L\right)^\top \boldsymbol{\theta} = \left(\boldsymbol{\Pi}^{1/2}\sqrt{\boldsymbol{\pi}}\right)^\top \boldsymbol{\theta} = \sum_i \pi_i\theta_i \equiv \bar{\theta}_\pi$ (the $\boldsymbol{\Pi}$-weighted average of $\boldsymbol{\theta}$, a genuine weighted average since $\sum_i \pi_i = \left\|\sqrt{\boldsymbol{\pi}}\right\|^2 = 1$), (62) gives $\boldsymbol{a} \to \tau_1\boldsymbol{u}_1^R = \bar{\theta}_\pi\mathbf{1}$. For $\boldsymbol{\Omega}\mathbf{1} = \sum_m \Phi(\lambda_m)\,\tau_m\boldsymbol{u}_m^R$ (with $\Phi(\lambda) \equiv (1-\lambda)\,L(\lambda) - \alpha$ as in Proposition 1), $\Phi(\lambda_1) \to \alpha \cdot 1 - \alpha = 0$ and $\Phi(\lambda_m) \to -\alpha$ for $m \neq 1$, so

$$\boldsymbol{\Omega}\mathbf{1} \to \sum_{m\neq 1} (-\alpha)\,\tau_m\boldsymbol{u}_m^R = -\alpha\left(\boldsymbol{\theta} - \tau_1\boldsymbol{u}_1^R\right) = \alpha\left(\bar{\theta}_\pi\mathbf{1} - \boldsymbol{\theta}\right),$$

using the expansion $\boldsymbol{\theta} = \sum_m \tau_m\boldsymbol{u}_m^R$ from Fact 2. These are the natural $\boldsymbol{\Pi}$-weighted analogues of the symmetric-network limits $\boldsymbol{a} \to \bar{\theta}\mathbf{1}$, $\boldsymbol{\Omega}\mathbf{1} \to \alpha\left(\bar{\theta}\mathbf{1} - \boldsymbol{\theta}\right)$, consistent with the $\boldsymbol{\Pi}$-weighted conservativeness established in Part 1(b)(iii).

Part 2: spectral representation. Already established in (62) above, with $L(\cdot)$ the same function as in Proposition 4, since the eigenvalues λ_m of $\boldsymbol{G}$ and $\tilde{\boldsymbol{H}}$ coincide (Fact 2) and $L(\cdot)$ depends only on κ, α, and the eigenvalue. □

I Proof of Proposition 8

We begin by establishing the following lemma.

Lemma A.1. *Under Assumption 2,*

$$\boldsymbol{\Sigma}_\theta = \sum_m \sigma_m^2\,\boldsymbol{u}_m^R\left(\boldsymbol{u}_m^R\right)^\top, \qquad \boldsymbol{\Sigma}_a = \sum_m L(\lambda_m)^2\,\sigma_m^2\,\boldsymbol{u}_m^R\left(\boldsymbol{u}_m^R\right)^\top,$$

and, writing $\widetilde{\boldsymbol{\Sigma}}_a \equiv \boldsymbol{\Pi}^{1/2}\boldsymbol{\Sigma}_a\boldsymbol{\Pi}^{1/2}$,

$$\widetilde{\boldsymbol{\Sigma}}_a = \sum_m L(\lambda_m)^2\,\sigma_m^2\,\boldsymbol{v}_m\boldsymbol{v}_m^\top.$$

Proof. Recall that $\tilde{\boldsymbol{H}} \equiv (1-\alpha)\,\boldsymbol{\Pi}^{-1/2}\boldsymbol{H}\boldsymbol{\Pi}^{-1/2}$ denotes the symmetrized network, with $\boldsymbol{G} = \boldsymbol{\Pi}^{-1/2}\tilde{\boldsymbol{H}}\boldsymbol{\Pi}^{1/2}$. By Assumption 2, $\boldsymbol{\Sigma}_\theta = \sum_m \sigma_m^2 \boldsymbol{u}_m^R \left(\boldsymbol{u}_m^R\right)^\top$ directly. Writing $\boldsymbol{v}_m \equiv \boldsymbol{\Pi}^{1/2}\boldsymbol{u}_m^R$, which are orthonormal since $\left(\boldsymbol{u}_m^R\right)^\top \boldsymbol{\Pi}\boldsymbol{u}_p^R = \mathbf{1}_{m=p}$, it follows that

$$\boldsymbol{\Pi}^{1/2}\boldsymbol{\Sigma}_\theta\boldsymbol{\Pi}^{1/2} = \sum_m \sigma_m^2 \left(\boldsymbol{\Pi}^{1/2}\boldsymbol{u}_m^R\right)\left(\boldsymbol{\Pi}^{1/2}\boldsymbol{u}_m^R\right)^\top = \sum_m \sigma_m^2 \boldsymbol{v}_m\boldsymbol{v}_m^\top.$$

