

A Network Approach to Geopolitics^{*}

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Abstract

We develop a tractable framework for analyzing geopolitics as a network of coordination and strategic influence. Countries choose geopolitical actions that trade off domestic interests against incentives to align with connected countries. They can exert costly bilateral influence to affect one another's actions. We characterize equilibrium actions and bilateral influence in closed form. Cross-country differences in domestic interests decompose into latent geopolitical cleavages, each of which the network transmits into geopolitical actions independently of the others. The cross-country covariance matrices of domestic interests and of geopolitical actions identify the cleavages and their importance, from which we recover the network. We implement the framework using data on United Nations voting and democratic orientation. The inferred network is well approximated by a few dimensions, aligns with external measures of geopolitical ties, and recovers well-known historical cleavages. We undertake counterfactuals for domestic interests and network connections and quantify three notions of geopolitical power.

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1 Introduction

Geopolitics is a network phenomenon. What a country does in international affairs depends not only on its own interests but also on what its partners do. A sanction works only if others enforce it; a military threat is credible only if allies stand behind it; and a diplomatic initiative succeeds or fails depending on whether pivotal states align, hedge, or defect. The defining confrontations of the Cold War, from the Berlin Airlift and the Korean and Vietnam Wars to the Cuban Missile Crisis, turned on how the two blocs of countries lined up, not only on the superpowers themselves. Countries anticipate this and invest in shaping one another's choices, and that influence rarely stops at its immediate target. US export controls on advanced chipmaking equipment, for example, depend for their effect on complementary restrictions by the Netherlands and Japan, home to key suppliers.

This interdependence creates a fundamental challenge for anyone who studies geopolitics: what countries do is an equilibrium outcome, not a direct expression of what they would choose if they acted alone. When the UN General Assembly voted in March 2022 to condemn Russia's invasion of Ukraine, 141 countries voted in favor and 35 abstained, including India and South Africa. India's abstention may reflect its domestic interests, the pull of its long-standing ties to Russia, or both, and the vote by itself cannot distinguish among these.

The network that generates this pull is itself hard to observe, because geopolitical influence travels through many channels at once. The response to Russia's invasion combined financial sanctions, export controls, military assistance, the reallocation of energy supplies, and diplomatic pressure, deployed together by the US, the European Union, and their partners. Channels can also pull in opposite directions. In the maritime disputes of the South China Sea, regional states weigh economic ties to China against security ties to the US, and whether they coordinate their positions or fragment depends on both.

In this paper, we develop a tractable framework for analyzing geopolitics from a network perspective, allowing us to separate these forces and recover the network as a whole. Countries choose geopolitical actions that balance their domestic interests against the actions of the countries they are connected to. Before acting, they make costly investments to influence one another, either directly or through third countries. The central implication is that the network filters disagreement unevenly: disagreements that run between blocs pass through into actions, whereas disagreements among close partners are dampened by the incentive to coordinate. Under conditions we characterize, comparing how strongly each dimension of disagreement in domestic interests shows up in actions reveals the blocs, how much each matters, and hence the network itself. As such, the framework delivers three results: a closed-form equilibrium of actions and bilateral influence, a decomposition of that equilibrium along the network's geopolitical cleavages, and identification of the network from the covariances of domestic interests and actions.

Because the network is inferred from how domestic interests translate into actions, rather than

from any particular tie, it does not require specifying the channels of influence in advance. The recovered network aggregates the many channels that shape the actions we observe, including those that are hard to measure directly, together with the ways they interact. The approach requires only two empirical inputs, a measure of what countries want and a measure of what they do. We demonstrate the empirical implementation of our framework using data that are readily available and frequently used in existing research.

This network approach expands our understanding of geopolitics. Power becomes positional as well as material: a country can matter because of its network location as well as because of its size: if it brokers between blocs, controls a chokepoint, or swings between sides. Whether the network is unipolar, bipolar or non-polar depends on how coordination is organized across the system rather than a ranking of national capabilities. Which international issues lead to a polarization instead of a convergence in actions depends less on how much countries disagree than on whether their disagreements line up with cleavages in the network: two issues with the same spread of domestic interests can therefore produce very different degrees of division in actions.

Our theoretical model considers a world that consists of N countries connected by a geopolitical network G . This geopolitical network captures bilateral considerations for coordination in foreign policy, which may reflect alliances, economic dependence, or other strategic linkages. For a given international issue, each country i has a domestic interest θ_i and chooses a geopolitical action a_i . The geopolitical action is the outcome of a network “beauty-contest” coordination game: countries balance alignment with their own interest and alignment with the actions of their network neighbors. Prior to the coordination game, each country engages in costly bilateral strategic influence to shift other countries’ incentives for geopolitical actions. Country i may strategically influence j not because i wants to coordinate with j , but because doing so indirectly influences a third country k —who coordinates with j either directly or indirectly through the network—with whom i seeks alignment. The global equilibrium is a fixed point of these bilateral choices, where each country internalizes the full cascading impact of its influence on geopolitical actions across the geopolitical network.

We provide a closed-form characterization of the equilibrium profile of geopolitical actions and bilateral influence. This allows us to study how changes in domestic interests or network connections alter countries’ incentives to influence one another, and how the resulting adjustments in influence shape geopolitical actions. The model admits a unique equilibrium, in which each country’s equilibrium action is a convex combination of all countries’ interests. This equilibrium is “conservative”: across countries, the average geopolitical actions and average domestic interests coincide; and net geopolitical influence sums to zero across countries. These properties formalize a basic feature of multilateral influence: strategic influence reallocates incentives across countries, but does not create net influence in aggregate.

We next characterize why some international issues generate greater disagreement in geopolit-

ical actions than others. The extent of disagreement depends on how differences in domestic interests are distributed across the network. Disagreements that divide one bloc from another persist in geopolitical actions, whereas disagreements among close network partners are attenuated by coordination. Thus, two issues with the same overall dispersion of domestic interests can generate different degrees of geopolitical polarization. We characterize this relationship by decomposing domestic interests along the network’s eigenvectors, which we interpret as geopolitical cleavages. The equilibrium transmits each cleavage independently of the others (a property we call “spectral decoupling”): the profile of actions is the sum of the cleavages in domestic interests, each scaled by a factor that depends only on its own eigenvalue. Cleavages with larger eigenvalues, those that divide one bloc from another, pass through more fully into geopolitical actions. Strategic influence attenuates disagreement relative to coordination alone, with cheaper influence leading to greater convergence.

To use our framework to study counterfactual changes in domestic interests and network connections, we need to recover the geopolitical network. We provide conditions under which this unobserved geopolitical network can be identified from observed domestic interests and geopolitical actions. We consider an environment in which countries repeatedly play the coordination and strategic influence game over a fixed network, with domestic interests for each international issue drawn from a common distribution. We show that geopolitical actions are a linear function of domestic interests that depends on the geopolitical network, which imposes a tight relationship between the covariance matrices of geopolitical actions and domestic interests. If the covariance matrix of domestic interests is non-singular, the geopolitical network can be recovered from this relationship using matrix inversion. In contrast, if this covariance matrix is singular, the geopolitical network can still be recovered under an additional assumption of graph stationarity. This additional assumption implies that domestic interests and the geopolitical network share the same eigenbasis and hence have the same principal components.

This theoretical identification result motivates an empirical implementation based on estimating the joint principal components of the covariance matrices of θ and α . We illustrate our framework quantitatively with two widely used sources of cross-country political data. We measure domestic interests (Σ_θ) using the Varieties of Democracy (V-Dem) data, aggregating 37 variables that measure domestic interests and political institutions. We measure geopolitical actions (α) using UN General Assembly votes.¹ We treat the geopolitical network (G) as evolving slowly and estimate it using the joint principal components of the cross-country covariance matrices of domestic interests

¹Our paper is related to empirical research in economics and political science that uses the UN voting data to measure geopolitical alignment, including Signorino and Ritter (1999), Voeten (2000) and Bailey et al. (2017). Our use of the V-Dem data follows Coppedge et al. (2011) and Teorell et al. (2019), and our use of bilateral state visits data follows Kinne (2014). Russett (1966) and Kim and Russett (1996) extract a small number of dimensions from the UN voting data by factor analysis. UN ideal points are widely used as measures of state preferences (Bailey et al. (2017)). Our framework interprets them as actions that combine domestic interests with the pull of the network, and provides a way to separate the two.

and geopolitical actions for each decade. Although the assumption of the same principal components does not perfectly reconstruct the observed covariance matrices, we show that it provides a reasonable approximation to the data. The resulting decomposition is well approximated by a few dimensions: the top five components account for 97 and 96 percent of the covariance matrices of geopolitical actions and domestic interests, respectively. Geopolitical actions and domestic interests load on the joint principal components differently and the difference is informative about the geopolitical network. Under our identification result, the variance of geopolitical actions relative to the variance of domestic interests along a cleavage is increasing in that cleavage's importance (i.e., its eigenvalue) in the geopolitical network.

The eigenvector with the largest eigenvalue captures global consensus among all countries. The eigenvector with the second-largest eigenvalue captures the dominant source of division in the geopolitical network and tracks major shifts in the structure of international politics. In the 1960s, it aligns closely with the Cold War division between the Western and Soviet blocs. During the 1970s, however, the leading cleavage shifts to a divide between the major powers—including the US and the Soviet Union—and the developing world, reflecting the growing political salience of the “Third World.” Notably, China aligns with the major-power bloc rather than the developing world, despite its self-identification as a leader of the Third World. In the 1980s, the leading cleavage returns to the East–West divide before shifting sharply after the collapse of the Soviet Union. Since the 1990s, it has aligned with a geopolitical cleavage in which the US and Western Europe occupy one pole, China the other, and many countries in the Global South lie in between. Overall, these patterns do not simply reflect the familiar East–West split in UN votes: they capture the fact that votes diverge along this dimension by more, relative to the distribution of domestic interests, than along any other.

Next, we use the estimated eigenvectors and eigenvalues to construct an inferred geopolitical network for each decade. Our inferred dyadic links are systematically related to a range of empirical proxies for geopolitical connections between countries used in the economics and political science literature, including bilateral trade, geographical distance, and indicators of common religion, language and historical colonizers. Our model's predictions for strategic influence between countries are also related to separate data on bilateral state visits, which are not used in the quantification of our model. Our inferred geopolitical network is asymmetric, with the degree of asymmetry estimated from data on bilateral influence—countries' outward connections (how much they care about other countries) can differ substantially from their inward connections (how much other countries care about them). We find that geopolitical networks are typically stable over time, but large-scale geopolitical realignments such as the fall of the Iron Curtain can occur. Our estimates imply that the cascade of strategic influence through indirect connections in the network makes a substantial contribution to the overall strategic influence of countries.

Our framework provides a tractable platform to examine counterfactual questions that run

through core debates about geopolitics. Would a China with US-style domestic institutions align with the West? Do alliances extend a great power’s reach or tie its hands? Which countries’ absence would most change how the rest of the world lines up? Answering these questions yields three notions of power: autonomy, how much of a change in a country’s interests shows up in its own actions; control, how much other countries’ actions move when its own action moves; and the ability to shape the game, how much everyone else’s actions would change without it. We find that the US has the highest autonomy, whereas China has the highest control, and the two superpowers have the largest effects on the actions of other countries when removed from the network. A convergence in China’s domestic interests toward those of the US would pass through only partly into its own actions, by less than for the US, because its network partners anchor it. The change that does occur moves partners such as Syria, North Korea, and Iran, whose own interests are unchanged. Weakening US ties with its European NATO allies would make the US freer to act on its own interests but less able to move others, consistent with the view that alliances that constrain the US also extend its reach. Countries such as Iran and Russia can affect the equilibrium configuration of actions by more than implied by their size, because of their positions in the network.

Our paper is related to several strands of existing research. First, our work is related to recent research on geoeconomics within the economics literature. Much of this existing research examines one channel of influence at a time.² Trade dependence can be converted into leverage (Hirschman 1945; Clayton et al. 2026a; Liu and Yang 2026) and is closely tied to political alignment (Broner et al. 2024; Kleinman et al. 2024). Sanctions, including those that exploit control over payment networks, turn economic ties into instruments of coercion (Hufbauer et al. 1990; Farrell and Newman 2019; Cipriani et al. 2023; Clayton et al. 2026b). Currency dominance and state-directed lending confer influence of their own (Eichengreen 2011; Horn et al. 2021; Clayton et al. 2026b), as do foreign aid (Kuziemko and Werker 2006) and military alliances (Morrow 1991; Snyder 1997; Fearon and Hansen 2018). Recent work has begun to study how pairs of channels interact, including trade and war (Martin et al. 2008), trade and military alliances (Jackson and Nei 2015), trade and financial sanctions (Bayer et al. 2024), and military power and borrowing costs (Pflueger and Yared 2024). Our approach is holistic: we estimate the geopolitical network and infer bilateral influence from countries’ observed actions, so we recover how countries influence one another through the many channels that shape those actions, rather than through any single instrument.³

Second, we contribute to research on networks in economics, including Jackson and Wolinsky

²For recent reviews, see Clayton et al. (2025) and Mohr and Trebesch (2025); Mohr and Trebesch (2025) calls for a more holistic view of the full instruments of geopolitical influence and how they interact.

³Existing research typically assumes a single hegemon or models competition between dual hegemons. In contrast, we model a global geopolitical network, in which each country can strategically influence any other country, both directly and indirectly. Rather than modeling the specific economic instruments through which a hegemon exerts power (Clayton et al. 2026b) or relating political alignment to trade exposure (Kleinman et al. 2024), we recover the network of geopolitical ties from countries’ actions as a whole and study how domestic interests are transmitted through it.

(1996), Jackson (2008), Acemoglu et al. (2012), Chaney (2014), Galeotti et al. (2020), Goyal (2023), and Elliott et al. (2022). Our model builds on the theory literature on network games and beauty contests, including Morris and Shin (2002), Galeotti et al. (2010), Bramoullé et al. (2014), and Allen et al. (2006). We extend a network coordination game to include an influence stage of endogenous strategic investments that distort coordination incentives. Galeotti et al. (2020) also use the eigenvectors of the network to characterize how a planner targets changes in incentives. In our setting, each country chooses bilateral influence to advance its own objectives, internalizing how its influence propagates through the actions of others. With quadratic influence costs, the equilibrium is in closed form, and both influence and actions adjust to counterfactual changes.

Third, our empirical approach contributes to research estimating networks using observational data, including Jackson and Nei (2015), Graham (2017), König et al. (2017), Mele (2017), Trebbi and Weese (2019), de Paula (2020), Manresa (2013), Blume et al. (2015), Canen et al. (2023), Francois et al. (2023), and Gopinath et al. (2025). We provide conditions under which the cross-country covariances of domestic interests and geopolitical actions identify the geopolitical network. Rather than estimating individual link weights directly, our approach recovers the network through its eigenvectors and eigenvalues, using the joint spectral structure of these two covariance matrices. Our third notion of power, the effect of removing a country from the network, parallels the key-player intercentrality of Ballester et al. (2006).

Fourth, our research connects with antecedents in political economy. The notion of international relations as a network accords with Hoff and Ward (2004) and Hafner-Burton et al. (2009). The trade-off between a country’s own interests and the actions of its allies is a theme in alliance theory following Morrow (1991) and Snyder (1997). The asymmetry of power is emphasized in Hirschman (1945) and Keohane and Nye (1977). The role of chokepoints is stressed in Farrell and Newman (2019). The property that power is positional and issue-dependent is supported by Baldwin (1985) and Barnett and Duvall (2005). The presence of costly investments in influence follows Grossman and Helpman (1994), Antràs and Padró i Miquel (2011), Berger et al. (2013), Antràs and Padró i Miquel (2023), and the empirical lobbying literature reviewed in Bombardini and Trebbi (2020). The coordination of independent decisions echoes Schelling (1960) and the presence of spheres of influence aligns with Lake (2009). Our main contribution relative to this research is to develop a tractable theoretical model of a network coordination game with strategic influence and show how it can be used to identify the unobserved geopolitical network. We use a low-dimensional representation of this network in terms of its eigenvectors and eigenvalues to identify latent geopolitical cleavages and undertake counterfactuals.

The remainder of the paper is structured as follows. Section 2 sets up the model and derives our main theoretical results regarding equilibrium influence, spectral decoupling, and identification. Section 3 introduces our data and implements our methodology empirically. Section 4 provides fur-

ther evidence on the key empirical features of the geopolitical network. Section 5 undertakes counterfactuals to explore the role of domestic interests and network connections in shaping geopolitical actions. Section 6 summarizes our conclusions.

2 Theory

We consider a world that consists of N countries indexed by $i, j \in \{1, \dots, N\}$. The countries are connected by a weighted graph that can be represented by the matrix $\mathbf{G}$, where we use boldface throughout the paper to denote vectors (lowercase) and matrices (uppercase). The elements of this matrix $G_{ij} \in \mathbb{R}_{\geq 0}$ capture the magnitude of the political connections between countries i and j . The representative domestic policymaker in each country (the national government) has a domestic interest $\theta_i \in \mathbb{R}$ and can take a geopolitical action $a_i \in \mathbb{R}$.

We interpret $\mathbf{G}$ as the geopolitical network. It captures bilateral considerations for coordination in foreign policy. It could reflect strategic alliances (e.g., G_{ij} is large between allies), economic dependence (e.g., G_{ij} is increasing in bilateral trade), or other considerations for coordination (e.g., the US and Russia deliberating over a nuclear deal). We view $\mathbf{G}$ as reflecting medium to long-run geopolitical relationships between countries.

We interpret $\boldsymbol{\theta}$ as the heterogeneous domestic interests of countries over a specific international issue. For example, countries around the world may react differently to the capture of President Maduro of Venezuela or President Trump's threat of invading Greenland. We begin by analyzing the equilibrium for a given $\boldsymbol{\theta}$. Later we allow for multiple international issues, each of which corresponds to different $\boldsymbol{\theta}$, by assuming that $\boldsymbol{\theta}$ is drawn from a distribution across issues.

We interpret $\mathbf{a}$ as the geopolitical actions that countries adopt in response to the international issue. These geopolitical actions are influenced not only by domestic interests but also by global considerations of coordinating geopolitical actions with other nations. We model the choice of geopolitical actions $\mathbf{a}$ as the solution of a beauty-contest coordination game over the network, where each country i chooses its geopolitical action (a_i) to balance its alignment with its own domestic interests (θ_i) and the geopolitical actions of its network neighbors (a_j for $j \neq i$).

Prior to the coordination game, each country i can engage in costly strategic influence (ω_{ij}) to alter any other country j 's incentives to choose geopolitical actions. The game proceeds in two stages for each issue $\boldsymbol{\theta}$. In the influence stage, countries observe $\boldsymbol{\theta}$ and choose bilateral strategic influence. In the coordination stage, countries choose geopolitical actions, taking bilateral strategic influence as given. We characterize how bilateral strategic influence ($\boldsymbol{\Omega} \equiv [\omega_{ij}]$) and geopolitical actions ($\mathbf{a}$) depend on the geopolitical network ($\mathbf{G}$) and domestic interests $\boldsymbol{\theta}$. The game has perfect information: each country knows the domestic interests ($\boldsymbol{\theta}$), strategic influence ($\boldsymbol{\Omega}$) and geopolitical network connections ($\mathbf{G}$) for all countries.

In what follows, we first introduce a special case of the model with only the coordination stage

without any bilateral strategic influence. We then develop the full model with both the influence and coordination stages. In our baseline model, we assume for expositional simplicity that the geopolitical network is undirected ($\mathbf{G}$ is symmetric) and regular (every row of $\mathbf{G}$ sums to the same value). We denote the common row sum of $\mathbf{G}$ by $1 - \alpha$, where $\alpha \in (0, 1)$, and we assume throughout that the geopolitical network is strongly connected.⁴ In a later subsection, we extend our baseline model to introduce asymmetries in the geopolitical network.

2.1 Model without Strategic Influence

In the special case of the model without strategic influence, countries play a beauty-contest coordination game over the geopolitical network. The representative policymaker in each country i chooses its geopolitical action a_i to minimize its misalignment with its domestic interest (θ_i) and the geopolitical actions of its network neighbors (a_j), taking the geopolitical actions chosen by other countries as given. We assume that country i solves the following problem:

$$\max_{a_i} \left\{ - \left[\alpha (a_i - \theta_i)^2 + \sum_{j=1}^N G_{ij} (a_i - a_j)^2 \right] \right\}, \quad (1)$$

where the bilateral geopolitical links (G_{ij}) capture the extent to which countries i and j want their geopolitical actions to align.⁵

Taking the first-order condition with respect to a_i , we obtain country i 's best-response function:

$$a_i = \alpha \theta_i + \sum_{j=1}^N G_{ij} a_j. \quad (2)$$

Noting that $\alpha + \sum_j G_{ij} = 1$, each country's best response is a weighted average of its own interest (θ_i) and the geopolitical actions of its network neighbors (a_j). In vector form, this best-response function can be written as:

$$\mathbf{a} = \alpha \boldsymbol{\theta} + \mathbf{G} \mathbf{a}. \quad (3)$$

The equilibrium in geopolitical actions corresponds to the fixed point in which each country's best-response is consistent with the best-responses of all other countries:

$$\mathbf{a} \equiv \alpha (\mathbf{I} - \mathbf{G})^{-1} \boldsymbol{\theta}. \quad (4)$$

2.2 Model with Strategic Influence

The full model includes both the influence and coordination stages of the game. In the influence stage, each country i can choose to engage in costly bilateral strategic influence to change the incentives of any country j to choose geopolitical actions. After strategic influence has occurred, the game

⁴That is, for any i, j , there exists $k > 0$ where $[\mathbf{G}^k]_{ij} > 0$.

⁵Although the summation in the second term includes the own country $i = j$, the own-country entries in this second term are necessarily equal to zero (since $a_i = a_j$ for $i = j$).

moves on to the coordination stage, in which each country i chooses its geopolitical action a_i , taking as given the geopolitical actions of its network neighbors (a_j) and bilateral strategic influence (ω_{ij}). We assume that country i 's payoff takes the following form:

$$\begin{aligned}
U_i(\mathbf{a}, \mathbf{\Omega}) = & - \left[\alpha (a_i - \theta_i)^2 + \sum_{j=1}^N G_{ij} (a_i - a_j)^2 \right] \\
& - \kappa \sum_{k=1}^N \omega_{ki}^2 \\
& + 2 \sum_{m=1}^N \omega_{im} (a_i - \theta_i),
\end{aligned} \tag{5}$$

where $\omega_{ki} \in \mathbb{R}$ is the influence exerted by country i on country k —the first subscript indicates the country receiving influence and the second subscript indicates the country exerting influence—and $\mathbf{\Omega} \equiv [\omega_{ki}]$ is the matrix of bilateral influence.

The first line in (5) is country i 's payoff from the network coordination game. The second line is country i 's convex cost of exerting strategic influence. The parameter $\kappa > 1/\alpha$ is a cost shifter for strategic influence. Strategic influence can be either positive or negative. In both cases, the costs of strategic influence increase quadratically with its distance from zero. The third line is the payoff from receiving foreign influence ω_{im} exerted by each country m . Receiving positive net strategic influence ($\sum_{m=1}^N \omega_{im} > 0$) raises the marginal benefit of choosing an action a_i greater than domestic interests θ_i . Conversely, receiving negative net strategic influence ($\sum_{m=1}^N \omega_{im} < 0$) increases the incentive to choose an action a_i lower than domestic interests θ_i .

Therefore, strategic influence is valuable to country i insofar as it shifts its actions away from what it would choose based purely on domestic interests. This formulation of the impact of strategic influence on actions using domestic interests as the benchmark has several convenient properties. First, it implies translation invariance in $\boldsymbol{\theta}$: adding a constant c to every country's interest θ_i shifts every equilibrium action by c without changing welfare, since the deviation $a_i - \theta_i$ is invariant to uniform translations. Second, this specification permits self-influence—country i can exert influence on itself ($\omega_{ii} > 0$). Our concavity assumption $\kappa > 1/\alpha$ ensures that the returns to self-influence are bounded. This condition is sufficient for concavity and binds only for a country with no network connections (Online Appendix J.4). Incorporating self-influence allows us to apply linear algebra tools without separately dealing with the diagonals $\{\omega_{ii}\}$ of the influence matrix $\mathbf{\Omega}$. Finally, because θ_i is exogenous to country i , the term $\sum_{m=1}^N \omega_{im} \theta_i$ in the third line does not affect country i 's best-response choice of actions in the coordination stage, nor does it enter the payoff of any influencing country m . Therefore, it only affects the equilibrium through country i 's choice of self-influence ω_{ii} .

To illustrate the impact of strategic influence on countries' objectives, we can rewrite country i 's payoff in equation (5) in the following form:

Lemma 1. *The payoff of country i in equation (5) can be rewritten as:*

$$U_i(\mathbf{a}, \mathbf{\Omega}) = - \left[\alpha \left(a_i - \left(\theta_i + \frac{\sum_{m=1}^N \omega_{im}}{\alpha} \right) \right)^2 + \sum_{j=1}^N G_{ij} (a_i - a_j)^2 \right] + \frac{\left(\sum_{m=1}^N \omega_{im} \right)^2}{\alpha} - \kappa \sum_{k=1}^N \omega_{ki}^2. \quad (6)$$

Comparing equations (6) and (1), the net strategic influence received by country i acts like a shift in its interests from θ_i to $\theta_i + \sum_{m=1}^N \omega_{im}/\alpha$, with net transfers of $\left(\sum_{m=1}^N \omega_{im} \right)^2 / \alpha$.

2.3 Equilibrium

We solve the game by backward induction. We first solve for equilibrium geopolitical actions in the second stage of the game taking bilateral influence as given. We then solve for equilibrium bilateral strategic influence in the first stage of the game.

2.3.1 Equilibrium at the Coordination Stage

In the coordination stage, country i chooses its geopolitical action (a_i) to maximize its payoff (6), taking as given bilateral strategic influence ($\mathbf{\Omega}$) and the geopolitical actions of all other countries (a_j). The first-order condition or best-response function for country i takes a similar form as in equation (2), but modified by the impact of net strategic influence:

$$a_i = \alpha \theta_i + \sum_{j=1}^N G_{ij} a_j + \sum_{m=1}^N \omega_{im}. \quad (7)$$

In vector form, this best-response function can be written as:

$$\mathbf{a} = \alpha \boldsymbol{\theta} + \mathbf{G} \mathbf{a} + \mathbf{\Omega} \mathbf{1}. \quad (8)$$

Therefore, relative to the pure beauty-contest game in equation (3), country m 's strategic influence shifts the target country i 's geopolitical action by ω_{im} , taking the actions of all other countries as given. Moving $\mathbf{G} \mathbf{a}$ to the left-hand side, and multiplying both sides by $(\mathbf{I} - \mathbf{G})^{-1}$, we can write equilibrium actions in the coordination stage as an explicit function of domestic interests and influence:

$$\mathbf{a} = (\mathbf{I} - \mathbf{G})^{-1} (\alpha \boldsymbol{\theta} + \mathbf{\Omega} \mathbf{1}). \quad (9)$$

The ik -th entry of the matrix $(\mathbf{I} - \mathbf{G})^{-1}$ captures how country k 's domestic interest (θ_k) and net strategic influence received ($[\mathbf{\Omega} \mathbf{1}]_k = \sum_{j=1}^N \omega_{kj}$) affect the equilibrium geopolitical actions chosen by country i . Note that this matrix inverse has the following Neumann series representation: $(\mathbf{I} - \mathbf{G})^{-1} = \mathbf{I} + \mathbf{G} + \mathbf{G}^2 + \dots$. In this representation, each successive power of the matrix $\mathbf{G}$ captures a higher-order network connection: $\mathbf{G}_{ik}$ is the connection between i and k , $\mathbf{G}_{ik}^2 = \sum_{j=1}^N G_{ij} G_{jk}$ is the indirect connection between i and k through a common neighbor j , and so on. Therefore, foreign influence on country k affects country i 's action if they are either directly or indirectly connected through the network.