Since $\mathbb{E}\left[\boldsymbol{\theta}\right] = \mathbf{0}$ and $\boldsymbol{a} = \boldsymbol{M}\boldsymbol{\theta}$ is linear, $\mathbb{E}\left[\boldsymbol{a}\right] = \mathbf{0}$ and $\boldsymbol{\Sigma}_a = \boldsymbol{M}\boldsymbol{\Sigma}_\theta\boldsymbol{M}^\top$. Using $\boldsymbol{M} = \sum_m L\left(\lambda_m\right)\boldsymbol{u}_m^R\left(\boldsymbol{u}_m^L\right)^\top$ and biorthonormality $\left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{u}_p^R = \mathbf{1}_{m=p}$,

$$\boldsymbol{M}\boldsymbol{\Sigma}_\theta = \left(\sum_m L\left(\lambda_m\right)\boldsymbol{u}_m^R\left(\boldsymbol{u}_m^L\right)^\top\right)\left(\sum_p \sigma_p^2\boldsymbol{u}_p^R\left(\boldsymbol{u}_p^R\right)^\top\right) = \sum_m L\left(\lambda_m\right)\sigma_m^2\boldsymbol{u}_m^R\left(\boldsymbol{u}_m^R\right)^\top,$$

since the cross terms $\left(\boldsymbol{u}_m^L\right)^\top \boldsymbol{u}_p^R = \mathbf{1}_{m=p}$ collapse the double sum to $p = m$. Multiplying on the right by $\boldsymbol{M}^\top = \sum_q L\left(\lambda_q\right)\boldsymbol{u}_q^L\left(\boldsymbol{u}_q^R\right)^\top$ and using $\left(\boldsymbol{u}_m^R\right)^\top\boldsymbol{u}_q^L = \left(\boldsymbol{u}_q^L\right)^\top\boldsymbol{u}_m^R = \mathbf{1}_{q=m}$ (the same biorthonormality, with the roles of the two vectors in the inner product exchanged) collapses the sum to $q = m$:

$$\boldsymbol{\Sigma}_a = \sum_m L\left(\lambda_m\right)^2\sigma_m^2\,\boldsymbol{u}_m^R\left(\boldsymbol{u}_m^R\right)^\top.$$

Finally, $\widetilde{\boldsymbol{\Sigma}}_a = \boldsymbol{\Pi}^{1/2}\boldsymbol{\Sigma}_a\boldsymbol{\Pi}^{1/2} = \sum_m L\left(\lambda_m\right)^2\sigma_m^2\left(\boldsymbol{\Pi}^{1/2}\boldsymbol{u}_m^R\right)\left(\boldsymbol{\Pi}^{1/2}\boldsymbol{u}_m^R\right)^\top = \sum_m L\left(\lambda_m\right)^2\sigma_m^2\boldsymbol{v}_m\boldsymbol{v}_m^\top$, using $\boldsymbol{\Pi}^{1/2}\boldsymbol{u}_m^R = \boldsymbol{\Pi}^{1/2}\boldsymbol{\Pi}^{-1/2}\boldsymbol{v}_m = \boldsymbol{v}_m$. □

We now prove Proposition 8 in the paper.

Proposition. ***(Proposition 8 in the paper)*** *Under Assumption 2, the eigenvalue λ_m and eigenvector $\boldsymbol{u}_m^R$ are identified whenever $\sigma_m^2 > 0$, via:*

$$\lambda_m = L^{-1}\left(\sqrt{\frac{[\widetilde{\Sigma}_a]_{mm}}{[\widetilde{\Sigma}_\theta]_{mm}}}\right),$$

where the diagonal entries are taken after projection onto the shared eigenbasis $\{\boldsymbol{v}_m\}$ of $\widetilde{\boldsymbol{\Sigma}}_a$ and $\widetilde{\boldsymbol{\Sigma}}_\theta$.

Proof. **Recovering $\boldsymbol{v}_m$.** By Assumption 2, $\widetilde{\boldsymbol{\Sigma}}_\theta = \boldsymbol{\Pi}^{1/2}\boldsymbol{\Sigma}_\theta\boldsymbol{\Pi}^{1/2}$ is, as shown in the proof of Lemma A.1, diagonalized by $\{\boldsymbol{v}_m\}$ with diagonal entries σ_m^2. Given $\boldsymbol{\Pi}$ (equivalently $\boldsymbol{\pi}$, the vector of structural importance), $\widetilde{\boldsymbol{\Sigma}}_\theta$ is directly computable from the observable matrix $\boldsymbol{\Sigma}_\theta$, and by the spectral theorem its eigenvectors $\{\boldsymbol{v}_m\}$ — and hence $\boldsymbol{u}_m^R = \boldsymbol{\Pi}^{-1/2}\boldsymbol{v}_m$ — are recoverable directly from data, up to the same sign/rotation ambiguity as in the symmetric case (Proposition 5), which does not affect recovery of $\tilde{\boldsymbol{H}} = \sum_m \lambda_m\boldsymbol{v}_m\boldsymbol{v}_m^\top$ or $\boldsymbol{G} = \boldsymbol{\Pi}^{-1/2}\tilde{\boldsymbol{H}}\boldsymbol{\Pi}^{1/2}$ since both depend on $\boldsymbol{v}_m$ only through the sign-invariant outer product $\boldsymbol{v}_m\boldsymbol{v}_m^\top$.

Recovering λ_m. By Lemma A.1, projecting $\widetilde{\boldsymbol{\Sigma}}_a = \boldsymbol{\Pi}^{1/2}\boldsymbol{\Sigma}_a\boldsymbol{\Pi}^{1/2}$ onto the eigenbasis $\{\boldsymbol{v}_m\}$ gives diagonal entries

$$\left[\widetilde{\boldsymbol{\Sigma}}_a\right]_{mm} = \boldsymbol{v}_m^\top\widetilde{\boldsymbol{\Sigma}}_a\boldsymbol{v}_m = L\left(\lambda_m\right)^2\sigma_m^2,$$

using orthonormality of $\{\boldsymbol{v}_m\}$ to isolate the m-th term of the sum in Lemma A.1. If $\sigma_m^2 \neq 0$, divide by $\left[\widetilde{\boldsymbol{\Sigma}}_\theta\right]_{mm} = \sigma_m^2$ and take the nonnegative square root (justified since $L(\lambda) \geq 0$ for every $\lambda \in [-(1-\alpha), 1-\alpha]$ by Proposition 4, so $|L(\lambda_m)| = L(\lambda_m)$):

$$L(\lambda_m) = \sqrt{\frac{\left[\widetilde{\boldsymbol{\Sigma}}_a\right]_{mm}}{\left[\widetilde{\boldsymbol{\Sigma}}_\theta\right]_{mm}}}.$$

Since $L(\cdot)$ is strictly increasing on this range (Proposition 4), it is invertible on its image, so

$$\lambda_m = L^{-1}\left(\sqrt{\frac{\left[\widetilde{\boldsymbol{\Sigma}}_a\right]_{mm}}{\left[\widetilde{\boldsymbol{\Sigma}}_\theta\right]_{mm}}}\right),$$

uniquely determined by the observable (given $\boldsymbol{\Pi}$) quantities $\left[\widetilde{\boldsymbol{\Sigma}}_a\right]_{mm}$ and $\left[\widetilde{\boldsymbol{\Sigma}}_\theta\right]_{mm}$. Together with the recovered $\boldsymbol{v}_m$, this identifies $(\lambda_m, \boldsymbol{v}_m)$ and hence the term $\lambda_m \boldsymbol{v}_m \boldsymbol{v}_m^\top$ of $\tilde{\boldsymbol{H}} = \sum_m \lambda_m \boldsymbol{v}_m \boldsymbol{v}_m^\top$, and thus of $\boldsymbol{G} = \boldsymbol{\Pi}^{-1/2} \tilde{\boldsymbol{H}} \boldsymbol{\Pi}^{1/2}$, whenever $\sigma_m^2 \neq 0$.