2.3.2 Equilibrium at the Influence Stage

In the influence stage, country i chooses its bilateral strategic influence $\{\omega_{ki}\}_k$ to maximize its objective $U_i(\mathbf{a}(\boldsymbol{\Omega}), \boldsymbol{\Omega})$. In the first-order condition for country i 's optimal choice of strategic influence, the total derivative of its objective with respect to its bilateral strategic influence (ω_{ki}) can be decomposed into the following three terms:

$$0 = \frac{dU_i}{d\omega_{ki}} = \underbrace{\frac{\partial U_i}{\partial a_i} \frac{\partial a_i}{\partial \omega_{ki}}}_{\text{effect through } a_i} + \underbrace{\sum_{j \neq i} \frac{\partial U_i}{\partial a_j} \frac{\partial a_j}{\partial \omega_{ki}}}_{\text{effect through } a_j, j \neq i} + \underbrace{\frac{\partial U_i}{\partial \omega_{ki}} \Big|_{\mathbf{a}}}_{\text{direct effect}}. \quad (10)$$

The first term on the right-hand side is the indirect effect of ω_{ki} on country i 's payoff through the change in i 's own action a_i . Because a_i is chosen optimally at the coordination stage, this term vanishes by the envelope theorem. The second term is the indirect effect through changes in other countries' actions a_j ($j \neq i$). Since country i does not control these actions, the envelope theorem does not apply, and this second term does not zero out. The third term is the direct effect of ω_{ki} on country i 's payoff holding all actions fixed, which consists of the marginal cost of strategic influence ($-2\kappa \omega_{ki}$), and the marginal benefit of self-influence $2(a_i - \theta_i)$ for $k = i$.

Evaluating the non-zero terms in the first-order condition (10) yields the best-response function for country i 's strategic influence toward country k :

$$\kappa \omega_{ki} = \sum_{j=1}^N [(\mathbf{I} - \mathbf{G})^{-1}]_{jk} G_{ij} (a_i - a_j) + \mathbf{1}_{k=i} (a_i - \theta_i). \quad (11)$$

The left-hand side is the marginal cost of the strategic influence exerted by country i on country k . The right-hand side is the marginal benefit of this strategic influence. The first term on the right-hand side captures country i 's incentive to influence country k to take a higher action, taking into account the equilibrium propagation of higher a_k through the network. Country i 's payoff is higher when its neighbors' actions are closer to its own, as captured by the gap $(a_i - a_j)$. If this gap is positive, country i benefits from a higher a_j , and vice versa. The object $[(\mathbf{I} - \mathbf{G})^{-1}]_{jk}$ in this first term captures how influencing country k raises country j 's action in the coordination equilibrium, accounting for both the direct effect on k and all indirect network propagation. The object G_{ij} in this first term captures how important country j 's action is for country i 's payoff. Summing across countries j , we obtain country i 's overall incentive to influence country k , taking into account the full network of policy coordination. The second term on the right-hand side arises only for self-influence ($k = i$) and reflects the direct benefit of shifting country i 's own action further from its interest θ_i .

We can re-write the best-response function (11) in vector form by summing across i and noting that $\mathbf{G}$ is symmetric with a constant row sum of $1 - \alpha$:

$$\kappa \boldsymbol{\Omega} \mathbf{1} = \alpha (\mathbf{I} - \mathbf{G})^{-1} \mathbf{a} - \boldsymbol{\theta}, \quad (12)$$

where the left-hand side captures the net strategic influence received by each country. Therefore, the

ability of a country to obtain transfers through strategic influence depends on its interests relative to equilibrium actions in the network. Country k receives net influence to take a higher action if its interest is below the equilibrium actions of itself, its neighbors, its neighbors' neighbors, and so on.

Substituting for the equilibrium actions from the coordination game $\mathbf{a}(\Omega)$ in equation (9), we obtain the following closed-form solutions for equilibrium net influence and geopolitical actions in terms of domestic interests (θ) and the geopolitical network (G).

Proposition 1. *The equilibrium vectors of net influence and geopolitical actions satisfy:*

$$\Omega \mathbf{1} = - \left(\kappa (\mathbf{I} - G)^2 - \alpha \mathbf{I} \right)^{-1} \left((\mathbf{I} - G)^2 - \alpha^2 \mathbf{I} \right) \theta. \quad (13)$$

$$\mathbf{a} = \mathcal{L}(G) \theta, \quad \mathcal{L}(G) \equiv \left(\kappa (\mathbf{I} - G)^2 - \alpha \mathbf{I} \right)^{-1} (\mathbf{I} - G) (\kappa \alpha - 1). \quad (14)$$

Moreover, as $\kappa \rightarrow \infty$, $\Omega \mathbf{1} \rightarrow \mathbf{0}$ and $\mathbf{a} \rightarrow \alpha (\mathbf{I} - G)^{-1} \theta$; and as $\kappa \downarrow 1/\alpha$,

$$\mathbf{a} \rightarrow \bar{\theta} \mathbf{1}, \quad \Omega \mathbf{1} \rightarrow \alpha (\bar{\theta} \mathbf{1} - \theta), \quad \text{where } \bar{\theta} \equiv \frac{1}{N} \sum_i \theta_i.$$

Proof. See Online Appendix A. □

Proposition 1 shows that the equilibrium geopolitical action profile $\mathbf{a}$ is a linear transformation of the domestic interests θ : $\mathbf{a} = \mathcal{L}(G) \theta$. To gain further understanding for Proposition 1, it is useful to consider the two limits of the influence cost shifter (κ). As influence becomes infinitely costly ($\kappa \rightarrow \infty$), net influence vanishes and the game converges back to the beauty contest without influence. As influence becomes as cheap as our concavity assumption permits— κ converges to $1/\alpha$ from above ($\kappa \downarrow 1/\alpha$)—equilibrium actions converge to the unweighted global average $\bar{\theta}$. Intuitively, when influence is cheap, each country is pushed toward a common action, which is pinned down by the equal-weighted average of domestic interests. Net influence acts as a corrective force: countries whose domestic interests lie above the global average ($\theta_k > \bar{\theta}$) receive negative net influence that pulls their actions down, while those with domestic interests below the global average receive positive net influence that pushes their actions up.

For any interior value of influence costs ($\kappa \in (1/\alpha, \infty)$), the two equilibrium forces of beauty-contest coordination and strategic influence are both active, and the equilibrium of the game smoothly interpolates between these two limits. Although countries exert strategic influence, its magnitude is strictly attenuated relative to that in the lower limit. A reduction in the cost of influence (a lower κ) reduces the differences in equilibrium geopolitical actions across countries. As strategic influence becomes cheaper, countries move further away from their idiosyncratic domestic interests and toward the network consensus $\bar{\theta}$. In the limit $\kappa \downarrow 1/\alpha$ actions coincide with this consensus, as in the consensus models of DeGroot (1974) and Golub and Jackson (2010).

2.3.3 Properties of the Equilibrium

We now summarize a number of convenient properties of the equilibrium in Proposition 1.

Proposition 2. *The equilibrium has the following properties.*

1. *The equilibrium is unique.*
2. *Each country's equilibrium action is a convex combination over the domestic interests of all countries.*
3. *The equilibrium is translation invariant to θ : adding a constant c to all θ_i 's results in a new equilibrium in which all actions are also shifted by c , without any change in payoffs.*
4. *The equilibrium is conservative: the average geopolitical action and the average domestic interest across countries coincide: $\mathbf{1}^\top \mathbf{a} = \mathbf{1}^\top \theta$.*
5. *The equilibrium strategic influence nets out to zero: $\mathbf{1}^\top \Omega \mathbf{1} = 0$.*

Proof. See Online Appendix B. □

Our model features a unique equilibrium despite the rich network interactions between countries. The linearity of the best-response function (11) in domestic interests implies that each country's equilibrium geopolitical action is a convex combination of the domestic interests of all countries. Our assumption that the return to strategic influence depends on the gap between geopolitical actions and domestic interests ensures that the equilibrium is translation invariant to domestic interests. Therefore, adding a constant to all domestic interests simply results in a new equilibrium, in which all actions are also shifted by c , without any change in payoffs. Equilibrium geopolitical actions are also conservative, in the sense that the average geopolitical action and average domestic interest coincide. Finally, equilibrium strategic influence is positive for some bilateral pairs and negative for others, but averages out to zero across all bilateral pairs.

2.4 Spectral Decomposition and Geopolitical Cleavages

We now formally investigate the underlying characteristics of domestic interests (θ) that determine whether there is policy divergence or convergence across countries. We use a spectral decomposition of the geopolitical network $\mathbf{G}$ to provide this characterization:

$$\mathbf{G} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^\top \quad (\text{equivalently, } \mathbf{G} = \sum_{k=1}^N \lambda_k \mathbf{u}_k \mathbf{u}_k^\top), \quad (15)$$

where $\mathbf{\Lambda}$ is the diagonal matrix of $\mathbf{G}$'s eigenvalues $\lambda_k \in \mathbb{R}$, and each column of $\mathbf{U}$ is an associated eigenvector $\mathbf{u}_k$. Since $\mathbf{G}$ is real and symmetric, this spectral decomposition necessarily exists. Furthermore, the symmetry of $\mathbf{G}$ implies that the vectors are orthonormal, $\mathbf{U}^{-1} = \mathbf{U}^\top$, and all eigenvalues are real. We rank these eigenvalues in decreasing order.

We next note that any vector of domestic interests (θ) can be projected onto G 's eigenbasis:

$$\theta = \sum_{k=1}^N \tilde{\theta}_k \mathbf{u}_k, \quad \tilde{\theta} \equiv U^\top \theta. \quad (16)$$

That is, any vector θ of domestic interests across countries can be written as a linear combination of G 's eigenvectors $\{\mathbf{u}_k\}$, with weights $\tilde{\theta}_k = \mathbf{u}_k^\top \theta$ given by the projection of θ onto each eigenvector.

Each eigenvector captures a particular axis of polarization, which can be interpreted as a latent geopolitical cleavage—a specific arrangement of domestic interests across countries. We now show that each latent geopolitical cleavage translates into a natural notion of disagreements among network neighbors. We define the following network disagreement measure:

$$\mathcal{S}(\theta) \equiv \frac{1}{2} \sum_{i,j} G_{ij} (\theta_i - \theta_j)^2,$$

which captures how dissimilar domestic interests are, on average, across network neighbors.

Proposition 3. *The network disagreement of an eigenvector $\mathbf{u}_k$ is $\mathcal{S}(\mathbf{u}_k) = 1 - \alpha - \lambda_k$. Moreover,*

$$\begin{aligned} \mathbf{u}_1 &= \arg \min_{\|\theta\|=1} \mathcal{S}(\theta) \propto \mathbf{1}, \quad \lambda_1 = 1 - \alpha. \\ \mathbf{u}_k &\in \arg \min_{\theta \perp \{\mathbf{u}_j\}_{j < k}, \|\theta\|=1} \mathcal{S}(\theta) \text{ for all } k > 1. \end{aligned}$$

Proof. See Online Appendix C. □

We use Proposition 3 to characterize the extent of policy divergence or convergence given domestic interests (θ). First, the network disagreement of an eigenvector $\mathbf{u}_k$ is directly and negatively tied to its associated eigenvalue λ_k . Second, the eigenvector with the largest eigenvalue is a constant vector (which we call the consensus mode), and each subsequent eigenvector $\mathbf{u}_k$ is the vector of domestic interests, among those orthogonal to all prior eigenvectors, that minimizes network disagreement.

Third, because the eigenvectors are pairwise orthogonal, each eigenvector $\mathbf{u}_k$ averages to zero across countries for all $k > 1$ (since $\mathbf{1}^\top \mathbf{u}_k = 0$). Therefore, each eigenvector captures a geopolitical cleavage—a particular configuration of disagreements in domestic interests across countries. Those with positive entries in $\mathbf{u}_k$ are on one side of the cleavage while those with negative entries are on the other side of the cleavage.

Fourth, for all $k > 1$, Proposition 3 shows that the geopolitical cleavages with large eigenvalues capture international issues where neighbors in tightly connected geopolitical blocs (with high G_{ij}) tend to have similar domestic interests, whereas the disagreements across blocs (with low G_{ij}) can be large. Conversely, the cleavages with small or negative eigenvalues capture issues where neighbors in tightly connected geopolitical blocs (with high G_{ij}) tend to have dissimilar domestic interests, whereas the disagreements across blocs (with low G_{ij}) can be small.

We now combine this spectral decomposition with our result from Proposition 1 that the equilibrium geopolitical action profile $\mathbf{a}$ can be characterized as a linear transformation of the domestic interests $\boldsymbol{\theta}$: $\mathbf{a} = \mathcal{L}(\mathbf{G}) \boldsymbol{\theta}$, where $\mathcal{L}(\mathbf{G}) \equiv (\kappa(\mathbf{I} - \mathbf{G})^2 - \alpha\mathbf{I})^{-1}(\mathbf{I} - \mathbf{G})(\kappa\alpha - 1)$.

Proposition 4. *The operator $\mathcal{L}(\mathbf{G})$, mapping from domestic interests to geopolitical actions, is a spectral function of $\mathbf{G}$: it shares the same eigenbasis as $\mathbf{G}$. This implies spectral decoupling: for domestic interests decomposed along $\mathbf{G}$'s eigenvectors, the equilibrium mapping scales each cleavage independently, without spilling over into others:*

$$\boldsymbol{\theta} = \sum_{\ell} \tilde{\theta}_{\ell} \mathbf{u}_{\ell} \implies \mathbf{a} = \sum_{\ell} L(\lambda_{\ell}) \tilde{\theta}_{\ell} \mathbf{u}_{\ell}, \quad L(\lambda) \equiv \frac{(1 - \lambda)(\kappa\alpha - 1)}{\kappa(1 - \lambda)^2 - \alpha};$$

or, equivalently, projecting the equilibrium action profile onto $\mathbf{G}$'s eigenbasis $\tilde{\mathbf{a}} \equiv \mathbf{U}^{\top} \mathbf{a}$, we have:

$$\tilde{a}_{\ell} = L(\lambda_{\ell}) \tilde{\theta}_{\ell}.$$

The scaling factor $L(\cdot)$ is nonnegative, bounded above by one ($0 \leq L(\cdot) \leq 1$), and is strictly increasing in the eigenvalue λ_{ℓ} ($L'(\cdot) > 0$), implying that eigenvectors with larger eigenvalues translate into larger differences in geopolitical actions. Finally, the scaling factor $L(\cdot)$ is increasing in κ .

Proof. See Online Appendix D. □

Therefore, equilibrium geopolitical actions exhibit a spectral decoupling along the network's cleavages. When domestic interests are decomposed along these eigenvectors, the equilibrium mapping scales each cleavage independently, without spilling over to the other cleavages. The scaling factor, which translates domestic interests into geopolitical actions, depends only on the eigenvalue of the geopolitical cleavage. International disagreements associated with large eigenvalues (where disagreements are small between geopolitically close countries and large between geopolitically distant countries) tend to translate into large differences in geopolitical actions. This divergence is driven by a geopolitical echo-chamber effect: because the coordination game incentivizes alignment with immediate neighbors, connected countries mutually reinforce each other's stances, so that cross-bloc frictions pass through into policy polarization. Conversely, disagreements associated with small eigenvalues (where disagreements are large between geopolitically close countries and small between geopolitically distant countries) are neutralized by local coordination incentives, leading to convergent policy outcomes.

A lower cost of influence systematically lowers polarization in equilibrium geopolitical actions. As strategic influence becomes cheaper, countries move further away from their idiosyncratic domestic interests and toward the network consensus. In Online Appendix J.1, we introduce a contestability index, which measures the share of the foreign influence directed at each country that is offset by influence in the opposing direction. A low value for a country's contestability index indicates a strong consensus of foreign influence, which pushes that country's action in a unified direction. In

contrast, a high value for a country’s contestability index reflects a geopolitical tug-of-war, in which that country is a highly contested swing state for foreign influence.

In Online Appendix J.2, we provide an example geopolitical network of four countries (US, EU, Global South and China) that illustrates our approach. The first eigenvector is a constant vector that captures global agreement by all countries. The second eigenvector captures the leading geopolitical cleavage, in which the US and the EU take one position, China takes the polar opposite position, and the Global South occupies the middle. The third eigenvector captures a different cleavage, in which China aligns with the US and the EU, but all three disagree with the Global South. The eigenvector associated with the smallest eigenvalue captures localized disagreement, in the form of a policy fracture between the US and the EU. For the same dispersion of domestic interests, the leading cleavage generates greater dispersion in geopolitical actions than the fracture between the US and the EU, because coordination attenuates disagreements among close network partners more strongly than disagreements between blocs.

2.5 Stochastic International Issues and Identification

We have so far focused on geopolitical actions in response to a single international issue, treating the vector of domestic interests for that international issue (θ) as fixed. We now extend our theoretical analysis to a setting in which international issues arrive stochastically, and the domestic interests for each international issue (θ) are drawn from a fixed distribution of geopolitical shocks. We interpret these stochastic draws for domestic interests as capturing the shifting landscape of international crises, such as the Berlin Airlift, the Cuban Missile Crisis, the US proposal to purchase Greenland, or Chinese military exercises in the Taiwan Strait. We use this stochastic specification to provide conditions under which the unobserved geopolitical network (G) can be identified from observed second moments of geopolitical actions (a) and domestic interests (θ).

We define the covariance matrix of domestic interests across international issues as $\Sigma_\theta \equiv \mathbb{E} [\theta\theta^\top]$, whose ij -th entry is large when countries i and j have similar domestic interests relative to other countries, and the covariance matrix of geopolitical actions as $\Sigma_a \equiv \mathbb{E} [aa^\top]$.⁶ From Proposition 1, equilibrium geopolitical actions are linearly related to domestic interests through $a = \mathcal{L}(G)\theta$. Therefore, the covariance matrices of geopolitical actions and domestic interests are linked through $\mathcal{L}(G)$ as follows: $\Sigma_a = \mathcal{L}(G)\Sigma_\theta\mathcal{L}(G)$.

Intuitively, countries’ geopolitical actions are determined by both domestic interests and geopolitical connections, which implies that the cross-country covariance of geopolitical actions relative to that of domestic interests contains information on the geopolitical network. For example, if two countries have similar domestic interests across international issues but consistently take different

⁶We do not remove the cross-country average of domestic interests or actions: a common shift in all countries’ domestic interests shifts all geopolitical actions by the same amount (Proposition 2), so it loads only on the consensus mode and carries no information about the geopolitical network.

geopolitical actions, they cannot be too strongly connected, either directly or indirectly through third countries, since coordination would otherwise bring their actions closer together. Conversely, if two countries have different domestic interests but consistently take similar geopolitical actions, they are likely to be connected. When the covariance matrix of domestic interests (Σ_θ) is nonsingular, this information is sufficient to identify the geopolitical network, as summarized in the following theoretical identification result.

Proposition 5. (Identification) *Suppose that Σ_θ is nonsingular. Under the assumption that $\mathbf{G}$ has non-negative entries, rows summing to $1 - \alpha$, and $\kappa > 1/\alpha$, $\mathcal{L}(\mathbf{G})$ is the unique symmetric positive definite solution of:*

$$\Sigma_a = \mathcal{L}(\mathbf{G}) \Sigma_\theta \mathcal{L}(\mathbf{G}), \quad \mathcal{L}(\mathbf{G}) = \Sigma_\theta^{-1/2} \left(\Sigma_\theta^{1/2} \Sigma_a \Sigma_\theta^{1/2} \right)^{1/2} \Sigma_\theta^{-1/2}.$$

The geopolitical cleavages are the eigenvectors of $\mathcal{L}(\mathbf{G})$ and the eigenvalues of the geopolitical network can be recovered by inverting the scaling function $\lambda_\ell = L^{-1}(\cdot)$ from Proposition 4.

Proof. See Online Appendix E. □

The formula in Proposition 5 takes a particularly simple form when the covariance matrices of geopolitical actions (Σ_a) and domestic interests (Σ_θ) commute, which is equivalent to these two covariance matrices sharing the same principal components. In this case,

$$\mathcal{L}(\mathbf{G}) = (\Sigma_a \Sigma_\theta^{-1})^{1/2}, \tag{17}$$

which generalizes the ratio of the standard deviation of geopolitical actions to that of domestic interests. Intuitively, this is the matrix analogue of the pass-through of geopolitical cleavages in Proposition 4: for each geopolitical cleavage, it captures how much of the disagreement in domestic interests along that cleavage survives into disagreement in geopolitical actions, which is larger for cleavages that separate tightly connected blocs than for those that divide countries within a bloc.

In our empirical application, the covariance matrices are non-invertible or ill-conditioned, but a small number of principal components account for most of the variation in both domestic interests and geopolitical actions. We now show that we can still recover the geopolitical network from the cross-country covariance of geopolitical actions and domestic interests, under the additional assumption that these two covariance matrices share common principal components, so that they commute. In this case, equation (17) can be evaluated dimension by dimension, that is, separately along each common principal component for which the variance of domestic interests is nonzero. This additional assumption corresponds to a restriction on the primitives of the model, which we refer to as graph stationarity: domestic interests and the geopolitical network have the same eigenbasis and hence share the same principal components.⁷ Intuitively, domestic interests and the geopoliti-

⁷An analogous assumption is common in network signal processing (e.g., Marques et al. 2017; Segarra et al. 2017).

cal network are both determined by a common underlying set of latent geopolitical issues, such that they have the same principal components.

Under graph stationarity, the variance of geopolitical actions relative to the variance of domestic interests then identifies the scaling function $L(\lambda_\ell)$ for each cleavage ℓ , which is monotonic in the eigenvalues of the geopolitical network (λ_ℓ) and hence can be inverted to recover these eigenvalues, as summarized in the following result.

Proposition 6. (Identification under Graph Stationarity) *Under the assumption of graph stationarity, Σ_θ shares the eigenbasis U of G . Let $\tilde{\Sigma}_a \equiv U^\top \Sigma_a U$, $\tilde{\Sigma}_\theta \equiv U^\top \Sigma_\theta U$ be the projection of Σ_a and Σ_θ onto this eigenbasis, with ℓ -th diagonal elements $\sigma_{a,\ell}^2$ and $\sigma_{\theta,\ell}^2$, respectively. Given α and κ , the eigenvalues (λ_ℓ) and eigenvectors ($\mathbf{u}_\ell$) of G are identified if $\sigma_{\theta,\ell}^2$ is nonzero, with*

$$\lambda_\ell = L^{-1} \left(\sqrt{\sigma_{a,\ell}^2 / \sigma_{\theta,\ell}^2} \right), \quad (18)$$

where $L(\cdot)$ is defined in Proposition 4.

Proof. See Online Appendix F. □

Graph stationarity means that the covariance matrix of domestic interests can be written as $\Sigma_\theta = \sum_\ell \sigma_{\theta,\ell}^2 \mathbf{u}_\ell \mathbf{u}_\ell^\top$. The assumption places no restriction on the variance $\sigma_{\theta,\ell}^2$ along each cleavage, nor on how this variance relates to the importance of the cleavage in the network, λ_ℓ . Combined with Proposition 4, which shows that the equilibrium rescales each cleavage by $L(\lambda_\ell)$ without rotating it, the assumption implies that the covariance matrix of geopolitical actions is diagonalized by the same cleavages:

$$\Sigma_a = \sum_\ell L(\lambda_\ell)^2 \sigma_{\theta,\ell}^2 \mathbf{u}_\ell \mathbf{u}_\ell^\top. \quad (19)$$

Suppose, for example, that domestic interests differ by the same amount along two cleavages, one separating Western from Soviet-aligned countries and one dividing Western Europe internally. If geopolitical actions divide sharply along the first and barely along the second, the model infers that the first separates blocs and the second operates within one bloc.

In general, the theoretical assumption of graph stationarity need not be satisfied in the data, because the observed cross-country covariances of geopolitical actions and domestic interests need not be perfectly explained by a common set of principal components. We provide evidence below that the common principal components provide a reasonable approximation to these observed cross-country covariance matrices. We also report additional validation checks on the geopolitical network that we recover from these common principal components.

The covariance matrix of domestic interests (Σ_θ) is singular exactly when domestic interests do not vary along some cleavages ($\sigma_{\theta,\ell}^2 = 0$). Proposition 6 shows that this does not prevent identification along the cleavages along which domestic interests vary. We therefore reconstruct the geopo-

litical network from these cleavages, $\mathbf{G} = \sum_{\ell} \lambda_{\ell} \mathbf{u}_{\ell} \mathbf{u}_{\ell}^{\top}$, and set the eigenvalues of the remaining cleavages to zero.⁸

With stochastic international issues, we measure each country's *ex ante* welfare as its expected payoff ($\mathbb{E}_{\theta} [U_i]$) over the distribution of domestic interests (θ). In Online Appendix J.3, we show that the *ex ante* welfare of each country depends on the network structure as well as the set of international issues that arise. We express each country i 's *ex ante* welfare in terms of the eigenvectors and eigenvalues of $\mathbf{G}$, as well as of the matrix Σ_{θ} capturing the covariance of domestic interests.

2.6 Introducing Network Asymmetry

For expositional simplicity, we have so far assumed that the geopolitical network is symmetric. We now relax this assumption to allow the geopolitical network to be directed ($G_{ij} \neq G_{ji}$), such that the effect of country i on policy in country j can differ from that of country j on policy in country i . We thus incorporate asymmetries in the geopolitical network, reflecting differences in economic dependence, military capacity, or diplomatic reach.

We incorporate these network asymmetries by allowing for a geopolitical shifter for each country (π_i) that captures its structural importance in the network. Formally, let $\mathbf{H}$ be a symmetric, nonnegative matrix of network connections between countries, which plays the same role as the symmetric network in our baseline model except that its row sums may differ across countries. We define country i 's structural importance as the total intensity of its network connections, $\pi_i \equiv \sum_{j=1}^N H_{ij}$, and let $\mathbf{\Pi} = \text{Diag}(\boldsymbol{\pi})$; we normalize the scale of $\mathbf{H}$ such that $\sum_{i=1}^N \pi_i = 1$, which is without loss of generality. We construct the directed network as:

$$\mathbf{G} = (1 - \alpha) \mathbf{\Pi}^{-1} \mathbf{H}, \quad (20)$$

so that $G_{ij} = (1 - \alpha) H_{ij} / \pi_i$. Hence $G_{ij} / G_{ji} = \pi_j / \pi_i$: for a given connection H_{ij} , the country with greater structural importance responds less to the relationship than its partner does. In other words, countries with greater structural importance have a disproportionate impact on their neighbors, while being less sensitive to any single neighbor themselves.

We scale the costs of strategic influence by structural importance, such that the cost of country i exerting influence on country k is given by $\kappa (\pi_k / \pi_i) \omega_{ki}^2$. We thus capture the idea that influencing a country with greater structural importance (large π_k) is expensive, because its actions are anchored by dense network connections. Additionally, projecting influence from a peripheral country (small π_i) is costly, reflecting its limited diplomatic and economic reach.