Failure of identification when $\sigma_m^2 = 0$. If $\sigma_m^2 = 0$, Lemma A.1 gives $\left[\widetilde{\boldsymbol{\Sigma}}_a\right]_{mm} = L(\lambda_m)^2 \cdot 0 = 0$ for *every* value of λ_m: the observed moment carries no information about λ_m, exactly as in the symmetric case (Proposition 5). Consequently $\tilde{\boldsymbol{H}}$ (and $\boldsymbol{G}$) is identified only on the subspace spanned by $\{\boldsymbol{v}_m : \sigma_m^2 > 0\}$. $\square$

J Economic Versus Security Ties

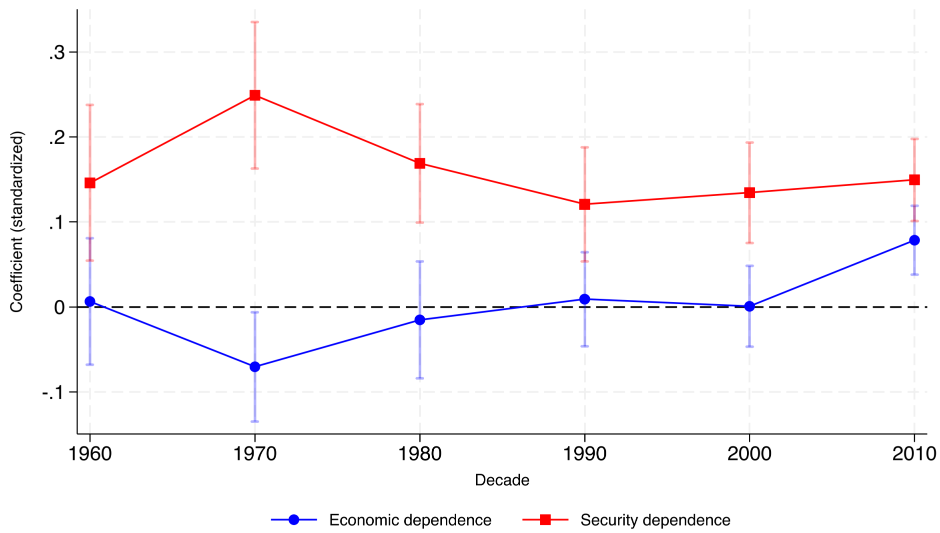
In this section of the Online Appendix, we provide additional empirical evidence on the relative importance of economic and security dependence for geopolitical network connections over time. The Formal Bilateral Influence Capacity (FBIC) dataset that we used to estimate the structural importance of countries ($\pi_{id} = GDP_{id}^{\beta}$) disaggregates overall dependence into separate measures of economic and security dependence. Using these disaggregated measures, we estimate the relationship between our geopolitical network connections (G_{ijd}), economic dependence (E_{ijd}) and security dependence (S_{ijd}) for each decade separately:

$$G_{ijd} = b_{1d}E_{ijd} + b_{2d}S_{ijd} + \alpha_{id} + \gamma_{jd} + u_{ijd}$$

where α_{id} and γ_{jd} are receiver-decade and sender-decade fixed effects, respectively, and u_{ijd} is a stochastic error.

In Figure A1, we display the estimated coefficients on economic dependence (b_{1d}) and security dependence (b_{2d}) over time. Again the inclusion of the receiver-decade and sender-decade fixed effects implies that these estimated coefficients are identified from bilateral variation. In the early decades of our sample period, we find that security dependence dominates economic dependence, which is consistent with the heightened geopolitical tension in these peak years of the Cold War. Over the course of our sample period, economic dependence becomes more important relative to security dependence, which is consistent with the globalization of the world economy increasing the relative importance of economic relationships between countries.

Figure A1: Economic vs. Security Dependence and Geopolitical Ties Over Time

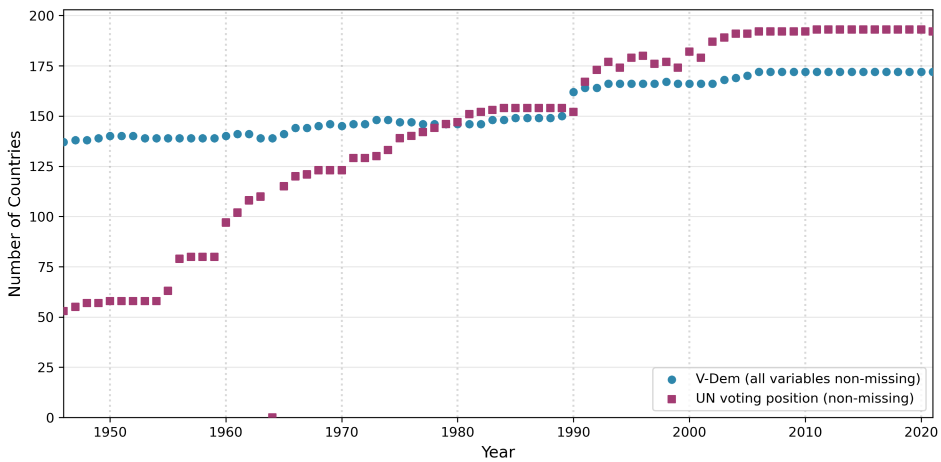


Note: Estimated coefficients from a regression of inferred geopolitical connections on both economic dependence and security dependence; regression estimated separately by decade; colored markers correspond to the point estimates; vertical bars correspond to the 95 percent confidence intervals.

K Additional Figures and Tables

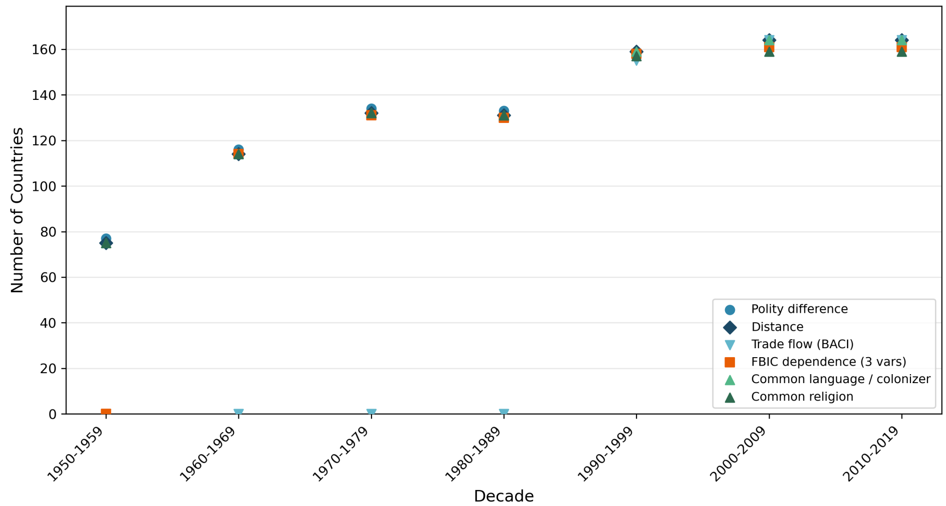
This section of the Online Appendix reports additional figures and tables that are discussed in the paper. Figures A2 and A3 display the country coverage of the sample for the main datasets used in our empirical analysis. Figure A4 displays the results of the two placebo exercises for the geopolitical network discussed in Section 3.4 of the paper: the permutation test and the bootstrap of the residuals. Figures A5 and A6 display the results of the counterfactual discussed in Section 5 of the paper, in which we change the domestic interests of China and the United States.

Figure A2: Country Coverage in the Varieties of Democracy (V-Dem) and UNGA Datasets



Note: Number of countries with non-missing data in the Varieties of Democracy (V-Dem) and UNGA datasets.

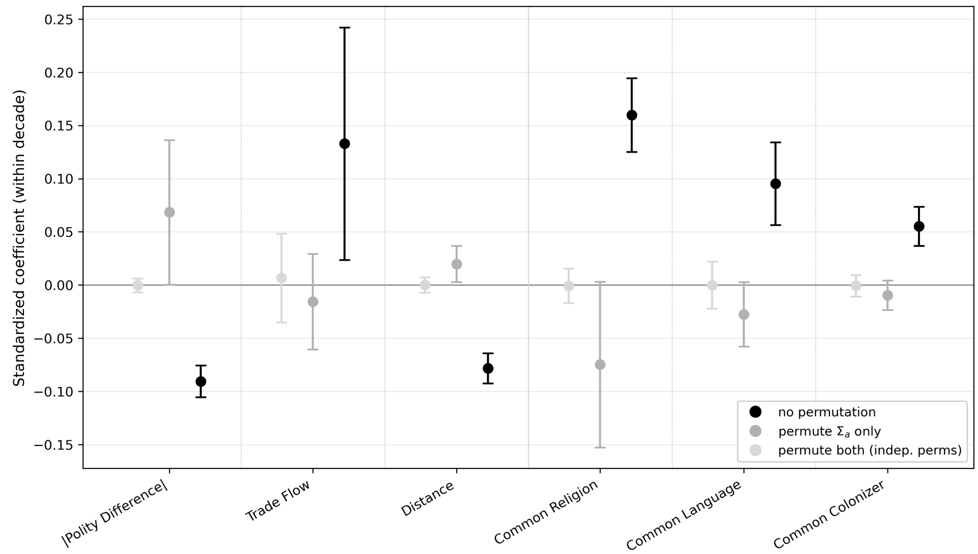
Figure A3: Country Coverage for Bilateral Proxies for Geopolitical Links