Combining these assumptions, country i 's payoff is now given by:

$$U_i(\mathbf{a}, \boldsymbol{\Omega}) = - \left[\alpha (a_i - \theta_i)^2 + \sum_j G_{ij} (a_i - a_j)^2 \right] - \kappa \sum_k \frac{\pi_k}{\pi_i} \omega_{ki}^2 + 2 \sum_m \omega_{im} (a_i - \theta_i),$$

⁸By spectral decoupling, the unidentified cleavages do not spill over into the identified ones, and we set their eigenvalues to zero, so that along these cleavages a country is pulled only toward the overall consensus, not toward any bloc.

where a symmetric network is the special case in which $\mathbf{H}$ has constant row sums, so that all countries have the same structural importance and $\pi_i = \pi_k$ for all i, k .

We now show that our main analytical results for the properties of the policy coordination and strategic influence game extend to this case of asymmetric networks.

Proposition 7. *Suppose $\mathbf{G} = (1 - \alpha) \mathbf{\Pi}^{-1} \mathbf{H}$ with influence costs $\kappa (\pi_k / \pi_i) \omega_{ki}^2$. Then:*

1. *Propositions 1 and 2 continue to hold. In particular:*

(a) *Equilibrium net influence and geopolitical actions satisfy:*

$$\begin{aligned} \Omega \mathbf{1} &= -(\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} ((\mathbf{I} - \mathbf{G})^2 - \alpha^2 \mathbf{I}) \boldsymbol{\theta}, \\ \mathbf{a} &= (\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} (\mathbf{I} - \mathbf{G}) (\kappa \alpha - 1) \boldsymbol{\theta}. \end{aligned}$$

(b) *The equilibrium inherits uniqueness, translation invariance, the convex combination property, $\mathbf{\Pi}$ -weighted conservativeness ($\sum_i \pi_i a_i = \sum_i \pi_i \theta_i$ and $\sum_i \pi_i \sum_m \omega_{im} = 0$), and the $\mathbf{\Pi}$ -weighted analogues of the two limiting benchmarks ($\kappa \rightarrow \infty$ and $\kappa \downarrow 1/\alpha$) from the symmetric baseline.*

2. *The equilibrium continues to admit a spectral representation in terms of the right and left eigenvectors of $\mathbf{G}$, $\mathbf{u}_m^R$ and $\mathbf{u}_m^L$, normalized so that $(\mathbf{u}_m^L)^\top \mathbf{u}_p^R = \mathbf{1}_{m=p}$:*

$$a_i = \sum_m L(\lambda_m) [\mathbf{u}_m^R]_i (\mathbf{u}_m^L)^\top \boldsymbol{\theta}, \quad (21)$$

where the scaling function $L(\lambda) \equiv (1 - \lambda) (\kappa \alpha - 1) / (\kappa (1 - \lambda)^2 - \alpha)$ is identical to that under a symmetric network.

Proof. See Online Appendix H. □

Therefore, our earlier propositions for a symmetric network extend to an asymmetric network by working in $\mathbf{\Pi}$ -weighted coordinates. Specifically, we define adjusted domestic interests and geopolitical actions as $\tilde{\theta}_m \equiv (\mathbf{u}_m^L)^\top \boldsymbol{\theta}$ and $\tilde{a}_m \equiv (\mathbf{u}_m^L)^\top \mathbf{a}$. These adjusted variables with a tilde are projections of domestic interests and equilibrium actions onto the eigenbasis of $\mathbf{G}$: $\boldsymbol{\theta} = \sum_m \tilde{\theta}_m \mathbf{u}_m^R$ and $\mathbf{a} = \sum_m \tilde{a}_m \mathbf{u}_m^R$. The equilibrium using these projected coordinates is structurally identical to that for a symmetric network. The closed-form expressions, the net influence equation, and the spectral scaling function $L(\lambda)$ all carry over unchanged.

We next extend our theoretical identification results to this case of an asymmetric network, showing that we can recover the spectral properties of the network from the observed covariance matrices for domestic interests (Σ_θ) and geopolitical actions (Σ_a). As in the symmetric case, we assume graph stationarity.

Assumption 1. (Graph Stationarity for Directed Networks) After weighting by structural importance, the covariance matrix of domestic interests shares the eigenbasis of the network: $\Pi^{1/2}\Sigma_\theta\Pi^{1/2}$ is diagonalized by the orthonormal eigenvectors $\Pi^{1/2}\mathbf{u}_m^R$ of the symmetric matrix $\Pi^{1/2}\mathbf{G}\Pi^{-1/2}$, so that $\Sigma_\theta = \sum_m \sigma_{\theta,m}^2 \mathbf{u}_m^R (\mathbf{u}_m^R)^\top$.

In the special case in which the network is symmetric ($\mathbf{H}$ has constant row sums, so that $\Pi \propto \mathbf{I}$ and $\mathbf{G} \propto \mathbf{H}$), Assumption 1 reduces to the graph stationarity assumption in Proposition 6. More generally, when the network is asymmetric, the Π -weighting means that the interests of structurally important countries carry more weight in each cleavage. As in the symmetric case, Assumption 1 corresponds to the Π -weighted covariance matrices of domestic interests and geopolitical actions sharing the same principal components (see Online Appendix G), which is the property that we use in our estimation in Section 3.2.

Combining Assumption 1 with the mapping from domestic interests to geopolitical actions from Proposition 7, we are again able to identify the eigenvalues (λ_m) and eigenvectors ($\mathbf{u}_m^R$) of the network, and hence recover the geopolitical network.

Proposition 8. (Identification) Under Assumption 1, given α , κ , and Π , the eigenvalue λ_m and eigenvector $\mathbf{u}_m^R$ are identified whenever $\sigma_{\theta,m}^2 > 0$, via:

$$\lambda_m = L^{-1} \left(\sqrt{\sigma_{a,m}^2 / \sigma_{\theta,m}^2} \right),$$

where $\tilde{\Sigma}_\theta \equiv \mathbf{U}_L^\top \Sigma_\theta \mathbf{U}_L$ and $\tilde{\Sigma}_a \equiv \mathbf{U}_L^\top \Sigma_a \mathbf{U}_L$ are the covariance matrices of the adjusted domestic interests and actions, with $\mathbf{U}_L \equiv [\mathbf{u}_1^L, \dots, \mathbf{u}_N^L]$ the matrix of left eigenvectors, so that $\sigma_{\theta,m}^2$ and $\sigma_{a,m}^2$ are the variances of $\tilde{\theta}_m$ and $\tilde{a}_m$.

Proof. See Online Appendix I. □

Proposition 8 identifies the eigenvalues λ_m and eigenvectors $\mathbf{u}_m^R$ (and hence recovers the geopolitical network) given the vector of structural importance ($\boldsymbol{\pi}$) and the parameters (α, κ) of the scaling function, up to the relative scale of geopolitical actions and domestic interests (Section 3.2). For the special case of a symmetric network, this eigenbasis is pinned down by graph stationarity alone. More generally, when the network is asymmetric, there is an additional unknown—the vector of structural importance $\boldsymbol{\pi}$ —since the eigenvectors of the network are only revealed after Π -weighting the covariance matrices. If the structural importance of a country (π_i) can be directly measured empirically, the vector $\boldsymbol{\pi}$ can be directly constructed using these empirical weights. Alternatively, the vector $\boldsymbol{\pi}$ can be estimated to match empirical moments, as discussed in Subsection 3.3 below.

Therefore, the methods that we developed in previous subsections for a symmetric network carry over to the more general case of an asymmetric network considered in this subsection.

3 Empirical Implementation

We now introduce the data on actions and domestic interests used in our empirical analysis, and use our identification result to estimate the underlying geopolitical network.

3.1 Data

We combine data from a number of different sources to measure the covariance matrices for geopolitical actions ($\mathbf{a}$) and domestic interests ($\boldsymbol{\theta}$), and to undertake validation checks on our estimated geopolitical network.

3.1.1 Measuring Geopolitical Actions ($\mathbf{a}$): UN General Assembly Voting

We measure geopolitical actions ($\mathbf{a}$) using the United Nations General Assembly (UNGA) voting data, which have a number of advantages for our application. First, they capture an observable geopolitical action that is taken after international coordination and influence, as assumed in the model. Second, they are available over a relatively long time period. Third, they are a widely used measure of geopolitical alignment in the political economy and international relations literatures, which allows us to illustrate our methodology using a standard data source.

A potential limitation of the UNGA voting data is heterogeneity in the resolutions that are voted on. To address this limitation, we use the estimated ideal points of Bailey et al. (2017), which are adjusted for issue composition and the baseline tendency for agreement on specific issues. We take the ideal points estimated for six distinct issue topics (human rights, nuclear weapons/materials, Middle East, disarmament, colonialism, and economic issues). We interpret these ideal points for the six topics as actions in the model. Our sample runs from 1960 to 2019, which we divide into six decades to allow for slow-moving changes in the geopolitical network;⁹ for each decade d , we estimate the cross-country covariance matrix of geopolitical actions across the six topics p as:

$$(\Sigma_{a,d})_{ij} = \sum_{t \in d} \sum_{p=1}^6 (a_{ipt} - \bar{a}_{pt}) (a_{jpt} - \bar{a}_{pt}), \quad (22)$$

where a_{ipt} denotes country i 's ideal point on topic p and $\bar{a}_{pt}$ denotes the Π -weighted average position on topic p : $\bar{a}_{pt} \equiv \sum_k \frac{\pi_{kd}}{\sum_i \pi_{id}} a_{kpt}$. The Π -weighted average is each topic's loading on the consensus mode; subtracting it removes the component of each topic that reflects the arbitrary level of UN coding rather than any geopolitical cleavage; and it is the demeaning under which the Π -weighted covariance matrices are consistent with Assumption 1. We thus capture both cross-country and cross-issue variation, as the ij -th entry of $\Sigma_{a,d}$ will be larger when countries i and j both deviate from the average action on each issue similarly. This covariance summarizes similarities in countries' ideal-point levels across topics and years, including persistent differences in positions.

⁹Issue-specific ideal points are not available for the 1960s, only the overall ideal point. Therefore, we construct $\Sigma_{a,1960s}$ from the overall ideal point, such that there is only variation across years within the 1960s decade.

3.1.2 Measuring Domestic Interests (θ): Varieties of Democracy (V-Dem)

We measure countries' domestic interests (θ) using orientation toward democracy from the Varieties of Democracy (V-Dem) dataset, which aggregates expert evaluations to score countries across hundreds of disaggregated domestic political and social dimensions, such as freedom of the press, civil liberties, and judicial independence. These data again have several advantages. First, they capture fundamental characteristics of countries that shape preferences before international coordination and influence. Second, they are available over a relatively long time period on a range of observable characteristics. Third, an influential line of research in political science argues that national preferences are shaped in powerful ways by domestic institutions (e.g., Moravcsik 1997; Gartzke 1998).

The V-Dem dataset contains over one thousand variables that record different dimensions of political institutions. These variables are aggregated into 48 separate index variables, each of which summarizes a country's political institutions relating to a given topic, such as freedom of expression. We use the 37 of these indices that are related to countries' positions on international issues, as listed in Table A1 and discussed in further detail in Online Appendix L.1.

Let x_{imt} denote country i 's V-Dem score on domestic issue m in year t . We scale each index by its range, so that indices measured on different scales are comparable. We construct the cross-country covariance matrix of domestic interests ($\Sigma_{\theta,d}$) by computing the covariance between the interests vectors of countries i and j over 37 distinct domestic issues and years t within each decade d :

$$(\Sigma_{\theta,d})_{ij} = \sum_{t \in d} \sum_{m=1}^{37} (x_{imt} - \bar{x}_{mt}) (x_{jmt} - \bar{x}_{mt}).$$

We again capture cross-sectional variation, since the ij -th entry of $\Sigma_{\theta,d}$ will be larger if the domestic interests of countries i and j deviate from the cross-country mean for a given dimension m similarly within that decade. As for $\Sigma_{a,d}$, we demean by the Π -weighted average value $\bar{x}_{mt} \equiv \sum_k \frac{\pi_{kd}}{\sum_i \pi_{id}} x_{kmt}$, where π_{id} is the structural importance weight constructed from GDP in Section 3.3. Finally, we add back the squares of these Π -weighted averages ($\bar{a}_{pt}^2$ and $\bar{x}_{mt}^2$) to every entry of $\Sigma_{a,d}$ and $\Sigma_{\theta,d}$, so that these matrices approximate the uncentered second moments including the consensus mode $\mathbb{E}[\mathbf{a}\mathbf{a}^\top]$ and $\mathbb{E}[\boldsymbol{\theta}\boldsymbol{\theta}^\top]$ in Section 2.5 (Proposition 2).

3.1.3 Other Data Sources and Coverage

In empirical validation checks of our inferred geopolitical network (G), we use a range of empirical proxies for bilateral geopolitical connections between countries. We use variables from the CEPII Gravity dataset (bilateral trade, distance and indicators for cultural and historical linkages from Conte et al. 2022), the Polity Project (the difference in polity scores between countries), the Formal Bilateral Influence Capacity (FBIC) dataset, and the Country and Organization Leader Travel (COLT) dataset on bilateral state visits.

The number of countries used to recover the geopolitical network varies across decades, because

of the emergence of new states and missing data for some years. As shown in Figure A2 in Online Appendix L.2, the coverage of V-Dem and UN voting data grows over time, ultimately converging to roughly 175 countries with non-missing data on both domestic interests and geopolitical actions. Figure A3 in Online Appendix L.2 reports country coverage for the variables used in empirical validation checks of our inferred geopolitical network. In general, we have high coverage throughout our sample period, except that bilateral trade flows are available only from 1990 onwards.

3.2 Estimating Geopolitical Cleavages

We implement our empirical analysis separately by decade, assuming that $\mathbf{G}$ is slow moving and stable within a decade. This subsection estimates the cleavages of the network $(\mathbf{u}_m^R, \lambda_m)$, taking the structural importance weights (π_i) as given; Section 3.3 estimates the weights. We calibrate the parameters (α, κ) of the scaling function $(L(\lambda))$. In our baseline specification, we set $\alpha = 0.4$ and $\kappa = 5$, which ensures that our theoretical parameter restrictions of $0 < \alpha < 1$ and $\kappa > 1/\alpha$ are satisfied. We report robustness checks to alternative values of these parameters below.

We estimate our asymmetric network specification by weighting the covariance matrices of geopolitical actions $(\Sigma_{a,d})$ and domestic interests $(\Sigma_{\theta,d})$ using the structural importance matrix $(\Pi_d \equiv \text{Diag}(\pi_d))$, which yields $\Sigma_{\theta,d}^\Pi = \Pi_d^{1/2} \Sigma_{\theta,d} \Pi_d^{1/2}$ and $\Sigma_{a,d}^\Pi = \Pi_d^{1/2} \Sigma_{a,d} \Pi_d^{1/2}$. Under Assumption 1, domestic interests and geopolitical actions share the same principal components. We therefore estimate the cleavages as the eigenvectors of the average of the two matrices, each normalized by its trace, computed with the consensus component retained and imposed as the first component. When the two matrices share an eigenbasis exactly, this procedure recovers that eigenbasis. When they do not, this procedure recovers the directions that account for the most variance in domestic interests and geopolitical actions combined, which is the standard approximation to common principal components (Krzanowski 1984; Flury 1988). Given the resulting common basis $\mathbf{U}_d$, the ratio of the variance of actions to the variance of interests along each eigenvector identifies its eigenvalue through Proposition 8.

Geopolitical actions $(\mathbf{a})$ and domestic interests $(\boldsymbol{\theta})$ are measured in different units, so the data pin down the eigenvalues of $\mathbf{G}$ only up to the relative scale of actions and interests. The model bounds this scale but does not pin it down. For a nonnegative network, the Perron–Frobenius theorem places every cleavage eigenvalue strictly below the consensus eigenvalue $1 - \alpha$, and we impose this ordering as a normalization. We select the largest admissible scale, setting the largest cleavage eigenvalue to $1 - \alpha - \epsilon$ with $\epsilon = 0.01$, so that the leading cleavage is as pronounced as the model permits, decade by decade. Along each cleavage, counterfactual actions in response to changes in domestic interests are invariant to this choice.¹⁰ This invariance does not extend to shocks along the

¹⁰Along each cleavage ℓ , the data pin down the measured gain $\hat{L}_\ell = \sigma_{a,\ell}/\sigma_{\theta,\ell}$, the ratio of the action to interest dispersion. The model interprets this ratio as $L(\lambda_\ell) = \hat{L}_\ell$, and since $L(\cdot)$ is strictly monotone given α, κ , it can be inverted to recover the eigenvalue λ_ℓ . The covariance matrices determine $L(\lambda_\ell)^2$, and hence this ratio only up to sign;

consensus mode or to counterfactuals that alter G directly. Section 3.4 and Online Appendix L.3.4 show that the results are robust to our choice of the scale by varying ϵ .

We reconstruct the geopolitical network using the consensus mode and the four components that explain the most variance in geopolitical actions, setting the remaining eigenvalues to zero.¹¹ Among the retained components, we rank the cleavages by their eigenvalues. We examine the share of the variance of domestic interests and geopolitical actions explained by these top-five components. We report this share both for $\Sigma_{a,d}$ and $\Sigma_{\theta,d}$ as defined above, which include the consensus component, and for the same matrices with the consensus component removed. For the former, the top-five components account for 97 and 96 percent of the variation in geopolitical actions and domestic interests, respectively; with the consensus component removed, they account for 97 and 81 percent. The lower share for domestic interests is expected: the V-Dem indices measure the whole of a country's political institutions, and the joint basis weights more heavily the institutional dimensions that are transmitted more strongly into geopolitical actions. Online Appendix L.3.1 reports these statistics by decade, and Online Appendix L.3.3 shows that the leading cleavage is unchanged under alternative estimators of the common basis.

The covariance matrices thus identify the cleavages and their eigenvalues up to the relative scale of geopolitical actions and domestic interests, given the parameters (α, κ) of the scaling function. We set the eigenvalues of the cleavages along which domestic interests do not vary to zero, so that along these cleavages a country is pulled only toward the overall consensus, not toward any bloc.

3.3 Estimating Structural Importance

We now discuss the estimation of the vector of structural importance (π_d) . The asymmetry of the network—how much more a large country's ties matter to its partners than to itself—cannot be read off the covariance matrices for geopolitical actions and domestic interests, because they are symmetric. Therefore, we use the prediction of our model that more structurally important countries exert more influence than they receive. We choose the degree of asymmetry of the network so that the model reproduces the observed patterns of outward and inward influence in the FBIC data.

Specifically, we assume a constant elasticity relationship between structural importance and a country's size as measured by its gross domestic product (GDP): $\pi_{id} = \text{GDP}_{id}^\beta$, where β is a param-

our assumption $\kappa > 1/\alpha$, under which $L(\cdot)$ is positive along every cleavage, selects the positive root. To see that this leaves the scale free, fix the units of θ and suppose actions are rescaled by $s > 0$, so that estimation uses $s \cdot a$ in place of a . The eigenvectors are unaffected, because each matrix enters normalized by its trace, but each recovered eigenvalue shifts to $\lambda_\ell^{(s)} = L^{-1}(s \hat{L}_\ell)$. The equilibrium mapping from interests to actions, expressed in the original units, is unchanged, because the two appearances of s cancel: $\frac{1}{s} L(\lambda_\ell^{(s)}) = \frac{1}{s} L(L^{-1}(s \hat{L}_\ell)) = \frac{1}{s} \cdot s \hat{L}_\ell = \hat{L}_\ell$.

¹¹A rank-five reconstruction of the network need not be entrywise nonnegative, and the estimated networks contain some negative entries (14 percent of off-diagonal entries averaged across decades). Nevertheless, the condition for the influence problem to remain well-posed with negative entries (Online Appendix J.4) is satisfied for our estimated and counterfactual networks, and none of our quantitative results below relies on nonnegativity.

ter that we estimate to match an empirical moment in the data. We use GDP on a purchasing power parity (PPP) basis, which is available for all countries (including former communist countries) back to 1960.¹² Intuitively, β captures the asymmetry of the network; larger β exaggerates differences in country size, whereas $\beta \rightarrow 0$ returns a symmetric network. We choose the parameter β such that the model replicates the empirical relationship between the ratio of outward to inward influence and GDP in the following OLS regression specification:

$$\ln \left(\frac{\text{influence exerted}}{\text{influence received}} \right)_{id} = a_d + b_d \ln \text{GDP}_{id} + \epsilon_{id}, \quad (23)$$

where influence exerted is the average influence of a sender country on all receiver countries within each decade; influence received is the average influence of a receiver country from all sender countries within each decade; GDP_{id} is the average GDP of country i in decade d ; ϵ_{id} is a stochastic error; and we estimate this regression separately for each decade.

We first estimate this regression using actual data on bilateral foreign influence from the Formal Bilateral Influence Capacity (FBIC) dataset, which measures state-sanctioned or state-sponsored and publicly acknowledged bilateral dependence between countries. Bilateral dependence is measured in the data based on diplomatic exchange, arms transfers, and goods trade. Critically, bilateral dependence is asymmetric—in a given pair, one country can depend more on the other. Consistent with the predictions of our model, in which more important countries send more outward influence to other countries relative to the inward influence they receive from other countries, we find positive and statistically significant estimates ranging from $\hat{b}_d = 0.708$ to $\hat{b}_d = 0.777$ using these data. In Online Appendix L.3.2, we report the full set of decade-specific coefficients in Table A5.

We next estimate the same regression on model-predicted foreign influence. To obtain model-predicted foreign influence, we start with an assumed value for β_d , and generate the π_d vector using $\pi_{id} = \text{GDP}_{id}^{\beta_d}$. Given the resulting Π_d matrix, we estimate the common principal components for geopolitical actions and domestic interests as in the previous subsection, and construct the asymmetric geopolitical network (G_d) for our assumed value of β_d . Given this asymmetric geopolitical network (G_d) and the observed actions (a_d) from UN ideal points, we can invert our model's equilibrium conditions for actions and influence to recover the implied matrix of bilateral influence (Ω_d). Using this model-predicted influence matrix (Ω_d), we can compute the average influence exerted by a sender country on all receiver countries, and the average influence experienced by a receiver country from all sender countries. Given these predictions for outward and inward influence for our assumed value of β_d , we estimate the coefficient b_d in the regression (23). Finally, we choose β_d so that the coefficient estimated on model-generated influence equals the coefficient estimated on the FBIC data.

The decade-specific β_d exponents range from 0.687 to 0.814, with the full set of values reported

¹²Market exchange rates are unavailable for China, Russia, and other planned economies for much of our sample.

in Table A6; for simplicity, we use their median, $\beta = 0.75$, for all decades. Given this parameter value, we set $\pi_{id} = \text{GDP}_{id}^{0.75}$ for each decade, and use the resulting Π_d matrix to estimate the common principal components and recover the unobserved geopolitical network in each decade ($G_d = \Pi_d^{-1/2} U_d \Lambda_d U_d^\top \Pi_d^{1/2}$) using the top-five eigenvectors.

3.4 Validation of Inferred Geopolitical Network and Strategic Influence

3.4.1 Inferred Geopolitical Network

We implement a number of validation checks on our inferred geopolitical network. First, we show that our inferred network links are systematically related to empirical proxies for these connections. We estimate a bilateral regression of our inferred network connections for receiver i and sender j in decade d (G_{ijd}) on each of these empirical proxies (x_{ijd}), including receiver-decade (α_{id}) and sender-decade (γ_{jd}) fixed effects, so that b is identified from bilateral variation alone:

$$G_{ijd} = b \times x_{ijd} + \alpha_{id} + \gamma_{jd} + \epsilon_{ijd}.$$

Table 1 reports the estimation results using standard deviation units for all continuous variables. Countries that are more distant, both in terms of geographic distance (column 1) and in political/ideological differences (column 2), have significantly lower edge values, indicating lower weights in the geopolitical network. In contrast, countries that trade more (column 3) or share cultural/historical ties (columns 4–6) have stronger geopolitical connections. Therefore, our inferred geopolitical network is systematically related in the expected way to these empirical proxies for network connections between countries.

This pattern of results is not simply mechanical. The eigenvalues that determine the recovered geopolitical network are identified from the ratio of action dispersion to interest dispersion along each cleavage, rather than from the level of either, and it is far from obvious that this transformation of the data should be related in the expected way to these empirical proxies.¹³ Overall, these results confirm that our recovered geopolitical network exhibits sensible cross-sectional variation that qualitatively matches economic and cultural interactions between countries.

In Online Appendix L.3.3, we implement two further robustness checks. First, we undertake a permutation placebo (Figure A4a), in which we randomly relabel the countries in the action covariance matrix (or both the action and interests covariance matrices) before recovering G_d . This severs the true correspondence between a country's actions and its interests, so that any structure that survives is mechanical rather than substantive. This exercise shows that the regression coefficients

¹³The edges in our geopolitical network G_d are given by $G_{ijd} = \sqrt{\pi_{jd}/\pi_{id}} \sum_{m=1}^5 \lambda_{md} [U_d]_{im} [U_d]_{jm}$, where $\lambda_{1d} = 1 - \alpha$ is the imposed consensus eigenvalue and $\lambda_{md} = L^{-1} \left(\sqrt{\sigma_{a,md}^2 / \sigma_{\theta,md}^2} \right)$ for $m \geq 2$. Therefore, these edges involve a linear combination of (i) the structural importance weights (π_{id}), (ii) the eigenvalues of the geopolitical network (λ_{md}), and (iii) entries of the joint principal components (the columns of U_d). The eigenvalues in turn depend on the *relative* responsiveness of geopolitical actions and domestic interests for each geopolitical cleavage.

Table 1: Geopolitical Network Edges and Measures of Countries' Ties

	Outcome: G_{ijd}					
	log(Distance _{ij})	Polity _{id} - Polity _{jd}	log(Trade flow _{ijd})	Common religion _{ij}	Common language _{ij}	Common colonizer _{ij}
	(1)	(2)	(3)	(4)	(5)	(6)
x_{ijd}	-0.0879*** (0.00679)	-0.0937*** (0.00794)	0.0363*** (0.00882)	0.0416*** (0.00452)	0.0927*** (0.0199)	0.0567*** (0.00976)
Observations	112144	104846	61582	110218	112144	112144
Sender $\times$ decade FE:	✓	✓	✓	✓	✓	✓
Receiver $\times$ decade FE:	✓	✓	✓	✓	✓	✓

Note: This table shows the regression of the ij -th entry of the G network for decade d on decade-level averages of dyadic variables. The covariate in Column (1) is the log of geographical distance between country i and country j . The covariate in Column (2) is the distance between country i and j 's average polity scores in each decade d . The covariate in Column (3) is the log of exports from country i to j in decade d . Logs are taken after averaging the raw variables within each decade. Columns (4)–(6) have covariates describing cultural similarities: Column (4) is a measure of countries' religious proximity, and Columns (5)–(6) are indicators of common language or common colonizer. We standardize the dependent variable within decade and the continuous covariates (columns 1–4), whereas the covariates in Columns (5)–(6) are binary indicators. CEPII trade flow data (column 3) go back to 1990, whereas polity data (column 2) cover the entire panel. Fixed effects are separate sender ($j \times$ decade) and receiver ($i \times$ decade) effects. Standard errors are clustered on the unordered pair level, with significance levels denoted * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

shrink toward zero and most become statistically insignificant.

Second, we undertake a residual bootstrap exercise (Figure A4b), in which we examine the sensitivity of our validation regressions to the variation discarded by the rank-five reconstruction. Because our estimation procedure estimates a common basis for the two covariance matrices, imperfect fit in the joint basis means that the original covariance matrices have some portion that is explained by the top-five joint principal components, and some part that is a residual. Holding the retained components fixed, we repeatedly replace the residual with a random matrix that keeps its eigenvalues but replaces its eigenvectors with a random orthonormal set, so that the residual has the same overall magnitude but its weight is spread across arbitrary country pairs. We then re-estimate G_d and re-run our validation regressions.