Note: Number of countries with non-missing data for the bilateral proxies for geopolitical links.

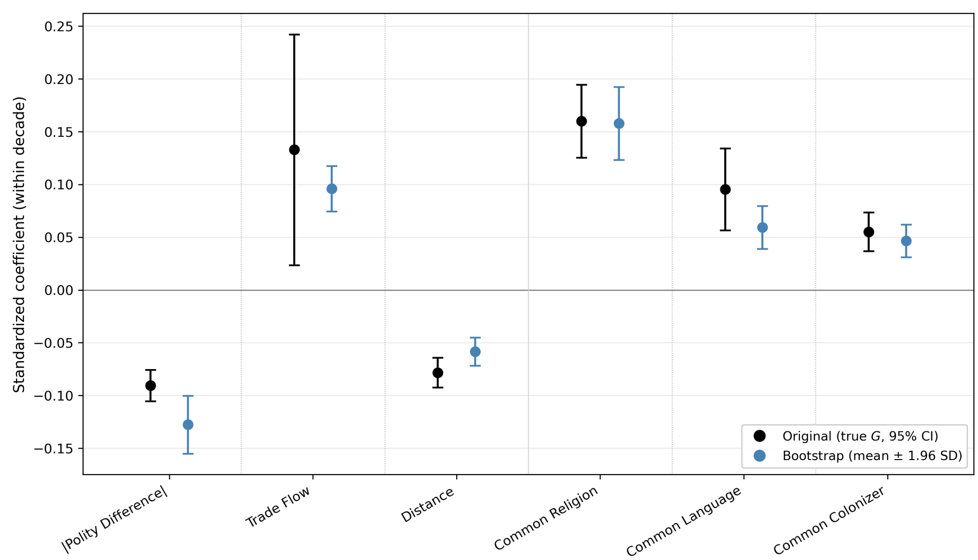
Figure A4: Robustness of $\boldsymbol{G}_d$

(a) Permutation of matrices



Note: Permutation placebo exercise. Recall that each decade's network $\boldsymbol{G}_d$ is recovered by jointly diagonalizing two cross-country covariance matrices—of preferences ($\Sigma_{\theta,d}$) and of actions ($\Sigma_{a,d}$)—weighted by economic size Π_d. To test whether the validation results require the true country-to-country correspondence among these inputs, we apply a random permutation to the countries of one covariance matrix before recovery, leaving the other matrix and Π_d at their true labels. The coefficients from two variants are shown—permuting $\Sigma_{a,d}$ only and permuting both $\Sigma_{a,d}$ and $\Sigma_{\theta,d}$ independently—alongside the unpermuted network. For each draw we re-recover $\boldsymbol{G}_d$, and re-run the specification of Table 2. Black points and lines mark the unpermuted (true) coefficient/CI; the gray markers plot the mean of the validation coefficient across permutations, and its bar spans ±1.96 times the standard deviation of those point estimates across permutations. The true coefficients are large and precisely estimated for every covariate, whereas the permuted coefficients are generally statistically indistinguishable from zero.