These two robustness checks behave as they should if G_d captures underlying geopolitical ties: the validation coefficients are not statistically distinguishable from zero when the inputs are scrambled, but the coefficients remain stable in sign and magnitude when we bootstrap the residual. Together these two exercises indicate that the estimated G_d networks do not simply reflect some mechanical consequences of our estimation procedure, but instead recover geopolitical structure whose relation to external measures like distance or trade behaves as expected.

We find that these results are robust across alternative values of our three calibrated parameters: (i) the weight α on domestic interests, (ii) the cost of influence κ and (iii) the eigenvalue gap ϵ between the Perron eigenvalue and the dominant non-Perron eigenvalue. In our baseline specification, we assume $\alpha = 0.4$, $\kappa = 5$, and $\epsilon = 0.01$. In Online Appendix L.3.4, we demonstrate a similar qualitative pattern of results across the following parameter values: $\epsilon \in [0.005, 0.1]$, $\alpha \in [0.3, 0.5]$, and $\kappa \in [4, 10]$. We find that the coefficients in Table 1 carry the same sign for parameter values in these intervals, and nearly all remain statistically significant (Figure A5). The between-country rankings of the three notions of power that we introduce in Section 5 below retain a correlation of at least 0.96 across all these parameter combinations (Table A7).

3.4.2 Inferred Strategic Influence

We now provide additional validation for our model’s predictions for bilateral strategic influence using separate data on state visits. We use the Country and Organization Leader Travel (COLT) dataset’s records of state visits between the heads of state/government of different nations. We measure strategic influence using the number of state visits for each country-pair-year, since many of the most important efforts at cross-country coordination happen in bilateral country visits or forums like the G7. These state visits are plausibly undirected, since France’s president traveling to the US versus the US president traveling to France does not carry a substantially different interpretation. Therefore, we aggregate state visits at the country-pair-year level.

We generate model predictions for strategic influence following a similar procedure as for the estimation of the vector of structural importance (π_d). We first take UN resolution votes in each year t as the empirical actions ($\mathbf{a}_t$) and back out domestic interests (θ_t) using the equilibrium condition for actions ($\mathbf{a}_t = \mathcal{L}(\mathbf{G}_d)\theta_t$). We then invert our model’s first-order condition for influence to recover the implied matrix of bilateral influence (Ω_t).

In the model, influence ω_{kit} can be negative or positive as countries may want others to take a higher or lower action. State visits, however, measure the magnitude of influence rather than the political intention/direction of influence. To make the model-implied object match the structure of state visits in the data, we compute the average absolute magnitude of inward and outward influence for each country pair in the model, such that its predictions are also symmetric/undirected. We denote the ij -th element of the matrix of averaged absolute values of influence as $\bar{\omega}_{(ij)t}$. We then estimate the following regression between empirical state visits for pair ij and time period t (which can be a year or window of years) and our model’s predictions ($\bar{\omega}_{(ij)t}$):

$$\text{State visits}_{(ij)t} = b\bar{\omega}_{(ij)t} + \gamma_{(i)t} + \gamma_{(j)t} + \mathbf{X}_{(ij)t} + \epsilon_{(ij)t}$$

where the subscript notation emphasizes that the regression is unordered, such that our outcome variable is only defined on the unordered pair involving i and j , and not on the ordered pair $i \rightarrow j$ versus $j \rightarrow i$. Consistent with this, we denote the country-period fixed effects as $\gamma_{(i)t}$ to indicate that we have a fixed effect for whenever country i appears in a pair in period t (and similarly for country j). Additionally, $\mathbf{X}_{(ij)t}$ denotes a set of bilateral controls: log distance, log trade, as well as measures of political and cultural similarity, as used in Table 1.

In our model, influence is nonzero for each country pair. In the data, many country pairs record zero state visits in a given year, which we interpret in terms of our model as state visits only being observed when influence is sufficiently large. In our baseline specification, we report results for country pairs with at least one state visit in a given year. To allow for the inclusion of country pairs with no visits, and thus incorporate both the intensive and extensive margins of state visits, we aggregate the data into non-overlapping three and five-year windows. In these specifications, country pairs

Table 2: State Visits and Model-implied Influence

	Outcome: state visits _{(ij),t}					
	Annual		3-year bins		5-year bins	
	(1)	(2)	(3)	(4)	(5)	(6)
Influence, $\bar{\omega}_{(ij)t}$	9.2180*** (2.9034)	4.5963** (1.9241)	31.3730*** (10.0312)	15.9142** (7.0897)	42.0385*** (13.5018)	18.5346* (9.4734)
Shared country-period FE	✓	✓	✓	✓	✓	✓
Bilateral controls	✗	✓	✗	✓	✗	✓
Observations	24,572	24,572	16,809	16,809	13,468	13,468

Note: This table shows OLS regressions of (positive) state visits on model-implied influence. Visits are averaged across both directions of each unordered country pair. Influence averages absolute model-implied influence over both directions and each year’s qualifying UN roll calls. Roll calls require observed votes from strictly more than 75% of network countries; remaining missing votes equal zero. Columns (1)–(2) use annual observations, Columns (3)–(4) use three-year bins, and Columns (5)–(6) use five-year bins. The data is binned as fixed, non-overlapping windows: for 3-year bins, this means the first bin is 1990–1992, the second is 1993–1995, etc. (and similarly for five year bins). Bins may cross decades. A pair enters a bin if it has a recorded visit in at least one year in that bin. State visits are summed, and influence and controls are averaged, over all years in that bin. Every column includes country-period fixed effects. Columns (2), (4), and (6) add log distance, log period-mean bilateral trade, the absolute gap between period-mean Polity scores, common language, common colonizer, and religious proximity. Standard errors are clustered by unordered country pair. Significance levels are denoted * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

are included if they have one or more visits in a three or five-year window.

Table 2 reports the estimation results. Columns (1)–(2) show the coefficients for year-level regressions, whereas Columns (3)–(4) use three-year windows, and Columns (5)–(6) consider five-year windows. As the length of the windows increases, we allow for more years within that window with zero visits. Therefore columns with larger window sizes are those which include more extensive margin variation. Across all of these different specifications, we find a strong relationship between our model’s predictions for the magnitude of foreign influence and state visits in the data.

These results are robust to the inclusion of fixed effects and bilateral controls. All specifications feature country-period fixed effects, which control for the fact that some countries may have more state visits for reasons other than those in the model. Columns (2), (4), and (6) include bilateral controls (e.g., log distance and log trade) for (possibly time-varying) pair-level determinants of state visits from sources other than model-implied influence. The results show that the model’s implied levels of influence are robustly associated with actual state visits.

4 Properties of the Geopolitical Network

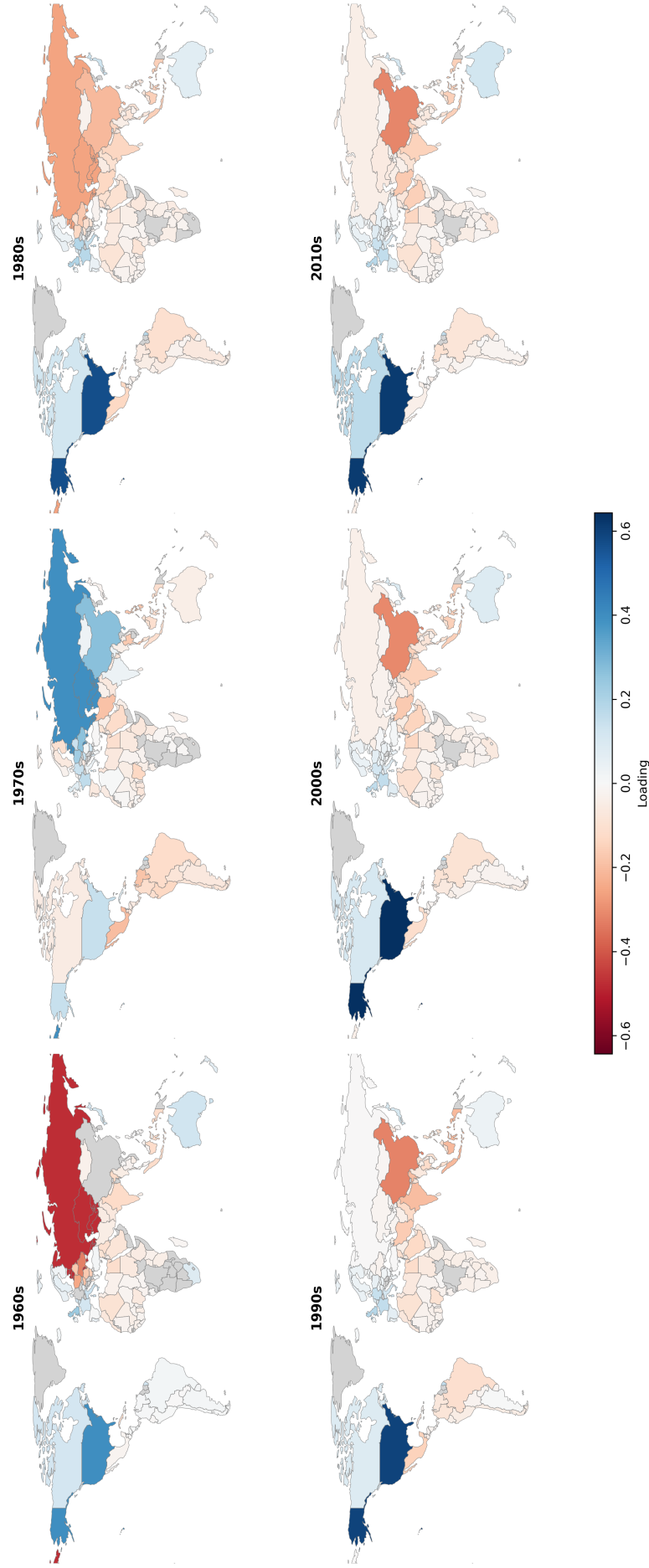
We now provide evidence on the empirical properties of our inferred geopolitical network.

4.1 Recovered Geopolitical Cleavages and Their Evolution

The leading cleavage of the estimated network—the dimension along which tightly connected blocs agree internally and disagree with one another—tracks the major realignments of the postwar period. Figure 1 maps u_2 , the eigenvector associated with the second-largest eigenvalue, by decade, with red and blue denoting opposite sides of the cleavage.

The evolution of the leading cleavage tracks major shifts in the structure of international politics. In the 1960s, it aligns closely with the bipolar Cold War division between the Western and Soviet blocs. During the 1970s, however, the leading cleavage shifts toward a divide between the major

Figure 1: The Leading Cleavage by Decade



Note: Figure shows the second eigenvector, which captures the network's leading cleavage, for which tightly connected blocs exhibit high internal consensus but cross-bloc polarization; second eigenvector displayed by decade; heatmap from red to blue, where the pair of countries with the darkest red and the darkest blue are those with the greatest geopolitical cleavage. All maps use Natural Earth's country-level dataset (version 5.1.1, released in May 2022), with the same geographic boundaries used across all decades. The country boundaries follow the source dataset and do not imply any position by the authors regarding sovereignty, territorial claims, or political status.

powers—including the US and the Soviet Union—and the developing world, reflecting the growing political salience of the Third World (e.g., Prashad 2007). This is the decade in which the General Assembly, at the initiative of developing countries, adopted the 1974 Declaration on the Establishment of a New International Economic Order. By the 1980s, the leading cleavage returns to the East–West divide, before changing sharply after the collapse of the Soviet Union.

The post-1989 realignment is especially visible in the spatial distribution of this cleavage. Former Soviet-aligned states in Eastern Europe, such as Poland and the Czech Republic, both of which joined NATO in 1999, move toward the Western bloc. At the same time, the former Soviet bloc no longer anchors the same extreme position it occupied during the Cold War. Since the 1990s, the leading cleavage has instead been organized around a configuration in which the US and Western Europe occupy one pole, China occupies the other, and many countries in the Global South lie in between.

This evolution also captures changes in the position of groups of countries relative to the Cold War divide. From the 1970s onward, countries in Asia, Africa and Latin America appear as a more cohesive Global South, including many former colonies of Western powers that are generally positioned away from the Western bloc. Notably, China loads with the major-power cleavage during this period, although Deng Xiaoping used a 1974 address to the General Assembly to place China in the Third World. China’s position is consistent with the PRC’s entry into the UN, with a permanent Security Council seat, in 1971, and with its rapprochement with the US. Over the post-2000 decades, we find a broader structural reorganization of the international network, with China emerging as the central pole of a geopolitical bloc opposed to the US and Western Europe. Thus, the estimated eigenvector recovers not only the Cold War cleavage itself, but also the subsequent realignments through which that cleavage was transformed.¹⁴

One might wonder whether this result is mechanical, since the East–West divide is visible in raw UN votes during the Cold War, and in domestic institutions as well. Neither fact alone produces the ranking. What makes the East–West divide the network’s leading cleavage is that votes diverge along it by more, relative to domestic interests, than along any other dimension (Proposition 8). Our procedure recovers this cleavage, and its later transformations, from the data without imposing them.

We construct our geopolitical network from the top-five geopolitical cleavages, as discussed above, setting the eigenvalues of the remaining cleavages to zero. In Online Appendix L.5, we provide further evidence on its descriptive properties. We map the network connections of the US and China, as the two leading powers at the end of our sample period. We show both the outward network (the importance of other countries to either the US or China) and the inward network (the importance of either the US or China to other countries). Because the US and China are structurally important,

¹⁴If we regress our geopolitical network connections on separate measures of economic and security dependence, we find that security dependence dominates in nearly all decades, with the gap growing smaller until the 2010s, when the coefficients on security and economic dependence are statistically indistinguishable from each other. This is consistent with globalization increasing the relative importance of economic links (see Online Appendix L.4).

the weight other countries place on them exceeds the weight they place on others. China has strong connections to countries such as Russia, North Korea, and India, with which the US has traditionally had weak links. The US has the strongest connections with Canada and Western Europe, while China has some of its strongest connections in Asia.

In Online Appendix L.6, we provide wider evidence on the heterogeneity in network connections. The US and China have negatively correlated outward networks and negatively correlated inward networks. France and Germany have highly correlated networks in both directions. China and Russia have negatively correlated outward networks but positively correlated inward networks. From our regressions of bilateral network connections on a range of empirical proxies that were discussed in Subsection 3.4, this heterogeneity is driven by differences in economic size, geography, and history, as well as cultural similarities.

4.2 Direct Versus Indirect Network Connections

Influence often reaches its target indirectly. A great power that courts a regional leader also moves the neighbors that coordinate with it, and a country may be worth influencing less for its own actions than for those of its partners.

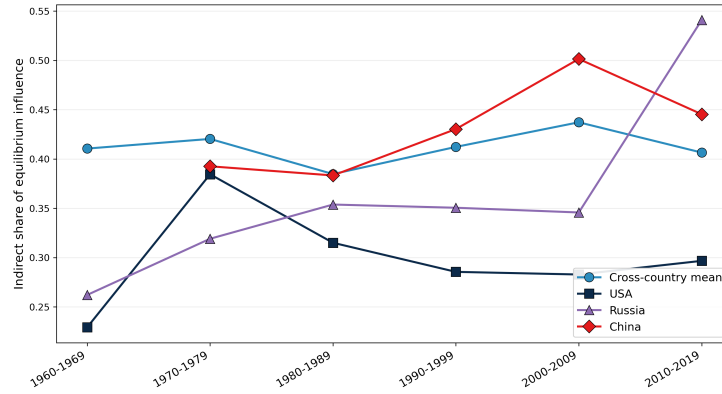
We next use our geopolitical network to quantify the importance of indirect versus direct connections between countries. We simulate alternative international issues by taking random draws for domestic interests from a normal distribution with mean zero and a variance-covariance matrix equal to our estimated cross-country covariance matrix for each decade ($\Sigma_{\theta,d}$). For each simulation, we use the vector of realizations for domestic interests (θ) and our geopolitical network (G_d), together with the first-order conditions in Proposition 7, to solve for equilibrium influence (Ω) and geopolitical actions (a). Finally, we use these first-order conditions to decompose the equilibrium influence from country i to country k as follows:

$$\kappa(\pi_k/\pi_i)\omega_{ki} = \underbrace{G_{ik}(a_i - a_k)}_{\text{direct}} + \underbrace{\sum_j G_{ij}(a_i - a_j)G_{jk} + \sum_j G_{ij}(a_i - a_j)(G^2)_{jk} + \dots}_{\text{indirect}}, \quad (24)$$

for $k \neq i$, where the first term captures the direct influence that country i has on country k ; the remaining terms capture the indirect or higher-order influence that country i has on country k (through k 's immediate neighbors, from the neighbors of its immediate neighbors, and so on).

We compute the average shares of direct and indirect influence in equation (24) across 1,000 simulations for each decade. In Figure 2, we display these average shares of indirect influence in overall influence for all countries (blue line with circles), the US (black line with squares), Russia (purple line with triangles), and China (red line with diamonds). Countries differ in how much of their influence runs through indirect connections. Relative to the cross-country average of about 40 percent, the US share is lower in every decade, at roughly 30 percent since the 1980s, consistent with a dominant

Figure 2: Share of Indirect Influence in Overall Influence



Note: Average shares of indirect influence in overall influence in each decade of our sample across 1,000 simulations. Shares of indirect influence computed by taking random draws for domestic interests from a normal distribution with mean zero and a variance-covariance matrix equal to our estimated cross-country covariance matrix for each decade ($\Sigma_{\theta, d}$). For each vector of realizations for domestic interests (θ), we use our inferred geopolitical network for that decade (G_d) and the first-order conditions in Proposition 7 to compute equilibrium influence (Ω) and geopolitical actions (a) and implement the decomposition in equation (24).

power achieving more through direct bilateral ties. China’s share, by contrast, exceeds the world average from the 1990s onward, and Russia’s rises above half in the 2010s. These patterns suggest that both countries’ influence runs disproportionately through their partners, consistent with China’s high control and Russia’s outsized footprint in the next section.

5 Counterfactuals

Debates about international politics are often debates about counterfactuals. Would a democratic China align with the West? Do America’s alliances extend its reach or constrain it? Could Washington pull Moscow away from Beijing? Which way will India lean, and how much would it matter? Each question requires two things: separating what countries want from the pull of their partners, and tracing how a change in one part of the network travels through direct and indirect connections. Observed alignments alone allow neither; the estimated network provides a way to do both.

Our counterfactuals fall into two families that correspond to the two forces in the model, and to two long-standing views of what drives international politics. The first changes domestic interests while holding the network fixed. This is in the spirit of theories in which foreign policy reflects domestic preferences and institutions (Moravcsik 1997). The second changes network connections while holding interests fixed, in the spirit of theories that emphasize alliances and the structure of the international system (Waltz 1979; Snyder 1997). Because the network aggregates all channels, these exercises change the relationship between two countries as a whole—economic, security, and diplomatic ties together—rather than any single instrument.

Within each family, we report an exercise motivated by a specific policy debate and an exercise applied to every country in turn. Our general exercises deliver three notions of power. “Autonomy” is the share of a change in a country’s own interests that shows up in its own actions. “Control” is how much other countries’ actions move per unit of movement in its own action. The “ability to shape

the game” is how much the configuration of all other countries’ actions would change if the country were removed from the network, analogous to key player intercentrality in Ballester et al. (2006).

Since equilibrium actions are linear in domestic interests, $\mathbf{a} = \mathcal{L}(\mathbf{G})\boldsymbol{\theta}$ (Proposition 1), a change $\Delta\boldsymbol{\theta}$ in interests moves actions by $\mathcal{L}(\mathbf{G})\Delta\boldsymbol{\theta}$, and a change in the network from $\mathbf{G}$ to $\mathbf{G}'$ moves actions from $\mathcal{L}(\mathbf{G})\boldsymbol{\theta}$ to $\mathcal{L}(\mathbf{G}')\boldsymbol{\theta}$. Therefore, our counterfactuals are determined by properties of the operator $\mathcal{L}(\mathbf{G})$ and the covariance matrix of domestic interests ($\boldsymbol{\Sigma}_\theta$). They describe what the estimated network implies within the model, holding fixed everything not explicitly changed. In particular, they do not capture the further rewiring of the network that a large shock to interests might trigger.

5.1 Counterfactuals for Domestic Interests

We begin by quantifying the geopolitical implications of changes in countries’ interests. We start with an illustrative example for the two most structurally important countries in the final decade of our sample, in which we replace the domestic interests of China with those of the US. We next implement a general exercise, in which we change the domestic interests of each country in turn, and quantify our first two notions of geopolitical power for each of them.

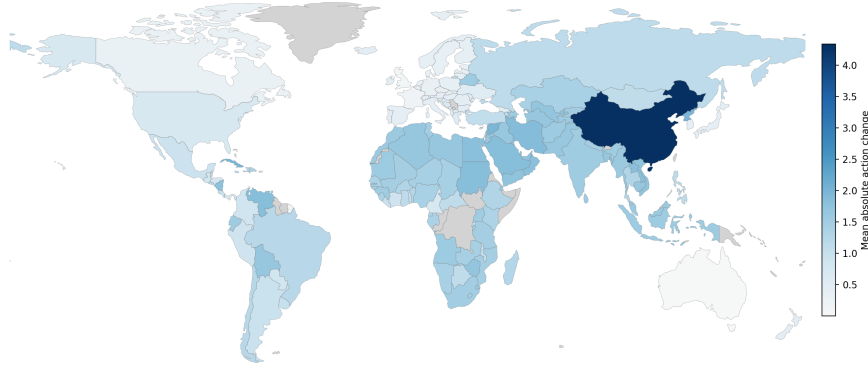
5.1.1 China with US Interests

For much of the post–Cold War period, Western policy toward China rested in part on a two-step forecast: economic growth would bring political liberalization, and political liberalization would bring China’s foreign policy closer to that of the West (e.g., Fukuyama 1992). Most of the debate has concerned the first step. We examine the second. Because domestic interests are measured by democratic orientation, we ask how China’s actions, and those of every other country, would change if China’s domestic political institutions matched those of the US. We hold China’s relationships with Russia, its neighbors, and its trading partners constant. The same question arises for any regime transition, from Eastern Europe after 1989 to more recent cases.

We set China’s interest equal to that of the US, holding all other countries’ interests and network connections unchanged at their values from 2010–2019. Since actions are linear in interests, the change in each country’s action is $\Delta a_i = \mathcal{L}(\mathbf{G})_{i,CHN} (\theta_{USA} - \theta_{CHN})$, and we summarize the response by its expected magnitude $\mathbb{E}[|\Delta a_i|] = |\mathcal{L}(\mathbf{G})_{i,CHN}| \mathbb{E}[|\theta_{USA} - \theta_{CHN}|]$, which captures the typical response of a country. We compute the expectation by drawing international issues from a normal distribution with mean zero and covariance matrix equal to our estimated covariance matrix of domestic interests ($\boldsymbol{\Sigma}_{\theta,d}$) for 2010–2019. We evaluate the expectations by averaging over 1,000 issues drawn from this distribution. Figure 3 shows a map of these counterfactual mean changes in actions, while Figure A10 displays the same information as a histogram.

China’s own action moves the most, but only part of the way. With full pass-through, China’s action would move as much as its interest does, by 11.3 on average ($\mathbb{E}[|\theta_{USA} - \theta_{CHN}|]$). Instead it moves by 4.3, or 38 percent of full pass-through. The remainder is absorbed by China’s incen-

Figure 3: Changes in Each Country’s Action in Response to Setting China’s Domestic Interest Equal to the US Domestic Interest



Note: Map of the mean absolute change $\mathbb{E}[|\Delta a_i|]$ in each country’s equilibrium action when China’s interest is set equal to that of the US, equal to the product of the absolute entry $|\mathcal{L}(\mathbf{G})_{i,CHN}|$ and the expected absolute gap between US and Chinese interests. All maps use Natural Earth’s country-level dataset (version 5.1.1, released in May 2022), with the same geographic boundaries used across all decades. The country boundaries follow the source dataset and do not imply any position by the authors regarding sovereignty, territorial claims, or political status.

tive to coordinate with partners—Russia, India, and other regional powers—whose interests are unchanged. This 38 percent is China’s autonomy, which Section 5.1.2 compares across countries. Convergence in domestic institutions would therefore deliver only partial convergence in geopolitical behavior, and China’s equilibrium actions remain anchored to its network partners. This result illustrates how established geopolitical relationships constrain countries’ behavior, limiting the extent to which changes in domestic interests translate into changes in actions.

The changes in actions that do occur spread through the network beyond China. Countries whose own interests are unchanged move as well, with the largest adjustments in Syria, North Korea, and Iran. Domestic political change in a major power thus reshapes its partners’ positions even when its own position remains anchored.

In Online Appendix L.8.1, we report the analogous counterfactual in which US interests converge toward those of China. The qualitative patterns are similar: changes in the domestic interests of a major power propagate throughout the geopolitical network, affecting the equilibrium actions of many countries whose own interests remain unchanged.

5.1.2 Autonomy and Control

The China exercise generalizes to every country. When a country’s interests change—through an election, a revolution, or a shift in public opinion—two questions follow. How much does its own behavior change (a measure of autonomy), and how much does the rest of the world follow (a measure of control)?

We consider a change in one country’s domestic interest, holding all other countries’ interests and network connections unchanged, and recompute equilibrium actions. We compute a country’s autonomy as the change in its actions relative to the change in its own interests: $|\Delta a_i|/|\Delta \theta_i|$, which equals the diagonal entry $|\mathcal{L}(\mathbf{G})_{ii}|$. We compute a country’s control as the average absolute change

in other countries' actions relative to the absolute change in its own action: $\frac{1}{N-1} \sum_{j \neq i} |\Delta a_j| / |\Delta a_i|$, which equals $\frac{1}{N-1} \sum_{j \neq i} |\mathcal{L}(\mathbf{G})_{ji}| / |\mathcal{L}(\mathbf{G})_{ii}|$. Both measures depend on entries of the operator $\mathcal{L}(\mathbf{G})$, and hence do not depend on the issue or the size of the change in interests.

Panel (a) of Figure 4 maps our autonomy measure: the ability to translate one's own interests into one's own actions. Even the US, the most autonomous country, passes only 60 percent of a change in its interests into its actions, followed by China (38 percent) and India (30 percent).

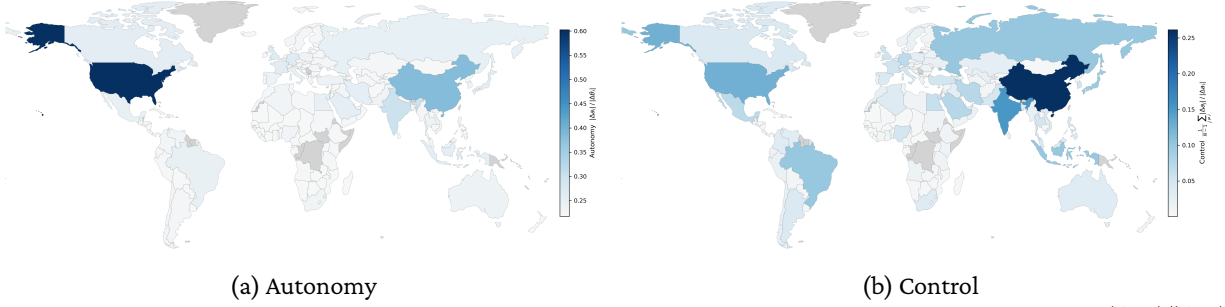
Panel (b) maps our control measure: the movement induced in the actions of others, per unit of one's own movement in action. China stands alone in control. Each unit of movement in China's action moves the average other country by 0.26 units, about twice the values for India (0.15) and the US (0.13). This contrast reflects the estimated pattern of connections rather than country size: China and the US have nearly the same structural importance in the 2010s, but other countries place more than twice the weight on China that they place on the US. This pattern of results is consistent with China's influence through the Belt and Road Program and other initiatives, and with China's having overtaken the US as the larger trade and investment partner of many countries.

To interpret these values of control in terms of real-world actions, we use the 766 UN General Assembly roll calls of 2010–2019 in which more than 75 percent of countries cast a vote. We treat each country's observed vote on a roll call (1 for yes, 0 for abstain, –1 for no) as its equilibrium action. We use the first-order conditions in Proposition 7 to recover the domestic interests that rationalize this roll call vote. We next move one country's recovered interest across the interquartile range of the cross-country distribution of interests for that roll call vote (to the 25th percentile if it lies closer to the 75th, and to the 75th percentile otherwise). Finally, we round the recomputed actions back to the nearest integer of –1, 0, and 1. Over the decade, moving China's interests changes 6,088 votes of other countries; moving US interests changes 3,408 votes; moving India's interests changes 249 votes; whereas there is no other country for which a shock to interests changes more than five votes. A vote changes only when the induced movement in another country's action exceeds one half, which requires both high control and high autonomy. This property explains why the US ranks above India in votes changed despite ranking below it in control. Autonomy and control are positively correlated across countries (0.69; Figure A12 in Online Appendix L.8.1). But the two superpowers differ on these dimensions: the US leads on autonomy, whereas China leads on control.

5.2 Counterfactuals for Network Connections

We next show how our framework can be used to quantify the geopolitical implications of changes in network connections. We start with some illustrative examples of changes to the geopolitical network. We then implement a general exercise in which we eliminate each country in turn from the network and recompute the equilibrium among the remaining countries, which allows us to illustrate our third notion of geopolitical power based on the structure of the game.

Figure 4: Autonomy and Control in the 2010–2019 Geopolitical Network



Note: Panel (a) maps a country's autonomy, measured as the size of its own action change relative to the size of its own interest change ($|\Delta a_i|/|\Delta \theta_i|$), which equals the diagonal entry $|\mathcal{L}(\mathbf{G})_{ii}|$ of the operator mapping interests to actions; complete pass-through of interests into actions equals one. Panel (b) maps a country's control, measured as the average across the other countries of the size of the change in their actions relative to the size of the change in the country's own actions in response to the change in its interests ($\frac{1}{N-1} \sum_{j \neq i} |\Delta a_j|/|\Delta a_i|$), which equals $\frac{1}{N-1} \sum_{j \neq i} |\mathcal{L}(\mathbf{G})_{ji}|/|\mathcal{L}(\mathbf{G})_{ii}|$; one-for-one movement of every other country equals one. Both are ratios of entries of the operator and do not depend on the issue or on the size of the change in interests. All maps use Natural Earth's country-level dataset (version 5.1.1, released in May 2022), with the same geographic boundaries used across all decades. The country boundaries follow the source dataset and do not imply any position by the authors regarding sovereignty, territorial claims, or political status.

5.2.1 Individual Network Connections

Recent disputes between the US and its European allies, over trade, defense spending, and Greenland, have revived an old question in alliance theory: do alliances extend a great power's reach or constrain its freedom of action (Morrow 1991; Snyder 1997)? First, we examine a 75 percent weakening of network connections between the US and its European NATO allies.¹⁵ Second, we evaluate the same 75 percent weakening of network connections between China and Russia, as the two leading Eastern powers. Third, we consider the implications of India (as a large country from the Global South) choosing between the Western and Eastern powers, by raising India's directed network weight on either the US and its European NATO allies or China and Russia. We add a total weight W to India's connections with these countries in $\mathbf{G}$, taken from its diagonal entry so that its row sum is unchanged. We choose W for each target group so that the overall size of the change in the network, measured by the Frobenius norm of the change in the off-diagonal entries of $\mathbf{G}$, equals that of the 75 percent weakening of the links between the US and its European NATO allies.¹⁶

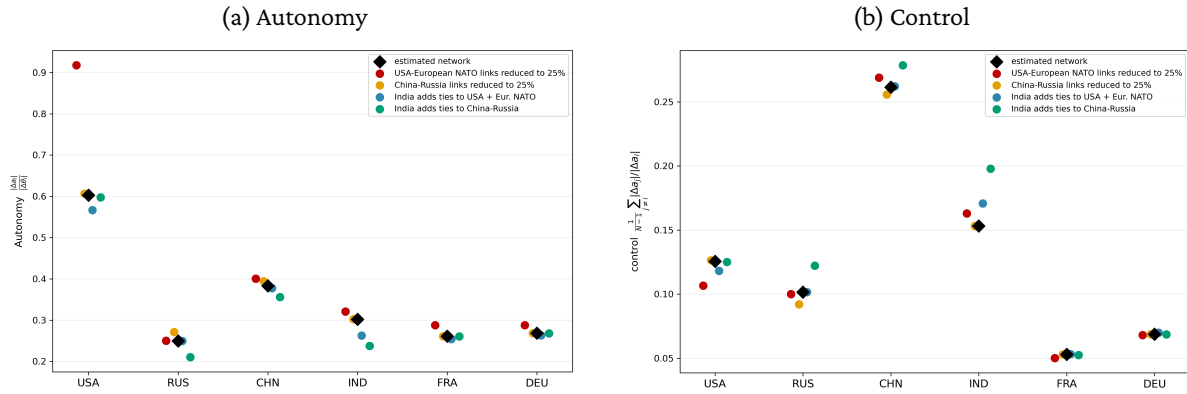
In Figure 5, we display the levels of prominent countries' geopolitical power (as measured by autonomy and control) at baseline and for each of these changes in network connections. Because autonomy and control are functions of the operator $\mathcal{L}(\mathbf{G})$, the counterfactual values are the diagonal elements (for autonomy) or ratios of elements (for control) of $\mathcal{L}(\mathbf{G}')$ for each edited network $\mathbf{G}'$. The

¹⁵We implement changes in network connections to the symmetric matrix $\mathbf{H}$ of Section 2.6, scaling the entries $H_{ij} = H_{ji}$ of the affected pairs to one quarter of their estimated values (a 75 percent reduction) and recomputing $\mathbf{G} = (1 - \alpha) \mathbf{\Pi}^{-1} \mathbf{H}$. After each change, each country's diagonal entry H_{ii} is reset so that $\sum_j H_{ij}$ returns to π_i , which keeps every row sum of $\mathbf{G}$ equal to $1 - \alpha$. Therefore, the removed bilateral weight is reassigned to the diagonal, with structural importance held fixed.

¹⁶We increase India's connection $H_{IND,j} = H_{j,IND}$ with each targeted country j , with the added directed weight $\Delta G_{IND,j} = (1 - \alpha) \Delta H_{IND,j} / \pi_{IND}$ proportional to $\sqrt{\pi_j}$ and scaled so that $\sum_{j \in T} \Delta G_{IND,j} = W$. Because $\mathbf{H}$ is symmetric, j 's weight on India rises by $\Delta G_{j,IND} = (\pi_{IND} / \pi_j) \Delta G_{IND,j}$, since $G_{ij} / G_{ji} = \pi_j / \pi_i$.

transatlantic link both constrains and empowers the US. When the link is weakened, US autonomy rises from 0.60 to about 0.90, while its control falls from 0.13 to about 0.11. European allies pull US actions toward their own interests, and the same ties carry US actions to the rest of the world. Weakening the alliance would leave the US freer to act on its own interests and less able to bring others along. This pattern is consistent with the view that the United States’ outsized geopolitical influence stems in part from its alliances. France, by contrast, gains some autonomy and loses almost no control, because its position depends mostly on its ties within the European Union.

Figure 5: Impact of Changes in Network Connections on Countries’ Autonomy and Control



Note: This figure plots the levels of autonomy (panel a) and control (panel b) (as defined in Subsection 5.1.2) for the US, Russia, China, India, France, and Germany under different network-edit counterfactuals. Each marker corresponds to a different exercise, as labeled in the legend; the black marker denotes the baseline value. The red marker denotes the weakening of network connections between the US and its European NATO allies; the orange marker for the weakening of network connections between China and Russia; the blue marker for the strengthening of India’s connections with the US and its European NATO allies; the green marker for the strengthening of India’s directed connections with China and Russia.

Whether Washington could pull Moscow away from Beijing—a “reverse Kissinger,” after the US opening to China in 1971–72 that exploited the Sino-Soviet split—has been a recurring question in US policy debates. Indeed, when we weaken the China–Russia link, such intra-Eastern schism raises autonomy and reduces control for both countries, but by less than in the transatlantic case, for two reasons. The China–Russia link is a single tie, whereas the transatlantic link spans the US and its European allies, so the same 75 percent weakening is less than a quarter as large a change in the network (see Table A8). Moreover, per unit of change, actions also respond less. This result is consistent with the view that the outsized geopolitical influence of the United States stems in part from its alliances and that weakening these alliances erodes its superpower status.

India belongs both to the Quad, with the US, Japan, and Australia, and to BRICS and the Shanghai Cooperation Organization, with China and Russia. We ask what India would gain and lose by tying itself more closely to either side. Within the model, this exercise captures the trade-off behind India’s pursuit of strategic autonomy, and the same logic applies to other states that sit between blocs. India’s choice to strengthen its links with either the West or the East reduces its autonomy (as it becomes more influenced by the domestic interests of others) and increases its control (as it gains indirect influence over other nations through its network connections). Notably, the increase

in control is particularly large when India strengthens its links with the East, reflecting its geographical location and geopolitical connections to other Asian countries oriented toward China and Russia. Neither of these changes in India’s connections, nor the weakening of the China–Russia link, changes any individual UN vote in the exercise of Section 5.1.2 (Table A8), because the induced movements in other countries’ actions remain below the threshold at which a discretized vote changes.

5.2.2 Changes in the Structure of the Game

Debates about isolating Russia after its 2022 invasion of Ukraine, and about US withdrawal from multilateral institutions, turn on a common counterfactual: how would the rest of the world line up without a given country? The measure we construct captures a country’s ability to shape the game—how different the world would look without it—and combines the influence of its interests with that of its network connections.

Specifically, we exclude each country i in turn from the network and recompute equilibrium actions. We evaluate the change in the equilibrium configuration of actions for the remaining countries, as measured by the expected Euclidean distance between their action vectors with and without the excluded country: $\mathbb{E} [\|\mathbf{a}_{-i} - \mathbf{a}(\mathbf{G})_{-i}\|]$. We denote $\mathbf{a}(\mathbf{G})_{-i}$ as the remaining countries’ actions in the full network and $\mathbf{a}_{-i} = \mathcal{L}(\mathbf{G}_{-i})\boldsymbol{\theta}_{-i}$ as the remaining countries’ actions for the network excluding i , where $\boldsymbol{\theta}_{-i}$ represents the domestic interests of these remaining countries and $\mathbf{G}_{-i}$ corresponds to the network after removing i , with the remaining countries’ network connections renormalized and diagonals reset.¹⁷ This change in the equilibrium action vector in response to the exclusion of each country reflects the impact of both its interests and its network connections, where its network connections depend on its GDP-based structural importance weight (π_i). Following the same procedure as in the counterfactual for China with US interests, we evaluate the expectation $\mathbb{E} [\|\mathbf{a}_{-i} - \mathbf{a}(\mathbf{G})_{-i}\|]$ over international issues drawn from a normal distribution with mean zero and variance-covariance matrix $\Sigma_{\theta,d}$ for 2010–2019.

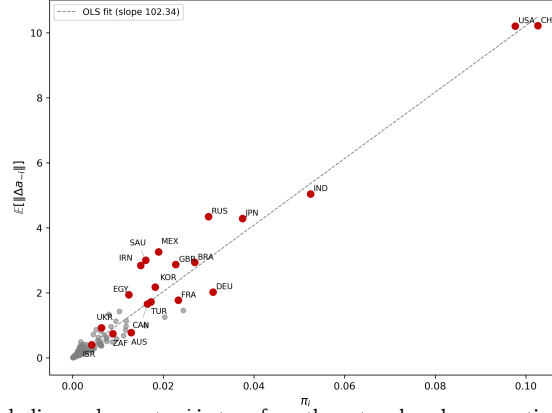
In Figure 6, we display this expected Euclidean distance, $\mathbb{E} [\|\mathbf{a}_{-i} - \mathbf{a}(\mathbf{G})_{-i}\|]$, against the structural importance of each excluded country. The superpowers dominate: excluding the US or China produces a mean action change of about 10.2, approximately twice the effect for India of 5.0, and about 2.4 times the effects for Japan and Russia of 4.3 each. The relationship between the change in the equilibrium action vector (vertical axis) and the GDP-based structural importance weight (horizontal axis) is approximately linear.¹⁸ Size accounts for most, but not all, of a country’s ability to shape the game. Among countries with similar GDP-based structural importance, some countries have a greater impact on the equilibrium action vector than others, because of their bilateral network

¹⁷When we remove a country from the network, we first edit the symmetric matrix $\mathbf{H}$ of Section 2.6. We then rebuild $\mathbf{G}$ under a convention in which each country’s diagonal entry compensates to keep the row sum constant, so as not to change the relative structural importance of other countries.

¹⁸China’s structural importance is greater than that of the US on the horizontal axis, because we estimate structural importance using GDP at purchasing power parity.

connections. For example, Russia and Iran both have a large impact relative to their size, whereas Australia has a small impact relative to its size.

Figure 6: Change in the Equilibrium Configuration of Actions from a Country’s Exclusion from the Network and the Excluded Country’s Structural Importance



Note: This figure shows the results of excluding each country i in turn from the network and recomputing the equilibrium action vector among the remaining countries. Vertical axis shows the change in the equilibrium configuration of actions, as measured by the expected Euclidean distance, over the estimated distribution of international issues, between the equilibrium action vectors for the remaining countries with and without the excluded country: $\mathbb{E} [\| \mathbf{a}_{-i} - \mathbf{a}(G)_{-i} \|]$. Horizontal axis shows the excluded country’s structural importance (π_i), as estimated using GDP at purchasing power parity. The dashed line shows the linear regression fit.

5.3 Which Shocks Matter Most?

Our two groups of counterfactuals speak to an old question in international relations: is the world shaped more by what countries want or by how they are connected? In Table A8 in Online Appendix L.8.3, we report two approaches to comparing the results of these counterfactuals in common terms. First, we scale the change in actions by the size of each shock, and compute an elasticity of actions with respect to each shock. Second, we evaluate the implied changes in UN voting outcomes, such that the counterfactuals can be compared in terms of their impact on UN votes. Measured in elasticity terms, shocks to great power interests are the most potent: the counterfactual assigning US interests to China, and the counterfactuals moving Chinese or US interests between the 25th and 75th percentiles have elasticities of around 2.3 and change more than 3,400 votes. In comparison, the network change with the largest elasticity, which involves a weakening of links between the US and its European NATO allies, has an elasticity of 0.75, and changes fewer than a tenth as many votes.

6 Conclusion

The central question of this paper is how domestic interests are translated into geopolitical actions in an interconnected world. We argue that this translation depends not only on countries’ own preferences but also on their positions in a geopolitical network and on the strategic influence exerted by other countries. To study this problem, we develop a tractable theoretical and empirical framework that combines network coordination, strategic influence, and spectral identification of the underlying geopolitical network.

Our framework yields three sets of results. Theoretically, it characterizes in closed form an equilibrium in which countries choose both bilateral influence and geopolitical actions. It shows that the mapping from domestic interests to actions decomposes along the network's cleavages, and it establishes that the cross-country covariances of interests and actions identify the network. Because the network is recovered from countries' actions as a whole, it encompasses the many channels of influence and their interactions, rather than any single instrument. Empirically, the recovered network is well approximated by a few dimensions, aligns with external measures of bilateral ties, and tracks the major realignments from the Cold War to the contemporary US–China rivalry. Finally, our counterfactuals yield three main empirical findings: the US has the most autonomy and China the most control; alliances trade a great power's freedom of action for its ability to move others; and, per unit of shock, changes in the domestic interests of the great powers move geopolitical actions more than changes in network connections.

More broadly, our findings suggest that incorporating network structure fundamentally changes how we think about international politics. Power is not simply a function of national capabilities or bilateral leverage, but also of where countries are positioned within the network and how influence propagates through it. Likewise, geopolitical outcomes are not determined solely by domestic interests, but emerge from equilibrium interactions among interconnected countries. This perspective provides a richer way to think about geopolitical power; unipolar, bipolar or non-polar international coordination; and polarization versus convergence in geopolitical actions.

Finally, we view our framework, which combines a tractable model of strategic interaction with a method for recovering latent network structure, as a foundation rather than a complete model of geopolitical networks. The approach requires only two empirical inputs—a measure of what actors want and a measure of what they do. It can therefore be applied wherever both are observed, from participation in sanctions coalitions and votes in other international organizations to the adoption of export controls and responses to individual crises. The same structure, in which actors' decisions are shaped by their own objectives, their network relationships, and their ability to influence others, also arises in technology competition, production networks, financial systems, and the diffusion of institutions. We believe that recovering the latent networks in these settings, and asking how they respond to shocks, is a promising direction for future research.

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Online Appendix for “A Network Approach to Geopolitics” (Not for Publication)^{*}

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Introduction

This appendix reports the technical derivations of the theoretical results reported in the paper (including the proofs of propositions) and presents supplementary empirical results that are discussed in the paper.

Section A provides the proof of Proposition 1 in the paper. Section B provides the proof of Proposition 2 in the paper. Section C provides the proof of Proposition 3 in the paper. Section D provides the proof of Proposition 4 in the paper. Section E provides the proof of Proposition 5 in the paper. Section F provides the proof of Proposition 6 in the paper. Section G shows that, when Σ_θ is nonsingular, the covariance matrices of domestic interests and geopolitical actions share the same principal components if and only if graph stationarity holds, as discussed in Section 2.5 of the paper. Section H provides the Proof of Proposition 7 in the paper. Section I provides the proof of Proposition 8 in the paper.

Section J contains additional theoretical results, as discussed in the paper. Subsection J.1 introduces a contestability index, which measures the share of the foreign influence directed at each country that is offset by influence in the opposing direction. Subsection J.2 uses an example geopolitical network to illustrate our approach. Subsection J.3 shows that with stochastic international issues each country's *ex ante* welfare can be measured as its expected payoff ($\mathbb{E}_\theta [U_i]$) over the distribution of the draws of domestic interests (θ). Subsection J.4 states the condition on the cost of influence under which the equilibrium is well posed when the geopolitical network has negative entries. Section K provides the proof of Proposition A.1 from Section J.3. Section L collects together additional empirical results that supplement the main empirical evidence reported in the paper.

A Proof of Proposition 1

Proposition. (*Proposition 1 in the paper*) *The equilibrium vectors of net influence and geopolitical actions satisfy:*

$$\begin{aligned}\Omega \mathbf{1} &= -(\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} ((\mathbf{I} - \mathbf{G})^2 - \alpha^2 \mathbf{I}) \boldsymbol{\theta} \\ \mathbf{a} &= (\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} (\mathbf{I} - \mathbf{G}) (\kappa \alpha - 1) \boldsymbol{\theta}\end{aligned}$$

Moreover, as $\kappa \rightarrow \infty$, $\Omega \mathbf{1} \rightarrow \mathbf{0}$ and $\mathbf{a} \rightarrow \alpha (\mathbf{I} - \mathbf{G})^{-1} \boldsymbol{\theta}$; and as $\kappa \downarrow 1/\alpha$,

$$\mathbf{a} \rightarrow \bar{\theta} \mathbf{1}, \quad \Omega \mathbf{1} \rightarrow \alpha (\bar{\theta} \mathbf{1} - \boldsymbol{\theta}),$$

where $\bar{\theta} \equiv \frac{1}{N} \sum_i \theta_i$.

Proof. The proof proceeds in four steps: (i) solve the coordination stage for $\mathbf{a}$ given Ω ; (ii) solve the influence stage for Ω given the coordination-stage response; (iii) combine the two fixed-point conditions into the closed-form solutions; and (iv) take the stated limits in κ .

Step 1: Coordination stage. Taking Ω as given, country i chooses a_i to maximize (5). The first-order

condition is

$$0 = \frac{\partial U_i}{\partial a_i} = -2\alpha (a_i - \theta_i) - 2 \sum_j G_{ij} (a_i - a_j) + 2 \sum_m \omega_{im},$$

i.e., $a_i = \alpha\theta_i + \sum_j G_{ij}a_j + \sum_m \omega_{im}$, or in vector form, using $[\Omega\mathbf{1}]_i = \sum_m \omega_{im}$,

$$\mathbf{a} = \alpha\boldsymbol{\theta} + \mathbf{G}\mathbf{a} + \Omega\mathbf{1}.$$

Since $\alpha \in (0, 1)$ and $\mathbf{G}$ has spectral radius $1 - \alpha < 1$ (by Perron–Frobenius, as $\mathbf{G}$ is nonnegative with constant row sums $1 - \alpha$), $(\mathbf{I} - \mathbf{G})$ is invertible, and the coordination-stage best responses admit the unique fixed point

$$\mathbf{a} = (\mathbf{I} - \mathbf{G})^{-1} (\alpha\boldsymbol{\theta} + \Omega\mathbf{1}). \quad (\text{A.1})$$

Step 2: Influence stage. Country i chooses $\{\omega_{ki}\}_k$ to maximize $U_i(\mathbf{a}(\Omega), \Omega)$, anticipating the coordination-stage response of all actions to Ω . By the chain rule, the total derivative with respect to ω_{ki} decomposes as

$$0 = \frac{dU_i}{d\omega_{ki}} = \underbrace{\frac{\partial U_i}{\partial a_i} \frac{\partial a_i}{\partial \omega_{ki}}}_{=0 \text{ (envelope)}} + \sum_{j \neq i} \frac{\partial U_i}{\partial a_j} \frac{\partial a_j}{\partial \omega_{ki}} + \left. \frac{\partial U_i}{\partial \omega_{ki}} \right|_{\mathbf{a}}.$$

The first term vanishes because a_i is chosen optimally in Step 1, so $\partial U_i / \partial a_i = 0$ at the equilibrium action (envelope theorem). The remaining two terms are computed directly from (5) and (A.1):

$$\frac{\partial U_i}{\partial a_j} = 2G_{ij} (a_i - a_j) \quad (j \neq i), \quad \frac{\partial a_j}{\partial \omega_{ki}} = [(\mathbf{I} - \mathbf{G})^{-1}]_{jk},$$

where the second equality follows because, from (A.1), $\partial \mathbf{a} / \partial \omega_{ki} = (\mathbf{I} - \mathbf{G})^{-1} \mathbf{e}_k$ (only the k -th entry of $\Omega\mathbf{1}$ responds to ω_{ki} , with unit coefficient). Since $G_{ii} (a_i - a_i) = 0$, the sum over $j \neq i$ can be extended to all j . For the direct effect, only the terms $-\kappa\omega_{ki}^2$ and, when $k = i$, $2\omega_{ii} (a_i - \theta_i)$ in (5) depend on ω_{ki} , so

$$\left. \frac{\partial U_i}{\partial \omega_{ki}} \right|_{\mathbf{a}} = -2\kappa\omega_{ki} + 2 \cdot \mathbf{1}_{k=i} (a_i - \theta_i).$$

Collecting terms and dividing by 2, the first-order condition for ω_{ki} is

$$\kappa \omega_{ki} = \sum_j G_{ij} (a_i - a_j) [(\mathbf{I} - \mathbf{G})^{-1}]_{jk} + \mathbf{1}_{k=i} (a_i - \theta_i). \quad (\text{A.2})$$

Summing (A.2) over $i = 1, \dots, N$ (i.e., over senders) and using $\sum_i \mathbf{1}_{k=i} (a_i - \theta_i) = a_k - \theta_k$:

$$\kappa \sum_i \omega_{ki} = \sum_i \sum_j G_{ij} (a_i - a_j) [(\mathbf{I} - \mathbf{G})^{-1}]_{jk} + a_k - \theta_k.$$

Because $\mathbf{G}$ is symmetric with constant row sums $1 - \alpha$, $\sum_i G_{ij} (a_i - a_j) = [\mathbf{G}\mathbf{a}]_j - (1 - \alpha) a_j$, so in vector form

$$\kappa \Omega\mathbf{1} = (\mathbf{I} - \mathbf{G})^{-1} [\mathbf{G}\mathbf{a} - (1 - \alpha) \mathbf{a}] + \mathbf{a} - \boldsymbol{\theta} = (\mathbf{I} - \mathbf{G})^{-1} [-(\mathbf{I} - \mathbf{G}) \mathbf{a} + \alpha \mathbf{a}] + \mathbf{a} - \boldsymbol{\theta},$$

which simplifies to

$$\kappa \Omega\mathbf{1} = \alpha (\mathbf{I} - \mathbf{G})^{-1} \mathbf{a} - \boldsymbol{\theta}. \quad (\text{A.3})$$

Step 3: Closed-form solution. Equations (A.1) and (A.3) are two linear equations in the unknowns $\mathbf{a}$ and $\Omega \mathbf{1}$. Substituting $\Omega \mathbf{1} = \frac{1}{\kappa} [\alpha (\mathbf{I} - \mathbf{G})^{-1} \mathbf{a} - \boldsymbol{\theta}]$ from (A.3) into (A.1) gives

$$\mathbf{a} = \alpha (\mathbf{I} - \mathbf{G})^{-1} \boldsymbol{\theta} + \frac{\alpha}{\kappa} (\mathbf{I} - \mathbf{G})^{-2} \mathbf{a} - \frac{1}{\kappa} (\mathbf{I} - \mathbf{G})^{-1} \boldsymbol{\theta}.$$

Since $\mathbf{G}$ is symmetric, all matrices appearing here are polynomials or rational functions of $\mathbf{G}$ and therefore commute. Left-multiplying both sides by $\kappa (\mathbf{I} - \mathbf{G})^2$ and collecting the $\mathbf{a}$ terms on the left and the $\boldsymbol{\theta}$ terms on the right:

$$[\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I}] \mathbf{a} = (\kappa \alpha - 1) (\mathbf{I} - \mathbf{G}) \boldsymbol{\theta},$$

so that

$$\mathbf{a} = (\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} (\mathbf{I} - \mathbf{G}) (\kappa \alpha - 1) \boldsymbol{\theta}, \quad (\text{A.4})$$

which is the second expression in the proposition. (The matrix $\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I}$ is invertible because $\kappa > 1/\alpha$ and $\mathbf{G}$'s eigenvalues lie in $[-(1 - \alpha), 1 - \alpha]$, so every eigenvalue of $\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I}$ is bounded below by $\kappa \alpha^2 - \alpha = \alpha (\kappa \alpha - 1) > 0$.) Substituting (A.4) back into (A.3), and using commutativity to write $(\mathbf{I} - \mathbf{G})^{-1} (\mathbf{I} - \mathbf{G}) = \mathbf{I}$ inside the resolvent:

$$\begin{aligned} \kappa \Omega \mathbf{1} &= \alpha (\kappa \alpha - 1) (\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} \boldsymbol{\theta} - \boldsymbol{\theta} \\ &= (\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} [\alpha (\kappa \alpha - 1) \mathbf{I} - \kappa (\mathbf{I} - \mathbf{G})^2 + \alpha \mathbf{I}] \boldsymbol{\theta} \\ &= (\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} \kappa [\alpha^2 \mathbf{I} - (\mathbf{I} - \mathbf{G})^2] \boldsymbol{\theta}, \end{aligned}$$

using $\alpha (\kappa \alpha - 1) + \alpha = \kappa \alpha^2$ in the last line. Dividing by κ :

$$\Omega \mathbf{1} = -(\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} ((\mathbf{I} - \mathbf{G})^2 - \alpha^2 \mathbf{I}) \boldsymbol{\theta}, \quad (\text{A.5})$$

which is the first expression in the proposition.