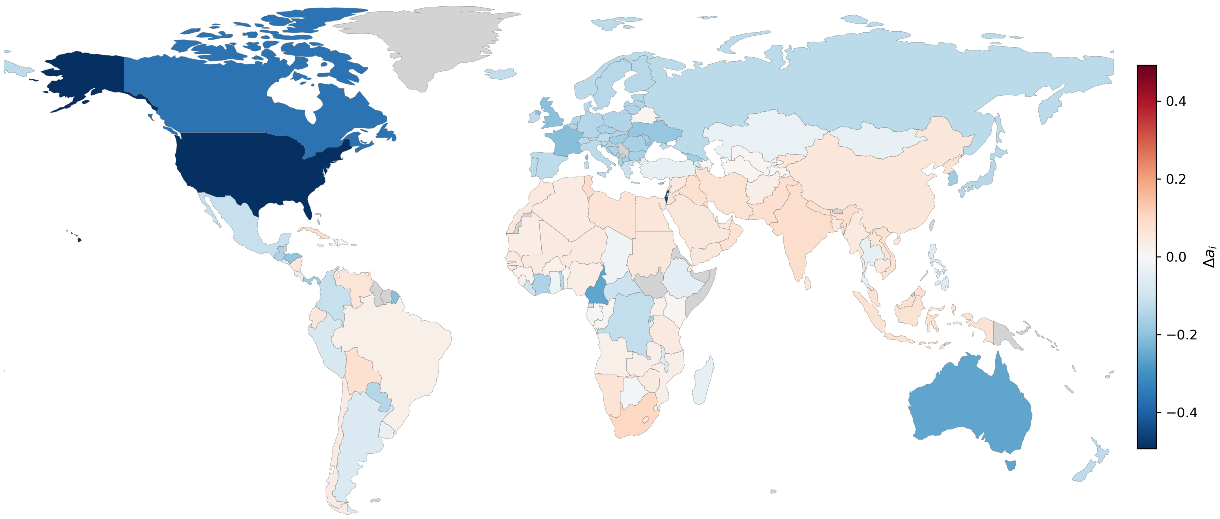
(b) Bootstrap of residuals



Note: Residual bootstrap. The network is recovered from only the leading joint modes of the preference and action covariance matrices; the remaining variation is unused. To test whether the validation results depend on this unused component, we split each Π_d-weighted covariance into its retained low-rank signal plus a residual, and form bootstrap draws that hold the signal fixed while replacing the residual with a random matrix of identical eigenvalue spectrum but randomly rotated eigenvectors—that is, noise of the same overall magnitude but arbitrary country loadings. For each of 100 draws we add the resampled residual back, re-recover $\boldsymbol{G}_d$, and re-run the validation regressions as in Table 2. The bootstrap coefficients cluster tightly around the true value, with the same sign and comparable magnitude, indicating that the findings are driven by the retained low-rank signal rather than the discarded residual.