Step 4: Limits in κ . Because $\mathbf{G}$ is real and symmetric, write its spectral decomposition $\mathbf{G} = \sum_m \lambda_m \mathbf{u}_m \mathbf{u}_m^\top$ with orthonormal eigenvectors $\{\mathbf{u}_m\}$. Expanding $\boldsymbol{\theta} = \sum_m \tilde{\theta}_m \mathbf{u}_m$ with $\tilde{\theta}_m = \mathbf{u}_m^\top \boldsymbol{\theta}$, equations (A.4) and (A.5) act mode by mode:

$$\begin{aligned} \mathbf{a} &= \sum_m L(\lambda_m) \tilde{\theta}_m \mathbf{u}_m, & \Omega \mathbf{1} &= \sum_m \Phi(\lambda_m) \tilde{\theta}_m \mathbf{u}_m, \\ L(\lambda) &\equiv \frac{(1 - \lambda)(\kappa \alpha - 1)}{\kappa (1 - \lambda)^2 - \alpha}, & \Phi(\lambda) &\equiv -\frac{(1 - \lambda)^2 - \alpha^2}{\kappa (1 - \lambda)^2 - \alpha}. \end{aligned}$$

Limit $\kappa \rightarrow \infty$. For every eigenvalue λ , $L(\lambda) \rightarrow \alpha (1 - \lambda)^{-1}$ and $\Phi(\lambda) \rightarrow 0$ as $\kappa \rightarrow \infty$ (divide numerator and denominator by κ). Hence $\Omega \mathbf{1} \rightarrow \mathbf{0}$ and $\mathbf{a} \rightarrow \sum_m \alpha (1 - \lambda_m)^{-1} \tilde{\theta}_m \mathbf{u}_m = \alpha (\mathbf{I} - \mathbf{G})^{-1} \boldsymbol{\theta}$.

Limit $\kappa \downarrow 1/\alpha$. Since $\mathbf{G}$ has nonnegative entries and constant row sums $1 - \alpha$, $\mathbf{G} \mathbf{1} = (1 - \alpha) \mathbf{1}$, so $\mathbf{u}_1 \equiv \mathbf{1}/\sqrt{N}$ is a unit eigenvector with eigenvalue $\lambda_1 = 1 - \alpha$; by Perron–Frobenius, λ_1 is a

simple eigenvalue and every other eigenvalue is strictly smaller whenever the network is connected, so $|1 - \lambda_m| > \alpha$ for every $m \neq 1$ while $1 - \lambda_1 = \alpha$. Evaluating L at λ_1 :

$$L(\lambda_1) = \frac{\alpha(\kappa\alpha - 1)}{\kappa\alpha^2 - \alpha} = \frac{\alpha(\kappa\alpha - 1)}{\alpha(\kappa\alpha - 1)} = 1 \quad \text{for all } \kappa > 1/\alpha,$$

so trivially $L(\lambda_1) \rightarrow 1$ as $\kappa \downarrow 1/\alpha$. For $m \neq 1$, the denominator $\kappa(1 - \lambda_m)^2 - \alpha \rightarrow \alpha^{-1}(1 - \lambda_m)^2 - \alpha > 0$ (strictly, since $(1 - \lambda_m)^2 > \alpha^2$) remains bounded away from zero as $\kappa \rightarrow 1/\alpha$, while the numerator $(1 - \lambda_m)(\kappa\alpha - 1) \rightarrow 0$; hence $L(\lambda_m) \rightarrow 0$ for $m \neq 1$. Therefore

$$\mathbf{a} \rightarrow L(\lambda_1) \tilde{\theta}_1 \mathbf{u}_1 = \tilde{\theta}_1 \mathbf{u}_1 = (\mathbf{u}_1^\top \boldsymbol{\theta}) \mathbf{u}_1 = \left(\frac{\mathbf{1}^\top \boldsymbol{\theta}}{N} \right) \mathbf{1} = \bar{\theta} \mathbf{1}.$$

For the influence term, $\Phi(\lambda) = (1 - \lambda)L(\lambda) - \alpha$ (a direct algebraic identity, since $(1 - \lambda)^2 - \alpha^2 = (1 - \lambda)(1 - \lambda) - \alpha^2$ and $(1 - \lambda)L(\lambda) - \alpha = [(1 - \lambda)^2(\kappa\alpha - 1) - \alpha(\kappa(1 - \lambda)^2 - \alpha)] / (\kappa(1 - \lambda)^2 - \alpha) = (\alpha^2 - (1 - \lambda)^2) / (\kappa(1 - \lambda)^2 - \alpha) = \Phi(\lambda)$). As $\kappa \downarrow 1/\alpha$, $\Phi(\lambda_1) \rightarrow (1 - \lambda_1) \cdot 1 - \alpha = \alpha - \alpha = 0$, while for $m \neq 1$, $\Phi(\lambda_m) \rightarrow (1 - \lambda_m) \cdot 0 - \alpha = -\alpha$. Hence

$$\Omega \mathbf{1} \rightarrow \sum_{m \neq 1} (-\alpha) \tilde{\theta}_m \mathbf{u}_m = -\alpha (\boldsymbol{\theta} - \tilde{\theta}_1 \mathbf{u}_1) = -\alpha (\boldsymbol{\theta} - \bar{\theta} \mathbf{1}) = \alpha (\bar{\theta} \mathbf{1} - \boldsymbol{\theta}).$$

This establishes both limits and completes the proof. $\square$

B Proof of Proposition 2

Proposition. (*Proposition 2 in the paper*) *The equilibrium has the following properties.*

1. *The equilibrium is unique.*
2. *Each country's equilibrium action is a convex combination over the domestic interests of all countries.*
3. *The equilibrium is translation invariant to $\boldsymbol{\theta}$: adding a constant c to all θ_i 's results in a new equilibrium in which all actions are also shifted by c , without any change in payoffs.*
4. *The equilibrium is conservative: the average geopolitical action and the average domestic interest across countries coincide: $\mathbf{1}^\top \mathbf{a} = \mathbf{1}^\top \boldsymbol{\theta}$.*
5. *The equilibrium strategic influence nets out to zero: $\mathbf{1}^\top \Omega \mathbf{1} = 0$.*

Proof. As established in Proposition 1, the equilibrium satisfies

$$\mathbf{a} = M\boldsymbol{\theta}, \quad M \equiv (\kappa(\mathbf{I} - \mathbf{G})^2 - \alpha\mathbf{I})^{-1}(\mathbf{I} - \mathbf{G})(\kappa\alpha - 1), \quad (\text{A.6})$$

$$\Omega \mathbf{1} = N\boldsymbol{\theta}, \quad N \equiv -(\kappa(\mathbf{I} - \mathbf{G})^2 - \alpha\mathbf{I})^{-1}((\mathbf{I} - \mathbf{G})^2 - \alpha^2\mathbf{I}). \quad (\text{A.7})$$

Two elementary facts about M and N will be used repeatedly.

Fact A (symmetry). M and N are symmetric. Indeed, $G = U\Lambda U^\top$ for an orthogonal U and diagonal Λ , so every matrix built from I, G , sums, products, and inverses thereof (as M and N are) can be written as $Uf(\Lambda)U^\top$ for a diagonal matrix $f(\Lambda)$; any such matrix is symmetric.

Fact B (action on 1). Since G has constant row sums $1 - \alpha$, $G\mathbf{1} = (1 - \alpha)\mathbf{1}$, so $(I - G)\mathbf{1} = \alpha\mathbf{1}$ and $(I - G)^2\mathbf{1} = \alpha^2\mathbf{1}$. Hence

$$(\kappa(I - G)^2 - \alpha I)\mathbf{1} = (\kappa\alpha^2 - \alpha)\mathbf{1} = \alpha(\kappa\alpha - 1)\mathbf{1},$$

so $\mathbf{1}$ is an eigenvector of $(\kappa(I - G)^2 - \alpha I)^{-1}$ with eigenvalue $[\alpha(\kappa\alpha - 1)]^{-1}$, and therefore

$$M\mathbf{1} = (\kappa\alpha - 1)[\alpha(\kappa\alpha - 1)]^{-1}\alpha\mathbf{1} = \mathbf{1}, \quad N\mathbf{1} = -[\alpha(\kappa\alpha - 1)]^{-1}(\alpha^2 - \alpha^2)\mathbf{1} = \mathbf{0}. \quad (\text{A.8})$$

1. Uniqueness. At the coordination stage, for any fixed Ω , U_i is strictly concave in a_i : from (5), $\partial^2 U_i / \partial a_i^2 = -2(\alpha + \sum_{j \neq i} G_{ij}) = -2(1 - G_{ii}) < 0$, since $\alpha > 0$ and $G_{ij} \geq 0$. Hence the first-order condition has a unique solution, and because $I - G$ is invertible (G 's spectral radius is $1 - \alpha < 1$ by Perron–Frobenius, as G is nonnegative with constant row sums $1 - \alpha$), the coordination-stage best response is the unique linear map $\mathbf{a} = (I - G)^{-1}(\alpha\theta + \Omega\mathbf{1})$. At the influence stage, anticipating this continuation, $U_i(\mathbf{a}(\Omega), \Omega)$ is quadratic in country i 's own controls $\{\omega_{ki}\}_k$ with Hessian equal to $-2\kappa I$ plus the terms that enter through $\mathbf{a}(\Omega)$. Online Appendix J.4 shows that this Hessian is negative definite whenever $\kappa > \kappa^*(G)$, and that $\kappa^*(G) \leq 1/\alpha$ when G is nonnegative, so it is negative definite under our assumption $\kappa > 1/\alpha$; hence country i 's best response at the influence stage is also unique and linear, given by the influence-stage best response displayed below. The full system of first-order conditions across all i and k is therefore a linear system in $(\mathbf{a}, \Omega)$, and it has a unique solution because the matrix $\kappa(I - G)^2 - \alpha I$ is invertible (every eigenvalue of G lies in $[-(1 - \alpha), 1 - \alpha]$, so every eigenvalue of $\kappa(I - G)^2 - \alpha I$ is at least $\kappa\alpha^2 - \alpha = \alpha(\kappa\alpha - 1) > 0$). Since each player's strategy is a strict best response to the (unique) strategies of others at every information set, this linear fixed point is the unique subgame-perfect equilibrium; existence is established by the explicit closed-form solution (A.6)–(A.7).

2. Convex combination. We show M has nonnegative entries and row sums equal to one; combined with $\mathbf{a} = M\theta$, this means $a_i = \sum_k M_{ik}\theta_k$ is a convex combination of $\{\theta_k\}$. The row-sum property is $M\mathbf{1} = \mathbf{1}$, already established in Fact B. For nonnegativity, work in G 's eigenbasis: M has eigenvalues

$$L(\lambda) = \frac{(1 - \lambda)(\kappa\alpha - 1)}{\kappa(1 - \lambda)^2 - \alpha}$$

at each eigenvalue λ of G . Write $z \equiv 1 - \lambda$ and $\varrho \equiv \sqrt{\alpha/\kappa}$; since $\kappa > 1/\alpha$, we have $\varrho^2 = \alpha/\kappa < \alpha^2$, so $\varrho < \alpha < 1$. By partial fractions,

$$L(\lambda) = \frac{\kappa\alpha - 1}{\kappa} \cdot \frac{z}{z^2 - \varrho^2} = \frac{\kappa\alpha - 1}{2\kappa} \left[\frac{1}{(1 - \varrho) - \lambda} + \frac{1}{(1 + \varrho) - \lambda} \right].$$

Because $|\lambda| \leq 1 - \alpha < 1 - \varrho < 1 + \varrho$ (the first inequality by Perron–Frobenius, the second because $\varrho < \alpha$), both terms admit convergent geometric expansions in λ :

$$\frac{1}{(1 - \varrho) - \lambda} = \sum_{n=0}^{\infty} \frac{\lambda^n}{(1 - \varrho)^{n+1}}, \quad \frac{1}{(1 + \varrho) - \lambda} = \sum_{n=0}^{\infty} \frac{\lambda^n}{(1 + \varrho)^{n+1}}.$$

Hence $L(\lambda) = \sum_{n \geq 0} d_n \lambda^n$ with

$$d_n = \frac{\kappa\alpha - 1}{2\kappa} \left[(1 - \varrho)^{-(n+1)} + (1 + \varrho)^{-(n+1)} \right] > 0 \quad \text{for every } n \geq 0,$$

since $\kappa\alpha - 1 > 0$ (as $\kappa > 1/\alpha$) and $0 < 1 - \varrho < 1 + \varrho$. Applying the same expansion to the matrix $\mathbf{G}$ (valid since $\mathbf{G}$'s eigenvalues satisfy the same bound), $\mathbf{M} = \sum_{n \geq 0} d_n \mathbf{G}^n$. Each $\mathbf{G}^n$ has nonnegative entries, because $\mathbf{G}$ does and products of nonnegative matrices are nonnegative; since each $d_n > 0$, every entry of $\mathbf{M}$ is a convergent sum of nonnegative terms, hence $M_{ik} \geq 0$ for all i, k . Together with $\mathbf{M}\mathbf{1} = \mathbf{1}$, this proves each row of $\mathbf{M}$ is a set of convex-combination weights.

3. Translation invariance. Let $\boldsymbol{\theta}' = \boldsymbol{\theta} + c\mathbf{1}$ for a constant c . We claim that $(\mathbf{a}', \boldsymbol{\Omega}') = (\mathbf{a} + c\mathbf{1}, \boldsymbol{\Omega})$ —the same influence matrix, with every action shifted by c —is the equilibrium associated with $\boldsymbol{\theta}'$. First, by (A.6) and $\mathbf{M}\mathbf{1} = \mathbf{1}$ (Fact B),

$$\mathbf{a}' = \mathbf{M}\boldsymbol{\theta}' = \mathbf{M}\boldsymbol{\theta} + c\mathbf{M}\mathbf{1} = \mathbf{a} + c\mathbf{1},$$

confirming the postulated shift in actions. Next, we verify this $(\mathbf{a}', \boldsymbol{\Omega}')$ satisfies the equilibrium conditions for $\boldsymbol{\theta}'$. The coordination-stage first-order condition, $a_i = \alpha\theta_i + \sum_j G_{ij}a_j + \sum_m \omega_{im}$, holds at $(\mathbf{a}', \boldsymbol{\theta}', \boldsymbol{\Omega}' = \boldsymbol{\Omega})$ because both sides increase by exactly $\alpha c + (1 - \alpha)c = c$ relative to the original equilibrium (using $\sum_j G_{ij} = 1 - \alpha$), matching $a'_i = a_i + c$. The influence-stage best response,

$$\kappa \omega_{ki} = \sum_j G_{ij} (a_i - a_j) \left[(\mathbf{I} - \mathbf{G})^{-1} \right]_{jk} + \mathbf{1}_{k=i} (a_i - \theta_i),$$

depends on $(\mathbf{a}, \boldsymbol{\theta})$ only through the differences $a_i - a_j$ and $a_i - \theta_i$, both of which are unchanged when $\mathbf{a} \rightarrow \mathbf{a} + c\mathbf{1}$ and $\boldsymbol{\theta} \rightarrow \boldsymbol{\theta} + c\mathbf{1}$. Hence $\boldsymbol{\Omega}' = \boldsymbol{\Omega}$ continues to satisfy every bilateral first-order condition unchanged, verifying the claim. Finally, payoffs in (5) depend on $(\mathbf{a}, \boldsymbol{\theta}, \boldsymbol{\Omega})$ only through $a_i - \theta_i$, $a_i - a_j$, and $\boldsymbol{\Omega}$ itself, all of which are identical at $(\mathbf{a}', \boldsymbol{\theta}', \boldsymbol{\Omega}')$ and $(\mathbf{a}, \boldsymbol{\theta}, \boldsymbol{\Omega})$; hence U_i is unchanged for every i .

4. Conservation. By Fact A, $\mathbf{M}$ is symmetric, so $\mathbf{M}^\top = \mathbf{M}$. By Fact B, $\mathbf{M}\mathbf{1} = \mathbf{1}$, so

$$\mathbf{1}^\top \mathbf{M} = (\mathbf{M}^\top \mathbf{1})^\top = (\mathbf{M}\mathbf{1})^\top = \mathbf{1}^\top.$$

Therefore $\mathbf{1}^\top \mathbf{a} = \mathbf{1}^\top \mathbf{M}\boldsymbol{\theta} = \mathbf{1}^\top \boldsymbol{\theta}$ for every $\boldsymbol{\theta}$.

5. Zero net influence. By Fact A, $\mathbf{N}$ is symmetric, so $\mathbf{N}^\top = \mathbf{N}$. By Fact B, $\mathbf{N}\mathbf{1} = \mathbf{0}$, so

$$\mathbf{1}^\top \mathbf{N} = (\mathbf{N}^\top \mathbf{1})^\top = (\mathbf{N}\mathbf{1})^\top = \mathbf{0}^\top.$$

Therefore $\mathbf{1}^\top \boldsymbol{\Omega}\mathbf{1} = \mathbf{1}^\top \mathbf{N}\boldsymbol{\theta} = 0$ for every $\boldsymbol{\theta}$. □

C Proof of Proposition 3

Proposition. (*Proposition 3 in the paper*) The network disagreement of an eigenvector $\mathbf{u}_k$ is $\mathcal{S}(\mathbf{u}_k) = 1 - \alpha - \lambda_k$. Moreover,

$$\begin{aligned} \mathbf{u}_1 &= \arg \min_{\|\boldsymbol{\theta}\|=1} \mathcal{S}(\boldsymbol{\theta}) \propto \mathbf{1}, \quad \lambda_1 = 1 - \alpha. \\ \mathbf{u}_k &\in \arg \min_{\boldsymbol{\theta} \perp \{\mathbf{u}_j\}_{j < k}, \|\boldsymbol{\theta}\|=1} \mathcal{S}(\boldsymbol{\theta}) \text{ for all } k > 1. \end{aligned}$$

Proof. **Part 1:** $\mathcal{S}(\mathbf{u}_k) = 1 - \alpha - \lambda_k$. Expand the square in the definition of $\mathcal{S}$:

$$\sum_{i,j} G_{ij} (\theta_i - \theta_j)^2 = \sum_{i,j} G_{ij} \theta_i^2 - 2 \sum_{i,j} G_{ij} \theta_i \theta_j + \sum_{i,j} G_{ij} \theta_j^2.$$

Since every row sum of $\mathbf{G}$ equals $1 - \alpha$, $\sum_j G_{ij} = 1 - \alpha$ for every i , so the first term is $\sum_i \theta_i^2 (1 - \alpha) = (1 - \alpha) \|\boldsymbol{\theta}\|^2$; by symmetry of $\mathbf{G}$, column sums equal row sums, so the third term is likewise $(1 - \alpha) \|\boldsymbol{\theta}\|^2$. The middle term is $2\boldsymbol{\theta}^\top \mathbf{G} \boldsymbol{\theta}$. Hence

$$\mathcal{S}(\boldsymbol{\theta}) = \frac{1}{2} [2(1 - \alpha) \|\boldsymbol{\theta}\|^2 - 2\boldsymbol{\theta}^\top \mathbf{G} \boldsymbol{\theta}] = (1 - \alpha) \|\boldsymbol{\theta}\|^2 - \boldsymbol{\theta}^\top \mathbf{G} \boldsymbol{\theta}. \quad (\text{A.9})$$

This identifies $\mathcal{S}(\boldsymbol{\theta}) = \boldsymbol{\theta}^\top \mathbf{L} \boldsymbol{\theta}$ with $\mathbf{L} \equiv (1 - \alpha) \mathbf{I} - \mathbf{G}$, the (weighted) graph Laplacian of $\mathbf{G}$, sharing $\mathbf{G}$'s eigenbasis with eigenvalues $1 - \alpha - \lambda_k$. In particular, for a unit eigenvector $\boldsymbol{\theta} = \mathbf{u}_k$, $\mathbf{u}_k^\top \mathbf{G} \mathbf{u}_k = \lambda_k$, so

$$\mathcal{S}(\mathbf{u}_k) = (1 - \alpha) - \lambda_k.$$

Part 2: $\mathbf{u}_1 \propto \mathbf{1}$ and $\lambda_1 = 1 - \alpha$. Since $\mathbf{G}\mathbf{1} = (1 - \alpha)\mathbf{1}$ (constant row sums), $\mathbf{1}$ is an eigenvector of $\mathbf{G}$ with eigenvalue $1 - \alpha$. For any eigenvalue λ of $\mathbf{G}$ with eigenvector $\mathbf{v}$, letting i index the largest entry of $|\mathbf{v}|$, the i -th row of $\mathbf{G}\mathbf{v} = \lambda\mathbf{v}$ gives $|\lambda| |v_i| = \left| \sum_j G_{ij} v_j \right| \leq \sum_j G_{ij} |v_j| \leq \left(\sum_j G_{ij} \right) |v_i| = (1 - \alpha) |v_i|$, so $|\lambda| \leq 1 - \alpha$ for every eigenvalue. Hence $1 - \alpha$ is the largest eigenvalue in absolute value, and since it is attained (by $\mathbf{1}$) it is also the algebraically largest eigenvalue: $\lambda_1 = 1 - \alpha$. Because the network is connected, $\mathbf{G}$ is an irreducible nonnegative matrix, so by the Perron–Frobenius theorem the Perron root $1 - \alpha$ is simple and its eigenspace is spanned by a strictly positive vector; since $\mathbf{1}$ is strictly positive and already an eigenvector for $\lambda_1 = 1 - \alpha$, it must (up to normalization) be this Perron eigenvector, i.e., $\mathbf{u}_1 = \mathbf{1}/\sqrt{N} \propto \mathbf{1}$.

Part 3: sequential variational characterization. By (A.9), for any unit vector $\boldsymbol{\theta}$, $\mathcal{S}(\boldsymbol{\theta}) = (1 - \alpha) - \boldsymbol{\theta}^\top \mathbf{G} \boldsymbol{\theta}$, an affine, strictly decreasing function of the Rayleigh quotient $\boldsymbol{\theta}^\top \mathbf{G} \boldsymbol{\theta}$. Hence minimizing $\mathcal{S}(\boldsymbol{\theta})$ over any constraint set is equivalent to maximizing $\boldsymbol{\theta}^\top \mathbf{G} \boldsymbol{\theta}$ over the same set, and it suffices to establish the following standard sequential characterization of the eigenbasis of a symmetric matrix.

Claim. For each $k = 1, \dots, N$,

$$\mathbf{u}_k \in \arg \max_{\boldsymbol{\theta} \perp \{\mathbf{u}_j\}_{j < k}, \|\boldsymbol{\theta}\|=1} \boldsymbol{\theta}^\top \mathbf{G} \boldsymbol{\theta}, \quad \text{with maximal value } \lambda_k.$$

(For $k = 1$ the orthogonality constraint is vacuous.)

Proof of Claim. Fix k and let θ be any unit vector orthogonal to $\mathbf{u}_1, \dots, \mathbf{u}_{k-1}$. Expand $\theta = \sum_{j \geq k} c_j \mathbf{u}_j$ in the orthonormal eigenbasis; orthogonality to $\mathbf{u}_1, \dots, \mathbf{u}_{k-1}$ forces the coefficients on those directions to vanish, and $\|\theta\| = 1$ gives $\sum_{j \geq k} c_j^2 = 1$ by Parseval's identity. Then

$$\theta^\top \mathbf{G} \theta = \sum_{j \geq k} \lambda_j c_j^2 \leq \lambda_k \sum_{j \geq k} c_j^2 = \lambda_k,$$

using $\lambda_j \leq \lambda_k$ for every $j \geq k$ (eigenvalues listed in decreasing algebraic order). The bound is attained at $\theta = \mathbf{u}_k$ (taking $c_k = 1$ and $c_j = 0$ for $j > k$), proving the claim; equality throughout requires $c_j = 0$ whenever $\lambda_j < \lambda_k$, so the maximizer is unique up to sign unless λ_k is a repeated eigenvalue, in which case any unit vector in its eigenspace (orthogonal to $\mathbf{u}_1, \dots, \mathbf{u}_{k-1}$) attains the same maximal value.

Applying the Claim with $k = 1$ and using the strictly decreasing affine relationship between $\mathcal{S}$ and the Rayleigh quotient gives $\mathbf{u}_1 = \arg \min_{\|\theta\|=1} \mathcal{S}(\theta)$, already identified as $1/\sqrt{N}$ in Part 2. Applying the Claim for general $k > 1$ gives $\mathbf{u}_k \in \arg \min_{\theta \perp \{\mathbf{u}_j\}_{j < k}, \|\theta\|=1} \mathcal{S}(\theta)$, completing the proof. $\square$

D Proof of Proposition 4

Proposition. (*Proposition 4 in the paper*) *The operator $\mathcal{L}(\mathbf{G})$, mapping from domestic interests to geopolitical actions, is a spectral function of $\mathbf{G}$: it shares the same eigenbasis as $\mathbf{G}$. This implies spectral decoupling: when domestic interests are decomposed along $\mathbf{G}$'s eigenvectors, the equilibrium mapping scales each cleavage independently, without spilling over into others:*

$$\mathbf{a} = \sum_{\ell} L(\lambda_{\ell}) \tilde{\theta}_{\ell} \mathbf{u}_{\ell}, \quad L(\lambda) \equiv \frac{(1 - \lambda)(\kappa\alpha - 1)}{\kappa(1 - \lambda)^2 - \alpha};$$

or, equivalently, projecting the equilibrium action profile onto $\mathbf{G}$'s eigenbasis $\tilde{\mathbf{a}} \equiv \mathbf{U}^\top \mathbf{a}$, we have:

$$\tilde{a}_{\ell} = L(\lambda_{\ell}) \tilde{\theta}_{\ell}.$$

The scaling factor $L(\cdot)$ is nonnegative, bounded above by one ($0 \leq L(\cdot) \leq 1$), and is strictly increasing in the eigenvalue λ_{ℓ} ($L'(\cdot) > 0$), implying that eigenvectors with larger eigenvalues translate into larger differences in geopolitical actions. Finally, the scaling factor $L(\cdot)$ is increasing in κ .

Proof. As established in Proposition 1, equilibrium geopolitical actions satisfy $\mathbf{a} = \mathcal{L}(\mathbf{G}) \theta$, where

$$\mathcal{L}(\mathbf{G}) \equiv (\kappa(\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} (\mathbf{I} - \mathbf{G}) (\kappa\alpha - 1), \quad \kappa > 1/\alpha. \quad (\text{A.10})$$

Domestic interests project onto $\mathbf{G}$'s eigenbasis as $\theta = \sum_{\ell} \tilde{\theta}_{\ell} \mathbf{u}_{\ell}$, $\tilde{\theta}_{\ell} = \mathbf{u}_{\ell}^\top \theta$.

Spectral decoupling. Because $\mathbf{G} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^\top$, every matrix built from $\mathbf{I}$ and $\mathbf{G}$ by addition, multiplication, and inversion is diagonalized by the same orthogonal matrix $\mathbf{U}$: for instance, $(\mathbf{I} - \mathbf{G})^n = \mathbf{U} (\mathbf{I} - \mathbf{\Lambda})^n \mathbf{U}^\top$ for any n , and $(\kappa(\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} = \mathbf{U} (\kappa(\mathbf{I} - \mathbf{\Lambda})^2 - \alpha \mathbf{I})^{-1} \mathbf{U}^\top$, since $\mathbf{U}$ is orthogonal ($\mathbf{U} \mathbf{U}^\top = \mathbf{I}$) so that sums, products, and inverses of diagonal matrices sandwiched

between U and $U^\top$ combine correctly (e.g., $UD_1U^\top \cdot UD_2U^\top = UD_1D_2U^\top$). Substituting into (A.10),

$\mathcal{L}(G) = U (\kappa(I - \Lambda)^2 - \alpha I)^{-1} (I - \Lambda) (\kappa\alpha - 1) U^\top = U \text{Diag}(L(\lambda_1), \dots, L(\lambda_N)) U^\top$, where $L(\lambda) \equiv (1 - \lambda)(\kappa\alpha - 1) / (\kappa(1 - \lambda)^2 - \alpha)$ is precisely the scalar function obtained by applying the same rational expression entrywise to each diagonal eigenvalue. Hence $\mathcal{L}(G)$ has eigenvectors u_ℓ with eigenvalues $L(\lambda_\ell)$: it is a spectral function of G , sharing G 's eigenbasis. Applying $\mathcal{L}(G)$ to $\theta = \sum_\ell \tilde{\theta}_\ell u_\ell$ term by term gives $a = \mathcal{L}(G) \theta = \sum_\ell L(\lambda_\ell) \tilde{\theta}_\ell u_\ell$, and projecting onto u_ℓ (using orthonormality) gives $\tilde{a}_\ell = u_\ell^\top a = L(\lambda_\ell) \tilde{\theta}_\ell$. This is the claimed spectral decoupling: each cleavage ℓ is scaled only by its own eigenvalue, with no mixing across cleavages.

For the remaining properties of $L(\cdot)$, substitute $z \equiv 1 - \lambda$, so that G 's eigenvalues $\lambda \in [-(1 - \alpha), 1 - \alpha]$ (a bound established in the proof of Proposition 3 via Perron–Frobenius) correspond to $z \in [\alpha, 2 - \alpha]$, and write

$$L(z) = \frac{(\kappa\alpha - 1)z}{\kappa z^2 - \alpha}.$$

Sign and denominator positivity. Since $\kappa > 1/\alpha$, $\kappa\alpha - 1 > 0$. For $z \geq \alpha > 0$, $\kappa z^2 - \alpha \geq \kappa\alpha^2 - \alpha = \alpha(\kappa\alpha - 1) > 0$ (the map $z \mapsto \kappa z^2 - \alpha$ is increasing for $z > 0$, so its minimum over $z \geq \alpha$ is attained at $z = \alpha$). Hence both numerator and denominator of $L(z)$ are strictly positive throughout the relevant range, so $L(\lambda) > 0$ for every eigenvalue λ of G ; in particular $L(\cdot) \geq 0$.

Upper bound $L(\cdot) \leq 1$. We must show $(\kappa\alpha - 1)z \leq \kappa z^2 - \alpha$ for $z \geq \alpha$, i.e., $\kappa z^2 - (\kappa\alpha - 1)z - \alpha \geq 0$. Factor:

$$\kappa z^2 - (\kappa\alpha - 1)z - \alpha = \kappa z(z - \alpha) + (z - \alpha) = (z - \alpha)(\kappa z + 1).$$

Since $z \geq \alpha$, $z - \alpha \geq 0$, and $\kappa z + 1 > 0$; hence the product is nonnegative, proving $L(z) \leq 1$ for all $z \geq \alpha$, with equality if and only if $z = \alpha$, i.e., $\lambda = 1 - \alpha$ (the top eigenvalue, associated with $u_1 \propto \mathbf{1}$). Combined with the previous step, $0 \leq L(\cdot) \leq 1$.

Monotonicity in λ . Differentiate L with respect to z using the quotient rule:

$$\frac{dL}{dz} = \frac{(\kappa\alpha - 1)(\kappa z^2 - \alpha) - (\kappa\alpha - 1)z(2\kappa z)}{(\kappa z^2 - \alpha)^2} = \frac{-(\kappa\alpha - 1)(\kappa z^2 + \alpha)}{(\kappa z^2 - \alpha)^2} < 0,$$

since $\kappa\alpha - 1 > 0$ and $\kappa z^2 + \alpha > 0$. Because $z = 1 - \lambda$, the chain rule gives $dL/d\lambda = -dL/dz > 0$: $L(\cdot)$ is strictly increasing in λ , so cleavages with larger eigenvalues are scaled more strongly into equilibrium actions.

Monotonicity in κ . Differentiate L with respect to κ at fixed z , using the quotient rule:

$$\begin{aligned} \frac{\partial L}{\partial \kappa} &= \frac{\alpha z(\kappa z^2 - \alpha) - (\kappa\alpha - 1)z \cdot z^2}{(\kappa z^2 - \alpha)^2} = \frac{z[\alpha\kappa z^2 - \alpha^2 - \kappa\alpha z^2 + z^2]}{(\kappa z^2 - \alpha)^2} \\ &= \frac{z(z^2 - \alpha^2)}{(\kappa z^2 - \alpha)^2} = \frac{z(z - \alpha)(z + \alpha)}{(\kappa z^2 - \alpha)^2}. \end{aligned}$$

Since $z \geq \alpha > 0$, every factor in the numerator is nonnegative, so $\partial L / \partial \kappa \geq 0$ for every eigenvalue $\lambda \leq 1 - \alpha$, with equality only at $z = \alpha$ (i.e., $\lambda = 1 - \alpha$, where $L \equiv 1$ regardless of κ , consistent with the equality case above). For every other eigenvalue, $L(\cdot)$ is strictly increasing in κ : as strategic influence becomes cheaper (lower κ), the scaling factor on every cleavage other than the consensus direction $\mathbf{1}$ falls, pulling equilibrium actions toward the network consensus. $\square$

E Proof of Proposition 5

Proposition. (Proposition 5 in the paper) Suppose that Σ_θ is nonsingular. Under the assumption that $\mathbf{G}$ has non-negative entries, rows summing to $1 - \alpha$, and $\kappa > 1/\alpha$, $\mathcal{L}(\mathbf{G})$ is the unique symmetric positive definite solution of:

$$\Sigma_a = \mathcal{L}(\mathbf{G}) \Sigma_\theta \mathcal{L}(\mathbf{G}), \quad \mathcal{L}(\mathbf{G}) = \Sigma_\theta^{-1/2} \left(\Sigma_\theta^{1/2} \Sigma_a \Sigma_\theta^{1/2} \right)^{1/2} \Sigma_\theta^{-1/2}. \quad (\text{A.11})$$

The geopolitical cleavages are the eigenvectors of $\mathcal{L}(\mathbf{G})$ and the eigenvalues of the geopolitical network can be recovered by inverting the scaling function $\lambda_\ell = L^{-1}(\cdot)$ from Proposition 4 in the paper.

Proof. Recall from Proposition 1 in the paper that equilibrium geopolitical actions are a linear transformation of domestic interests, $\mathbf{a} = \mathcal{L}(\mathbf{G}) \boldsymbol{\theta}$, with

$$\mathcal{L}(\mathbf{G}) \equiv [\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I}]^{-1} (\mathbf{I} - \mathbf{G}) (\kappa \alpha - 1). \quad (\text{A.12})$$

From Proposition 4, $\mathbf{G} = \mathbf{U} \boldsymbol{\Lambda} \mathbf{U}^\top$ with $\boldsymbol{\Lambda} = \text{Diag}(\lambda_1, \dots, \lambda_N)$, which implies that we can rewrite the above expression as:

$$\mathcal{L}(\mathbf{G}) = \mathbf{U} \text{Diag}(L(\lambda_1), \dots, L(\lambda_N)) \mathbf{U}^\top, \quad L(\lambda) \equiv \frac{(1 - \lambda)(\kappa \alpha - 1)}{\kappa(1 - \lambda)^2 - \alpha}. \quad (\text{A.13})$$

Since $\mathbf{a} = \mathcal{L}(\mathbf{G}) \boldsymbol{\theta}$, the covariance matrices of actions and interests are linked by

$$\Sigma_a = \mathcal{L}(\mathbf{G}) \Sigma_\theta \mathcal{L}(\mathbf{G})^\top. \quad (\text{A.14})$$

Step 1: $\mathcal{L}(\mathbf{G})$ is symmetric positive definite. Because $\mathbf{G}$ has nonnegative entries and rows summing to $1 - \alpha$, the Perron–Frobenius theorem implies that every eigenvalue satisfies $|\lambda_\ell| \leq 1 - \alpha$, so $\lambda_\ell \in [-(1 - \alpha), 1 - \alpha]$ (the bound used in the proofs of Propositions 3 and 4 in the paper). Write $z \equiv 1 - \lambda$, so that $z \in [\alpha, 2 - \alpha]$ on this range. Then

$$\kappa z^2 - \alpha \geq \kappa \alpha^2 - \alpha = \alpha(\kappa \alpha - 1) > 0,$$

using $\kappa > 1/\alpha$, so the denominator in (A.13) is strictly positive and $L(\lambda)$ is well defined. The numerator $z(\kappa \alpha - 1)$ is also strictly positive. Hence

$$\mathcal{L}(\mathbf{G}) > 0 \quad \text{for every eigenvalue } \lambda_\ell \text{ of } \mathbf{G}. \quad (\text{A.15})$$

From equation (A.13), $\mathcal{L}(\mathbf{G})$ is an orthogonal conjugate of a diagonal matrix with strictly positive entries. Therefore, $\mathcal{L}(\mathbf{G})$ is symmetric positive definite, which implies $\mathcal{L}(\mathbf{G})^\top = \mathcal{L}(\mathbf{G})$. Using this

property in equation ((A.14)), we have:

$$\Sigma_a = \mathcal{L}(G) \Sigma_\theta \mathcal{L}(G)^\top = \mathcal{L}(G) \Sigma_\theta \mathcal{L}(G). \quad (\text{A.16})$$

Since G is symmetric, we have $G = U \Lambda U^\top$ with U orthogonal and real eigenvalues $\lambda_1, \dots, \lambda_N$. For a symmetric positive definite matrix S , $S^{1/2}$ denotes its unique symmetric positive definite square root and $S^{-1/2} \equiv (S^{1/2})^{-1}$.

Step 2: $\mathcal{L}(G)$ is the unique symmetric positive definite solution of $\Sigma_a = X \Sigma_\theta X$. From Step 1 and equation (A.16), $X = \mathcal{L}(G)$ is a symmetric positive definite solution, so existence holds. For uniqueness, let X be any symmetric positive definite matrix with $X \Sigma_\theta X = \Sigma_a$. Since Σ_θ is non-singular, it is symmetric positive definite, and so $\Sigma_\theta^{1/2}$ exists and is invertible. Define

$$B \equiv \Sigma_\theta^{1/2} X \Sigma_\theta^{1/2}.$$

B is symmetric positive definite, being a congruence transformation of the symmetric positive definite matrix X by the invertible matrix $\Sigma_\theta^{1/2}$. Moreover,

$$B^2 = \Sigma_\theta^{1/2} X \Sigma_\theta^{1/2} \Sigma_\theta^{1/2} X \Sigma_\theta^{1/2} = \Sigma_\theta^{1/2} (X \Sigma_\theta X) \Sigma_\theta^{1/2} = \Sigma_\theta^{1/2} \Sigma_a \Sigma_\theta^{1/2}.$$

The right-hand side is symmetric positive definite (because Σ_a is: by (A.14) it is a congruence transformation of Σ_θ by the invertible matrix $\mathcal{L}(G)$). A symmetric positive definite matrix has a unique symmetric positive definite square root, so

$$B = \left(\Sigma_\theta^{1/2} \Sigma_a \Sigma_\theta^{1/2} \right)^{1/2}, \quad \text{and hence} \quad X = \Sigma_\theta^{-1/2} B \Sigma_\theta^{-1/2} = \Sigma_\theta^{-1/2} \left(\Sigma_\theta^{1/2} \Sigma_a \Sigma_\theta^{1/2} \right)^{1/2} \Sigma_\theta^{-1/2}.$$

Thus any symmetric positive definite solution coincides with the right-hand side of equation (A.11), and since $\mathcal{L}(G)$ is one such solution, $\mathcal{L}(G)$ equals this expression. This establishes Proposition 5 in the paper: the operator $\mathcal{L}(G)$ is identified from the observable covariance matrices $(\Sigma_a, \Sigma_\theta)$ without any assumption of graph stationarity.

Step 3: recovering the cleavages. From equation (A.13), $\mathcal{L}(G)$ and G share the eigenbasis U : the eigenvectors $\mathbf{u}_\ell$ of $\mathcal{L}(G)$ are the geopolitical cleavages, and the associated eigenvalues are $\mu_\ell \equiv L(\lambda_\ell)$. Since $\mathcal{L}(G)$ is identified by Step 2, its spectral decomposition is identified by the spectral theorem. When the μ_ℓ are pairwise distinct, each $\mathbf{u}_\ell$ is determined up to sign, and because $G = \sum_\ell \lambda_\ell \mathbf{u}_\ell \mathbf{u}_\ell^\top$ depends on $\mathbf{u}_\ell$ only through the sign-invariant outer product $\mathbf{u}_\ell \mathbf{u}_\ell^\top$, this sign ambiguity is immaterial. Repeated eigenvalues are handled at the end of Step 4.

Step 4: recovering the eigenvalues of G . From Proposition (4), $L(\cdot)$ is strictly increasing on $[-(1-\alpha), 1-\alpha]$ ($L'(\cdot) > 0$), hence injective, and therefore invertible on its image. Applying L^{-1} to each identified eigenvalue of $\mathcal{L}(G)$ gives

$$\lambda_\ell = L^{-1}(\mu_\ell), \quad \ell = 1, \dots, N,$$

and therefore

$$\mathbf{G} = \sum_{\ell} \lambda_{\ell} \mathbf{u}_{\ell} \mathbf{u}_{\ell}^{\top} = \mathbf{U} \text{Diag} \left(L^{-1}(\mu_1), \dots, L^{-1}(\mu_N) \right) \mathbf{U}^{\top} \equiv L^{-1}(\mathcal{L}(\mathbf{G})),$$

where the last expression is the spectral function L^{-1} applied to the identified matrix $\mathcal{L}(\mathbf{G})$. Every object on the right-hand side is determined by $(\Sigma_a, \Sigma_{\theta})$, so $\mathbf{G}$ is identified.

If some of the μ_{ℓ} coincide, the corresponding eigenvectors are determined only up to an orthogonal rotation within their shared eigenspace. Because L is injective, $\mu_m = \mu_p$ if and only if $\lambda_m = \lambda_p$, so the eigenspaces of $\mathcal{L}(\mathbf{G})$ and $\mathbf{G}$ coincide exactly, and any rotation within such an eigenspace leaves $\sum_{\ell} \lambda_{\ell} \mathbf{u}_{\ell} \mathbf{u}_{\ell}^{\top}$ unchanged. Hence $\mathbf{G}$ is identified regardless of whether its eigenvalues are distinct. $\square$

F Proof of Proposition 6

Proposition. (*Proposition 6 in the paper*) Under the assumption of graph stationarity, Σ_{θ} shares the eigenbasis $\mathbf{U}$ of $\mathbf{G}$. Let $\tilde{\Sigma}_a \equiv \mathbf{U}^{\top} \Sigma_a \mathbf{U}$, $\tilde{\Sigma}_{\theta} \equiv \mathbf{U}^{\top} \Sigma_{\theta} \mathbf{U}$ be the projection of Σ_a and Σ_{θ} onto the eigenbasis $\mathbf{U}$, with ℓ -th diagonal elements $\sigma_{a,\ell}^2$ and $\sigma_{\theta,\ell}^2$, respectively. Given α and κ , the eigenvalues (λ_{ℓ}) and eigenvectors $(\mathbf{u}_{\ell})$ of $\mathbf{G}$ are identified if $\sigma_{\theta,\ell}^2$ is nonzero, with

$$\lambda_{\ell} = L^{-1} \left(\sqrt{\sigma_{a,\ell}^2 / \sigma_{\theta,\ell}^2} \right), \quad (\text{A.17})$$

where $L(\cdot)$ is defined in Proposition 4.

Proof. The identification argument has two parts: recovering the eigenvectors $\mathbf{u}_{\ell}$ from Σ_{θ} alone, and then recovering the associated eigenvalues λ_{ℓ} by combining Σ_{θ} with Σ_a .

Recovering $\mathbf{u}_{\ell}$. Under the assumption of graph stationarity, $\Sigma_{\theta} = \mathbf{U} \tilde{\Sigma}_{\theta} \mathbf{U}^{\top}$ with $\tilde{\Sigma}_{\theta}$ diagonal: this is exactly the spectral decomposition of the (observable) matrix Σ_{θ} , with $\mathbf{u}_{\ell}$ the eigenvector associated with eigenvalue $\sigma_{\theta,\ell}^2$. By the spectral theorem, this decomposition is recoverable directly from data on Σ_{θ} : whenever the $\sigma_{\theta,\ell}^2$ are pairwise distinct, the eigenvectors $\{\mathbf{u}_{\ell}\}$ are uniquely determined up to sign, and since $\mathbf{G} = \sum_{\ell} \lambda_{\ell} \mathbf{u}_{\ell} \mathbf{u}_{\ell}^{\top}$ depends on each $\mathbf{u}_{\ell}$ only through the sign-invariant outer product $\mathbf{u}_{\ell} \mathbf{u}_{\ell}^{\top}$, this sign ambiguity is immaterial for recovering $\mathbf{G}$. (If some $\sigma_{\theta,\ell}^2$'s coincide, the corresponding eigenvectors are only determined up to an orthogonal rotation within their shared eigenspace; diagonalizing the covariance of actions within that eigenspace resolves them whenever the corresponding eigenvalues of $\mathbf{G}$ differ, and if those coincide too, any rotation within the eigenspace leaves $\mathbf{G}$ unchanged.)

Recovering λ_{ℓ} . By equation (19), which follows from $\mathbf{a} = \mathcal{L}(\mathbf{G}) \boldsymbol{\theta}$ and Proposition 4 because $\Sigma_a = \mathcal{L}(\mathbf{G}) \Sigma_{\theta} \mathcal{L}(\mathbf{G})^{\top}$,

$$\sigma_{a,\ell}^2 = L(\lambda_{\ell})^2 \sigma_{\theta,\ell}^2.$$

If $\sigma_{\theta,\ell}^2 \neq 0$, divide both sides by $\sigma_{\theta,\ell}^2$ and take the (nonnegative) square root:

$$|L(\lambda_\ell)| = \sqrt{\sigma_{a,\ell}^2 / \sigma_{\theta,\ell}^2}.$$

By Proposition 4, $L(\lambda) \geq 0$ for every eigenvalue λ of $\mathbf{G}$ in the relevant range $[-(1-\alpha), 1-\alpha]$, so $|L(\lambda_\ell)| = L(\lambda_\ell)$ and hence

$$L(\lambda_\ell) = \sqrt{\sigma_{a,\ell}^2 / \sigma_{\theta,\ell}^2}.$$

By Proposition 4, $L(\cdot)$ is strictly increasing on this range ($L'(\cdot) > 0$), hence injective, and therefore invertible on its image. Applying L^{-1} to both sides:

$$\lambda_\ell = L^{-1} \left(\sqrt{\sigma_{a,\ell}^2 / \sigma_{\theta,\ell}^2} \right),$$

which is uniquely determined by the observable quantities $\sigma_{a,\ell}^2$ and $\sigma_{\theta,\ell}^2$. Together with the recovered eigenvector $\mathbf{u}_\ell$, this identifies the pair $(\lambda_\ell, \mathbf{u}_\ell)$, and hence the term $\lambda_\ell \mathbf{u}_\ell \mathbf{u}_\ell^\top$ of $\mathbf{G} = \sum_\ell \lambda_\ell \mathbf{u}_\ell \mathbf{u}_\ell^\top$, whenever $\sigma_{\theta,\ell}^2 \neq 0$.

Failure of identification when $\sigma_{\theta,\ell}^2 = 0$. If instead $\sigma_{\theta,\ell}^2 = 0$, then by equation (19), $\sigma_{a,\ell}^2 = L(\lambda_\ell)^2 \cdot 0 = 0$ identically, for every value of λ_ℓ : the observed moment $\sigma_{a,\ell}^2$ carries no information about λ_ℓ , since domestic interests never vary along the cleavage $\mathbf{u}_\ell$ and hence this direction of $\mathbf{G}$ is never excited by any observed issue. Consequently, $\mathbf{G}$ is identified only on the subspace spanned by $\{\mathbf{u}_\ell : \sigma_{\theta,\ell}^2 > 0\}$; on the orthogonal complement, λ_ℓ remains unrestricted by $(\Sigma_a, \Sigma_\theta)$. $\square$

G Graph Stationarity

We now show that, when Σ_θ is nonsingular, the covariance matrices of domestic interests and geopolitical actions share the same principal components if and only if graph stationarity holds, as discussed in Section 2.5 of the paper.

Remark. Suppose that the eigenvalues of $\mathbf{G}$ are pairwise distinct and that Σ_θ is nonsingular. Then Σ_θ and Σ_a share a common set of principal components if and only if graph stationarity holds.

Proof. Two symmetric matrices share a common orthonormal eigenbasis if and only if they commute; we therefore characterize when $\Sigma_\theta \Sigma_a = \Sigma_a \Sigma_\theta$. If graph stationarity holds, then Σ_θ is a spectral function of $\mathbf{G}$ by assumption, and so is Σ_a by equation (19); hence they commute. For the converse, work in the eigenbasis of $\mathbf{G}$: write $\mathbf{G} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^\top$, $\mathbf{S} \equiv \mathbf{U}^\top \Sigma_\theta \mathbf{U}$, and $\mathbf{D} \equiv \text{Diag}(L(\lambda_1), \dots, L(\lambda_N))$, so that $\mathbf{U}^\top \Sigma_a \mathbf{U} = \mathbf{D} \mathbf{S} \mathbf{D}$ by Proposition 4. Direct computation gives

$$[\mathbf{U}^\top (\Sigma_\theta \Sigma_a - \Sigma_a \Sigma_\theta) \mathbf{U}]_{mp} = (L(\lambda_p) - L(\lambda_m)) [\mathbf{S} \mathbf{D} \mathbf{S}]_{mp}.$$

Because $L(\cdot)$ is strictly increasing (Proposition 4) and the eigenvalues are pairwise distinct, $L(\lambda_m) \neq L(\lambda_p)$ for $m \neq p$, so commutation requires $[\mathbf{S} \mathbf{D} \mathbf{S}]_{mp} = 0$ for all $m \neq p$: the matrix

SDS is diagonal. Now define $B \equiv D^{1/2}SD^{1/2}$, which is symmetric and positive definite, since S is positive definite (Σ_θ is a nonsingular covariance matrix) and D is positive definite ($L(\lambda) > 0$ on $[-(1-\alpha), 1-\alpha]$, by Proposition 4). Then $B^2 = D^{1/2}(SDS)D^{1/2}$ is diagonal with strictly positive entries. Since a positive definite matrix has a unique positive definite square root, and the diagonal matrix $(B^2)^{1/2}$ is one such root, B is diagonal; hence $S = D^{-1/2}BD^{-1/2}$ is diagonal, which corresponds precisely to graph stationarity. If some eigenvalues of G coincide, the same argument forces S to be block-diagonal across distinct eigenvalues, so that graph stationarity holds up to rotations within repeated eigenspaces, which leave G unchanged. The same argument in the Π -weighted coordinates of Section 2.6 delivers the analogous characterization of Assumption 1. $\square$

H Proof of Proposition 7

Proposition. (*Proposition 7 in the paper*) Suppose $G = (1-\alpha)\Pi^{-1}H$ with influence costs $\kappa(\pi_k/\pi_i)\omega_{ki}^2$. Then:

1. Propositions 1 and 2 continue to hold. In particular:

(a) Equilibrium net influence and geopolitical actions satisfy:

$$\begin{aligned}\Omega \mathbf{1} &= -(\kappa(\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1}((\mathbf{I} - \mathbf{G})^2 - \alpha^2 \mathbf{I}) \boldsymbol{\theta}, \\ \mathbf{a} &= (\kappa(\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1}(\mathbf{I} - \mathbf{G})(\kappa\alpha - 1) \boldsymbol{\theta}.\end{aligned}$$

(b) The equilibrium inherits uniqueness, translation invariance, the convex combination property, Π -weighted conservativeness ($\sum_i \pi_i a_i = \sum_i \pi_i \theta_i$ and $\sum_i \pi_i \sum_m \omega_{im} = 0$), and the Π -weighted analogues of the two limiting benchmarks ($\kappa \rightarrow \infty$ and $\kappa \downarrow 1/\alpha$) from the symmetric baseline.

2. The equilibrium continues to admit a spectral representation in terms of the right and left eigenvectors of G , $\mathbf{u}_m^R$ and $\mathbf{u}_m^L$, normalized so that $(\mathbf{u}_m^L)^\top \mathbf{u}_p^R = \mathbf{1}_{m=p}$:

$$a_i = \sum_m L(\lambda_m) [\mathbf{u}_m^R]_i (\mathbf{u}_m^L)^\top \boldsymbol{\theta}, \quad (\text{A.18})$$

where the scaling function $L(\lambda) \equiv (1-\lambda)(\kappa\alpha - 1) / (\kappa(1-\lambda)^2 - \alpha)$ is identical to that under a symmetric network.

Proof. Throughout this proof, write $\tilde{H} \equiv (1-\alpha)\Pi^{-1/2}H\Pi^{-1/2}$ for the symmetrized network and $\sqrt{\pi}$ for the vector with entries $\sqrt{\pi_i}$. By construction, $G = \Pi^{-1/2}\tilde{H}\Pi^{1/2}$, the matrix $\tilde{H}$ is symmetric and nonnegative, $\tilde{H}\sqrt{\pi} = (1-\alpha)\sqrt{\pi}$ (since $H\mathbf{1} = \pi$), and $\|\sqrt{\pi}\| = 1$ (since $\sum_i \pi_i = 1$). The influence cost of i exerting ω_{ki} on k is $\kappa(\pi_k/\pi_i)\omega_{ki}^2$, and country i 's payoff is:

$$U_i(\mathbf{a}, \boldsymbol{\Omega}) = - \left[\alpha(a_i - \theta_i)^2 + \sum_j G_{ij}(a_i - a_j)^2 \right] - \kappa \sum_k \frac{\pi_k}{\pi_i} \omega_{ki}^2 + 2 \sum_m \omega_{im}(a_i - \theta_i). \quad (\text{A.19})$$

We first collect four structural facts about the pair $(\mathbf{G}, \tilde{\mathbf{H}})$ that drive every part of the proof.

Fact 1 ($\mathbf{G}$ is nonnegative with constant row sums). $G_{ij} = (\sqrt{\pi_j}/\sqrt{\pi_i}) \tilde{H}_{ij} \geq 0$ since $\tilde{H}_{ij} \geq 0$ and $\sqrt{\pi_i}, \sqrt{\pi_j} > 0$. Since $\tilde{\mathbf{H}}\sqrt{\boldsymbol{\pi}} = (1 - \alpha)\sqrt{\boldsymbol{\pi}}$, $\mathbf{G}\mathbf{1} = \boldsymbol{\Pi}^{-1/2}\tilde{\mathbf{H}}\boldsymbol{\Pi}^{1/2}\mathbf{1} = \boldsymbol{\Pi}^{-1/2}\tilde{\mathbf{H}}\sqrt{\boldsymbol{\pi}} = (1 - \alpha)\boldsymbol{\Pi}^{-1/2}\sqrt{\boldsymbol{\pi}} = (1 - \alpha)\mathbf{1}$.