Figure A5: Equilibrium responses to US with Chinese preferences

(a) Map of action changes



(b) Histogram of action changes

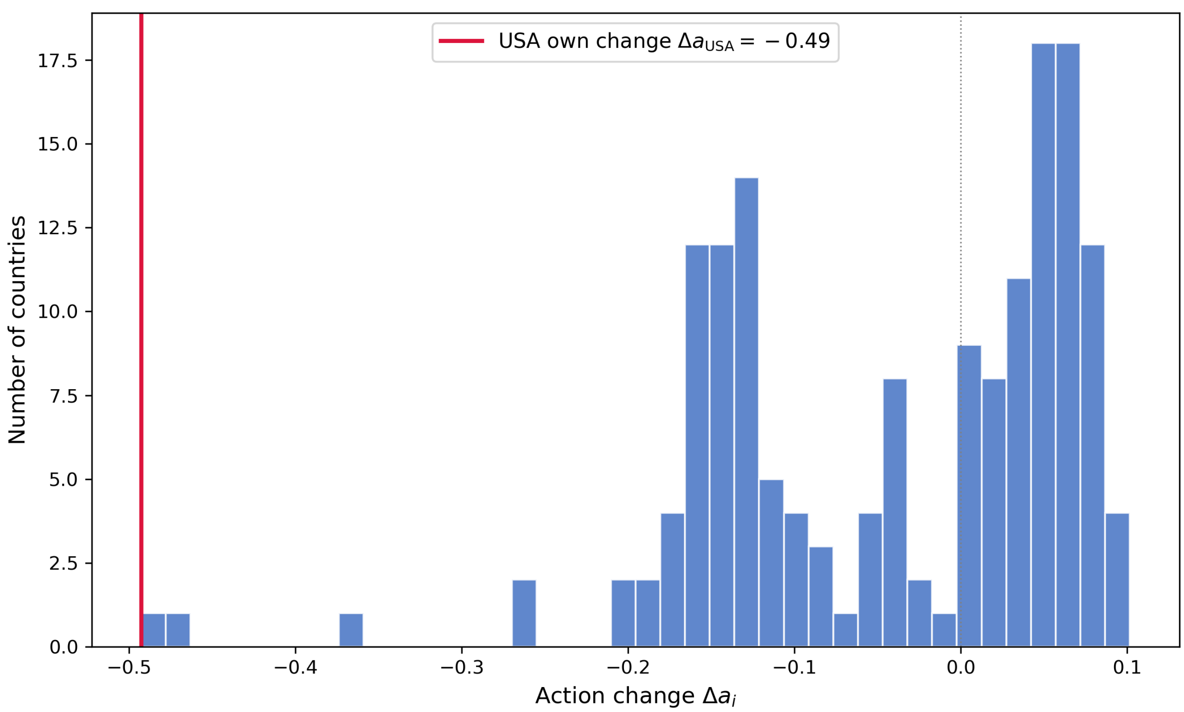
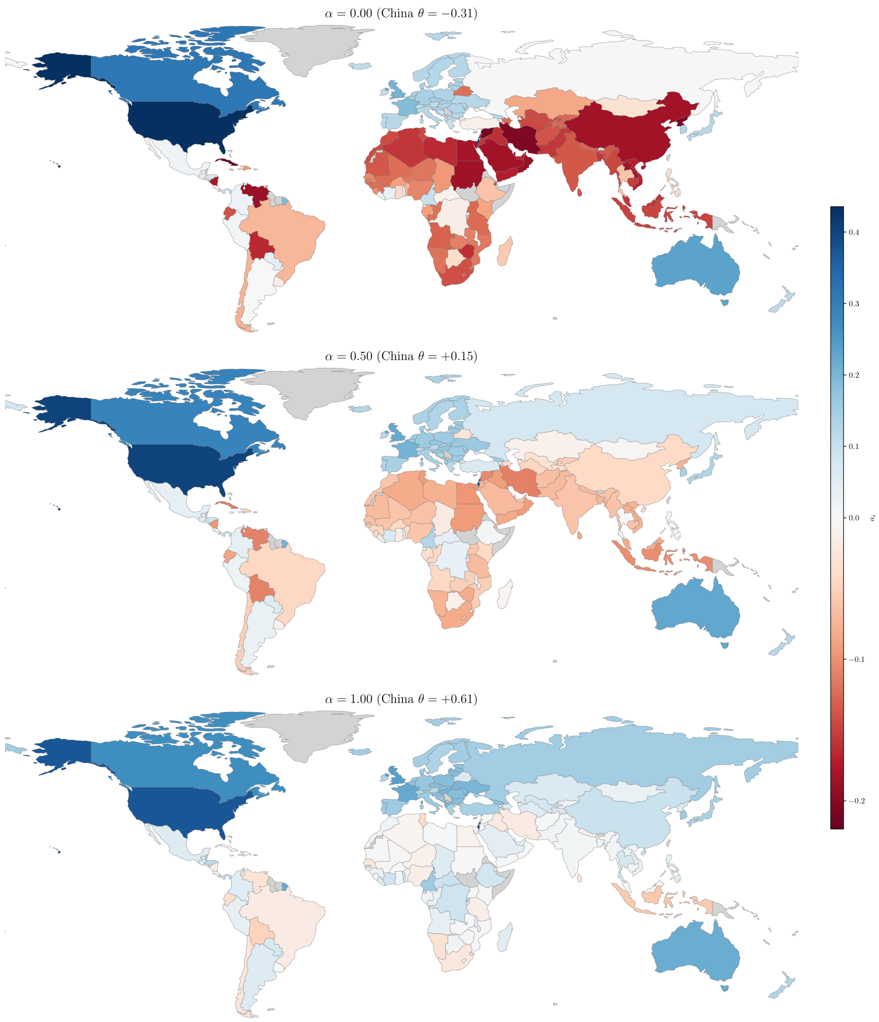


Figure A6: Equilibrium Actions Under Different Chinese Preferences in the 2010s



Note: Using our inferred geopolitical network ($\boldsymbol{G}$) in the 2010s, we evaluate countries' geopolitical actions ($\boldsymbol{a}$) in response to this international issue ($\boldsymbol{\theta}$) under three different scenarios. The top panel corresponds to the case in which the USA and China have their original preferences as given by the top non-Perron eigenmode in the 2010s network. The second panel corresponds to the case in which China lies at the midpoint between its original domestic interest and those of the United States. The third panel presents the case in which China and the USA have the same domestic interests.