Fact 2 (similarity transfer). Because $\mathbf{G} = \boldsymbol{\Pi}^{-1/2}\tilde{\mathbf{H}}\boldsymbol{\Pi}^{1/2}$, induction gives $\mathbf{G}^n = \boldsymbol{\Pi}^{-1/2}\tilde{\mathbf{H}}^n\boldsymbol{\Pi}^{1/2}$ for every $n \geq 0$, so for any polynomial (or, where the relevant inverse exists, rational function) $p(\cdot)$, $p(\mathbf{G}) = \boldsymbol{\Pi}^{-1/2}p(\tilde{\mathbf{H}})\boldsymbol{\Pi}^{1/2}$. In particular $\mathbf{G}$ and $\tilde{\mathbf{H}}$ are similar and share the same eigenvalues $\{\lambda_m\}$, all real (as $\tilde{\mathbf{H}}$ is symmetric) and satisfying $|\lambda_m| \leq 1 - \alpha$ by Perron–Frobenius applied to $\tilde{\mathbf{H}}$. Substituting the spectral form $\tilde{\mathbf{H}} = \sum_m \lambda_m \mathbf{v}_m \mathbf{v}_m^\top$,

$$\begin{aligned} p(\mathbf{G}) &= \boldsymbol{\Pi}^{-1/2} \left(\sum_m p(\lambda_m) \mathbf{v}_m \mathbf{v}_m^\top \right) \boldsymbol{\Pi}^{1/2} = \sum_m p(\lambda_m) (\boldsymbol{\Pi}^{-1/2} \mathbf{v}_m) (\boldsymbol{\Pi}^{1/2} \mathbf{v}_m)^\top \\ &= \sum_m p(\lambda_m) \mathbf{u}_m^R (\mathbf{u}_m^L)^\top, \end{aligned} \quad (\text{A.20})$$

with $\mathbf{u}_m^R \equiv \boldsymbol{\Pi}^{-1/2} \mathbf{v}_m$, $\mathbf{u}_m^L \equiv \boldsymbol{\Pi}^{1/2} \mathbf{v}_m$ satisfying biorthonormality $(\mathbf{u}_m^L)^\top \mathbf{u}_p^R = \mathbf{v}_m^\top \boldsymbol{\Pi}^{1/2} \boldsymbol{\Pi}^{-1/2} \mathbf{v}_p = \mathbf{v}_m^\top \mathbf{v}_p = \mathbf{1}_{m=p}$; since $\{\mathbf{v}_m\}$ is a basis and $\boldsymbol{\Pi}^{1/2}$ is invertible, $\{\mathbf{u}_m^R\}$ is also a basis, with $\{\mathbf{u}_m^L\}$ its (unique) dual basis, so every vector $\boldsymbol{\theta}$ has the expansion $\boldsymbol{\theta} = \sum_m \tau_m \mathbf{u}_m^R$, $\tau_m \equiv (\mathbf{u}_m^L)^\top \boldsymbol{\theta}$. Note $\mathbf{u}_1^R = \boldsymbol{\Pi}^{-1/2} \mathbf{v}_1 = \boldsymbol{\Pi}^{-1/2} \sqrt{\boldsymbol{\pi}} = \mathbf{1}$ (since $\mathbf{v}_1 = \sqrt{\boldsymbol{\pi}}$) and $\mathbf{u}_1^L = \boldsymbol{\Pi}^{1/2} \mathbf{v}_1 = \boldsymbol{\Pi}^{1/2} \sqrt{\boldsymbol{\pi}}$, i.e., $[\mathbf{u}_1^L]_i = \pi_i$.

Fact 3 ($\boldsymbol{\Pi}\mathbf{G}$ is symmetric). Define $\mathbf{W} \equiv \boldsymbol{\Pi}\mathbf{G} = \boldsymbol{\Pi}^{1/2}\tilde{\mathbf{H}}\boldsymbol{\Pi}^{1/2}$ (using $\mathbf{G} = \boldsymbol{\Pi}^{-1/2}\tilde{\mathbf{H}}\boldsymbol{\Pi}^{1/2}$). Since $\boldsymbol{\Pi}^{1/2}$ and $\tilde{\mathbf{H}}$ are both symmetric, $\mathbf{W}^\top = \boldsymbol{\Pi}^{1/2}\tilde{\mathbf{H}}^\top\boldsymbol{\Pi}^{1/2} = \boldsymbol{\Pi}^{1/2}\tilde{\mathbf{H}}\boldsymbol{\Pi}^{1/2} = \mathbf{W}$. Equivalently, $\pi_i G_{ij} = \sqrt{\pi_i} \sqrt{\pi_j} \tilde{H}_{ij} = \sqrt{\pi_j} \sqrt{\pi_i} \tilde{H}_{ji} = \pi_j G_{ji}$ for every i, j .

Fact 4 (row/column sums of $\mathbf{W}$). By Fact 1, $\mathbf{W}\mathbf{1} = \boldsymbol{\Pi}\mathbf{G}\mathbf{1} = (1 - \alpha)\boldsymbol{\Pi}\mathbf{1}$, i.e., row j of $\mathbf{W}$ sums to $(1 - \alpha)\pi_j$; by symmetry (Fact 3), column j sums to the same value: $\sum_i W_{ij} = (1 - \alpha)\pi_j$.

Part 1(a): the closed-form solutions. *Coordination stage.* The coordination-game component of (A.19) is unchanged from the symmetric model, so the first-order condition for a_i and the resulting fixed point are identical in form:

$$\mathbf{a} = (\mathbf{I} - \mathbf{G})^{-1} (\alpha \boldsymbol{\theta} + \boldsymbol{\Omega} \mathbf{1}), \quad (\text{A.21})$$

using invertibility of $\mathbf{I} - \mathbf{G}$, which holds because $|\lambda_m| \leq 1 - \alpha < 1$ for every eigenvalue of $\mathbf{G}$ (Fact 2).

Influence stage. The envelope-theorem argument is unchanged except for the direct effect, which now involves the asymmetric cost: $\partial U_i / \partial \omega_{ki} \big|_{\mathbf{a}} = -2\kappa (\pi_k / \pi_i) \omega_{ki} + 2 \cdot \mathbf{1}_{k=i} (a_i - \theta_i)$, while $\partial U_i / \partial a_j = 2G_{ij} (a_i - a_j)$ and $\partial a_j / \partial \omega_{ki} = [(\mathbf{I} - \mathbf{G})^{-1}]_{jk}$ are exactly as before (the latter because only the k -th entry of $\boldsymbol{\Omega} \mathbf{1} = \sum_i \omega_{ki}$ responds to ω_{ki} , with unit coefficient, regardless of the cost function). The

first-order condition is thus

$$\kappa \left(\frac{\pi_k}{\pi_i} \right) \omega_{ki} = \sum_j G_{ij} (a_i - a_j) \left[(\mathbf{I} - \mathbf{G})^{-1} \right]_{jk} + \mathbf{1}_{k=i} (a_i - \theta_i). \quad (\text{A.22})$$

Multiplying by π_i/π_k and summing over senders $i = 1, \dots, N$:

$$\kappa \sum_i \omega_{ki} = \frac{1}{\pi_k} \sum_j \left[(\mathbf{I} - \mathbf{G})^{-1} \right]_{jk} \left(\sum_i \pi_i G_{ij} (a_i - a_j) \right) + (a_k - \theta_k),$$

where the second term uses $\sum_i \pi_i \mathbf{1}_{k=i} (a_i - \theta_i) / \pi_k = a_k - \theta_k$. For the inner sum, use $W_{ij} = \pi_i G_{ij} = \pi_j G_{ji}$ (Fact 3) together with $\sum_i W_{ij} = (1 - \alpha) \pi_j$ (Fact 4):

$$\begin{aligned} \sum_i \pi_i G_{ij} (a_i - a_j) &= \sum_i W_{ij} a_i - a_j \sum_i W_{ij} = \sum_i W_{ji} a_i - (1 - \alpha) \pi_j a_j \\ &= \pi_j [\mathbf{G}\mathbf{a}]_j - (1 - \alpha) \pi_j a_j = \pi_j c_j, \end{aligned}$$

using symmetry $W_{ij} = W_{ji}$ so that $\sum_i W_{ji} a_i = [\mathbf{W}\mathbf{a}]_j = \pi_j [\mathbf{G}\mathbf{a}]_j$ (as $\mathbf{W} = \mathbf{\Pi}\mathbf{G}$), and writing $\mathbf{c} \equiv \mathbf{G}\mathbf{a} - (1 - \alpha) \mathbf{a} = -(\mathbf{I} - \mathbf{G}) \mathbf{a} + \alpha \mathbf{a}$. Substituting, and noting that the sum over j contracts the *first* index of $(\mathbf{I} - \mathbf{G})^{-1}$ (so this is genuinely a transpose contraction, since $\mathbf{G}$ need not be symmetric):

$$\kappa \Omega \mathbf{1} = \mathbf{\Pi}^{-1} (\mathbf{I} - \mathbf{G})^{-\top} \mathbf{\Pi} \mathbf{c} + \mathbf{a} - \boldsymbol{\theta}.$$

Since $\mathbf{G}^\top = \mathbf{\Pi}^{1/2} \tilde{\mathbf{H}} \mathbf{\Pi}^{-1/2}$ (using $\mathbf{G} = \mathbf{\Pi}^{-1/2} \tilde{\mathbf{H}} \mathbf{\Pi}^{1/2}$ and $\tilde{\mathbf{H}}^\top = \tilde{\mathbf{H}}$), $\mathbf{I} - \mathbf{G}^\top = \mathbf{\Pi}^{1/2} (\mathbf{I} - \tilde{\mathbf{H}}) \mathbf{\Pi}^{-1/2}$, so $(\mathbf{I} - \mathbf{G})^{-\top} = \mathbf{\Pi}^{1/2} (\mathbf{I} - \tilde{\mathbf{H}})^{-1} \mathbf{\Pi}^{-1/2}$ and, writing $\mathbf{a}^\Pi \equiv \mathbf{\Pi}^{1/2} \mathbf{a}$,

$$\begin{aligned} \mathbf{\Pi}^{-1} (\mathbf{I} - \mathbf{G})^{-\top} \mathbf{\Pi} \mathbf{c} &= \mathbf{\Pi}^{-1/2} (\mathbf{I} - \tilde{\mathbf{H}})^{-1} \mathbf{\Pi}^{1/2} \mathbf{c}, \\ \mathbf{\Pi}^{1/2} \mathbf{c} &= -(\mathbf{a}^\Pi - \tilde{\mathbf{H}} \mathbf{a}^\Pi) + \alpha \mathbf{a}^\Pi = (\tilde{\mathbf{H}} - (1 - \alpha) \mathbf{I}) \mathbf{a}^\Pi, \end{aligned}$$

using $\mathbf{\Pi}^{1/2} (\mathbf{I} - \mathbf{G}) \mathbf{a} = \mathbf{\Pi}^{1/2} \mathbf{a} - \tilde{\mathbf{H}} \mathbf{\Pi}^{1/2} \mathbf{a} = \mathbf{a}^\Pi - \tilde{\mathbf{H}} \mathbf{a}^\Pi$. Since $(\mathbf{I} - \tilde{\mathbf{H}})^{-1} (\tilde{\mathbf{H}} - (1 - \alpha) \mathbf{I}) = (\mathbf{I} - \tilde{\mathbf{H}})^{-1} [-(\mathbf{I} - \tilde{\mathbf{H}}) + \alpha \mathbf{I}] = -\mathbf{I} + \alpha (\mathbf{I} - \tilde{\mathbf{H}})^{-1}$,

$$\begin{aligned} \mathbf{\Pi}^{-1/2} (\mathbf{I} - \tilde{\mathbf{H}})^{-1} \mathbf{\Pi}^{1/2} \mathbf{c} &= \mathbf{\Pi}^{-1/2} \left[-\mathbf{I} + \alpha (\mathbf{I} - \tilde{\mathbf{H}})^{-1} \right] \mathbf{a}^\Pi \\ &= -\mathbf{a} + \alpha \mathbf{\Pi}^{-1/2} (\mathbf{I} - \tilde{\mathbf{H}})^{-1} \mathbf{\Pi}^{1/2} \mathbf{a} = -\mathbf{a} + \alpha (\mathbf{I} - \mathbf{G})^{-1} \mathbf{a}, \end{aligned}$$

using $(\mathbf{I} - \mathbf{G})^{-1} = \mathbf{\Pi}^{-1/2} (\mathbf{I} - \tilde{\mathbf{H}})^{-1} \mathbf{\Pi}^{1/2}$ (Fact 2) in the last step. Substituting back,

$$\kappa \Omega \mathbf{1} = [-\mathbf{a} + \alpha (\mathbf{I} - \mathbf{G})^{-1} \mathbf{a}] + \mathbf{a} - \boldsymbol{\theta} = \alpha (\mathbf{I} - \mathbf{G})^{-1} \mathbf{a} - \boldsymbol{\theta}, \quad (\text{A.23})$$

identical in form to the symmetric case. Equations (A.21) and (A.23) are *identical in form* to the corresponding coordination-stage and influence-stage fixed-point equations in the proof of Proposition 1, with $\mathbf{G}$ now asymmetric. Since the algebra combining these two equations into closed form (Step 3 of that proof) uses only that $(\mathbf{I} - \mathbf{G})$, $(\mathbf{I} - \mathbf{G})^{-1}$, and $(\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1}$ are all polynomials or

rational functions of the single matrix $\mathbf{G}$ —and such functions always commute, whether or not $\mathbf{G}$ is symmetric—that algebra applies verbatim here, yielding

$$\begin{aligned}\mathbf{a} &= (\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} (\mathbf{I} - \mathbf{G}) (\kappa \alpha - 1) \boldsymbol{\theta}, \\ \Omega \mathbf{1} &= -(\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})^{-1} ((\mathbf{I} - \mathbf{G})^2 - \alpha^2 \mathbf{I}) \boldsymbol{\theta},\end{aligned}$$

invertibility of $\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I}$ again following from $|\lambda_m| \leq 1 - \alpha$ for every eigenvalue of $\mathbf{G}$ (Fact 2), exactly as in Proposition 1. Write $\mathbf{a} = \mathbf{M}\boldsymbol{\theta}$, $\Omega \mathbf{1} = \mathbf{N}\boldsymbol{\theta}$ for these two matrices.

Part 1(b)(i): uniqueness. U_i is strictly concave in a_i (second derivative $-2(\alpha + \sum_{j \neq i} G_{ij}) = -2(1 - G_{ii}) < 0$, unchanged) and, at the influence stage, strictly concave in its own controls $\{\omega_{ki}\}_k$ whenever $\kappa > \kappa^*(\mathbf{G})$, which holds under $\kappa > 1/\alpha$ for nonnegative $\mathbf{H}$ by the argument of Online Appendix J.4 (third remark), applied to the Π -weighted influence cost. The same backward-induction argument as in the proof of Proposition 2 applies: each stage's best response is unique and linear, the resulting linear system in $(\mathbf{a}, \Omega)$ has a unique solution because $\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I}$ is invertible, and existence is given by the closed form just derived.

Part 1(b)(ii): convex combination. By Fact 2 with $p = L$, $\mathbf{M} = L(\mathbf{G}) = \sum_{n \geq 0} d_n \mathbf{G}^n$ using the same partial-fraction expansion as in the proof of Proposition 2, with the same coefficients $d_n > 0$ (the expansion depends only on κ, α and the eigenvalue bound $|\lambda_m| \leq 1 - \alpha$, both unchanged). Since $G_{ij} \geq 0$ (Fact 1, regardless of symmetry), $\mathbf{G}^n$ has nonnegative entries for every n , so $\mathbf{M}$ has nonnegative entries. Row sums equal one because $\mathbf{G}\mathbf{1} = (1 - \alpha)\mathbf{1}$ (Fact 1) gives $(\kappa (\mathbf{I} - \mathbf{G})^2 - \alpha \mathbf{I})\mathbf{1} = \alpha(\kappa \alpha - 1)\mathbf{1}$ exactly as in Fact B of the proof of Proposition 2, so $\mathbf{M}\mathbf{1} = \mathbf{1}$. Hence each $a_i = \sum_k M_{ik} \theta_k$ is a convex combination of $\{\theta_k\}$.

Part 1(b)(iii): Π -weighted conservativeness and zero net influence. By Fact 2, $\mathbf{M} = \Pi^{-1/2} \mathbf{M}_H \Pi^{1/2}$ and $\mathbf{N} = \Pi^{-1/2} \mathbf{N}_H \Pi^{1/2}$, where

$$\begin{aligned}\mathbf{M}_H &\equiv \left(\kappa (\mathbf{I} - \tilde{\mathbf{H}})^2 - \alpha \mathbf{I} \right)^{-1} (\mathbf{I} - \tilde{\mathbf{H}}) (\kappa \alpha - 1), \\ \mathbf{N}_H &\equiv - \left(\kappa (\mathbf{I} - \tilde{\mathbf{H}})^2 - \alpha \mathbf{I} \right)^{-1} \left((\mathbf{I} - \tilde{\mathbf{H}})^2 - \alpha^2 \mathbf{I} \right)\end{aligned}$$

are the same rational functions of $\tilde{\mathbf{H}}$ that $\mathbf{M}, \mathbf{N}$ are of $\mathbf{G}$; since $\tilde{\mathbf{H}}$ is symmetric, $\mathbf{M}_H$ and $\mathbf{N}_H$ are symmetric (Fact A of the proof of Proposition 2, applied to $\tilde{\mathbf{H}}$). Because $\tilde{\mathbf{H}}\sqrt{\boldsymbol{\pi}} = (1 - \alpha)\sqrt{\boldsymbol{\pi}}$, the same computation as Fact B of that proof (with $\sqrt{\boldsymbol{\pi}}$ in place of $\mathbf{1}$, and $\tilde{\mathbf{H}}$ in place of $\mathbf{G}$) gives

$$\mathbf{M}_H \sqrt{\boldsymbol{\pi}} = \sqrt{\boldsymbol{\pi}}, \quad \mathbf{N}_H \sqrt{\boldsymbol{\pi}} = \mathbf{0}. \quad (\text{A.24})$$

Now, $\Pi \mathbf{M} = \Pi^{1/2} \mathbf{M}_H \Pi^{1/2}$ (using $\Pi \Pi^{-1/2} = \Pi^{1/2}$), and this is symmetric since $\mathbf{M}_H$ and $\Pi^{1/2}$ are. For a symmetric matrix $\mathbf{S}$, $\mathbf{1}^\top \mathbf{S} = (\mathbf{S}\mathbf{1})^\top$; applying this with $\mathbf{S} = \Pi \mathbf{M} = \Pi^{1/2} \mathbf{M}_H \Pi^{1/2}$ and using $\Pi^{1/2} \mathbf{1} = \sqrt{\boldsymbol{\pi}}$ and (A.24),

$$\mathbf{1}^\top \Pi \mathbf{M} = (\Pi^{1/2} \mathbf{M}_H \Pi^{1/2} \mathbf{1})^\top = (\Pi^{1/2} \mathbf{M}_H \sqrt{\boldsymbol{\pi}})^\top = (\Pi^{1/2} \sqrt{\boldsymbol{\pi}})^\top = \mathbf{1}^\top \Pi,$$

since $\Pi^{1/2}\sqrt{\pi}$ has i -th entry π_i , matching $\Pi\mathbf{1}$. Therefore $(\Pi\mathbf{1})^\top \mathbf{a} = (\Pi\mathbf{1})^\top M\boldsymbol{\theta} = (\Pi\mathbf{1})^\top \boldsymbol{\theta}$ for every $\boldsymbol{\theta}$, i.e., $\sum_i \pi_i a_i = \sum_i \pi_i \theta_i$. By the identical argument applied to N , using $N_H\sqrt{\pi} = \mathbf{0}$ in place of $M_H\sqrt{\pi} = \sqrt{\pi}$,

$$\mathbf{1}^\top \Pi N = (\Pi^{1/2} N_H \sqrt{\pi})^\top = \mathbf{0}^\top,$$

so $\sum_i \pi_i [\Omega\mathbf{1}]_i = (\Pi\mathbf{1})^\top N\boldsymbol{\theta} = 0$ for every $\boldsymbol{\theta}$, i.e., $\sum_i \pi_i \sum_m \omega_{im} = 0$.

Part 1(b)(iv): the two limits. By (A.20) applied with $p = L$ (recalling $\mathbf{a} = M\boldsymbol{\theta} = L(\mathbf{G})\boldsymbol{\theta}$),

$$\mathbf{a} = \sum_m L(\lambda_m) \tau_m \mathbf{u}_m^R, \quad \tau_m = (\mathbf{u}_m^L)^\top \boldsymbol{\theta}. \quad (\text{A.25})$$

This is precisely the spectral representation claimed in Part 2 of the proposition; we establish it here since the two limits follow directly from it. As $\kappa \rightarrow \infty$, $L(\lambda_m) \rightarrow \alpha/(1 - \lambda_m)$ for every m (dividing numerator and denominator by κ), so by (A.20) applied to $p(\lambda) = \alpha/(1 - \lambda)$, $\mathbf{a} \rightarrow \sum_m \frac{\alpha}{1 - \lambda_m} \tau_m \mathbf{u}_m^R = \alpha(\mathbf{I} - \mathbf{G})^{-1} \boldsymbol{\theta}$; similarly $\Omega\mathbf{1} \rightarrow \mathbf{0}$, since the corresponding scaling function $\Phi(\lambda) \rightarrow 0$ for every eigenvalue. As $\kappa \downarrow 1/\alpha$: exactly as in the proof of Proposition 1, $L(\lambda_1) = L(1 - \alpha) \equiv 1$ for every κ (the numerator and denominator both equal $\alpha(\kappa\alpha - 1)$ at $\lambda = 1 - \alpha$), while $L(\lambda_m) \rightarrow 0$ for $m \neq 1$ (the denominator stays bounded away from zero as $\kappa \rightarrow 1/\alpha$ since $|1 - \lambda_m| \neq \alpha$ for $m \neq 1$, while the numerator $\rightarrow 0$). Using $\mathbf{u}_1^R = \mathbf{1}$ and $\tau_1 = (\mathbf{u}_1^L)^\top \boldsymbol{\theta} = (\Pi^{1/2}\sqrt{\pi})^\top \boldsymbol{\theta} = \sum_i \pi_i \theta_i \equiv \bar{\theta}_\pi$ (the Π -weighted average of $\boldsymbol{\theta}$, a genuine weighted average since $\sum_i \pi_i = \|\sqrt{\pi}\|^2 = 1$), (A.25) gives $\mathbf{a} \rightarrow \tau_1 \mathbf{u}_1^R = \bar{\theta}_\pi \mathbf{1}$. For $\Omega\mathbf{1} = \sum_m \Phi(\lambda_m) \tau_m \mathbf{u}_m^R$ (with $\Phi(\lambda) \equiv (1 - \lambda)L(\lambda) - \alpha$ as in Proposition 1), $\Phi(\lambda_1) \rightarrow \alpha \cdot 1 - \alpha = 0$ and $\Phi(\lambda_m) \rightarrow -\alpha$ for $m \neq 1$, so

$$\Omega\mathbf{1} \rightarrow \sum_{m \neq 1} (-\alpha) \tau_m \mathbf{u}_m^R = -\alpha(\boldsymbol{\theta} - \tau_1 \mathbf{u}_1^R) = \alpha(\bar{\theta}_\pi \mathbf{1} - \boldsymbol{\theta}),$$

using the expansion $\boldsymbol{\theta} = \sum_m \tau_m \mathbf{u}_m^R$ from Fact 2. These are the natural Π -weighted analogues of the symmetric-network limits $\mathbf{a} \rightarrow \bar{\theta}\mathbf{1}$, $\Omega\mathbf{1} \rightarrow \alpha(\bar{\theta}\mathbf{1} - \boldsymbol{\theta})$, consistent with the Π -weighted conservativeness established in Part 1(b)(iii).

Part 2: spectral representation. Already established in (A.25) above, with $L(\cdot)$ the same function as in Proposition 4, since the eigenvalues λ_m of $\mathbf{G}$ and $\tilde{\mathbf{H}}$ coincide (Fact 2) and $L(\cdot)$ depends only on κ, α , and the eigenvalue. $\square$

I Proof of Proposition 8

We begin by establishing the following lemma.

Lemma A.1. *Under Assumption 1,*

$$\Sigma_\theta = \sum_m \sigma_{\theta,m}^2 \mathbf{u}_m^R (\mathbf{u}_m^R)^\top, \quad \Sigma_a = \sum_m L(\lambda_m)^2 \sigma_{\theta,m}^2 \mathbf{u}_m^R (\mathbf{u}_m^R)^\top,$$

and, writing $\Sigma_a^\Pi \equiv \Pi^{1/2} \Sigma_a \Pi^{1/2}$,

$$\Sigma_a^\Pi = \sum_m L(\lambda_m)^2 \sigma_{\theta,m}^2 \mathbf{v}_m \mathbf{v}_m^\top.$$

Proof. Recall that $\tilde{\mathbf{H}} \equiv (1 - \alpha) \Pi^{-1/2} \mathbf{H} \Pi^{-1/2}$ denotes the symmetrized network, with $\mathbf{G} = \Pi^{-1/2} \tilde{\mathbf{H}} \Pi^{1/2}$. By Assumption 1, $\Sigma_\theta = \sum_m \sigma_{\theta,m}^2 \mathbf{u}_m^R (\mathbf{u}_m^R)^\top$ directly. Writing $\mathbf{v}_m \equiv \Pi^{1/2} \mathbf{u}_m^R$, which are orthonormal since $(\mathbf{u}_m^R)^\top \Pi \mathbf{u}_p^R = \mathbf{1}_{m=p}$, it follows that

$$\Pi^{1/2} \Sigma_\theta \Pi^{1/2} = \sum_m \sigma_{\theta,m}^2 (\Pi^{1/2} \mathbf{u}_m^R) (\Pi^{1/2} \mathbf{u}_m^R)^\top = \sum_m \sigma_{\theta,m}^2 \mathbf{v}_m \mathbf{v}_m^\top.$$

Since $\mathbb{E}[\boldsymbol{\theta}] = \mathbf{0}$ and $\mathbf{a} = \mathbf{M}\boldsymbol{\theta}$ is linear, $\mathbb{E}[\mathbf{a}] = \mathbf{0}$ and $\Sigma_a = \mathbf{M} \Sigma_\theta \mathbf{M}^\top$. Using $\mathbf{M} = \sum_m L(\lambda_m) \mathbf{u}_m^R (\mathbf{u}_m^L)^\top$ and biorthonormality $(\mathbf{u}_m^L)^\top \mathbf{u}_p^R = \mathbf{1}_{m=p}$,

$$\mathbf{M} \Sigma_\theta = \left(\sum_m L(\lambda_m) \mathbf{u}_m^R (\mathbf{u}_m^L)^\top \right) \left(\sum_p \sigma_{\theta,p}^2 \mathbf{u}_p^R (\mathbf{u}_p^R)^\top \right) = \sum_m L(\lambda_m) \sigma_{\theta,m}^2 \mathbf{u}_m^R (\mathbf{u}_m^R)^\top,$$

since the cross terms $(\mathbf{u}_m^L)^\top \mathbf{u}_p^R = \mathbf{1}_{m=p}$ collapse the double sum to $p = m$. Multiplying on the right by $\mathbf{M}^\top = \sum_q L(\lambda_q) \mathbf{u}_q^L (\mathbf{u}_q^R)^\top$ and using $(\mathbf{u}_m^R)^\top \mathbf{u}_q^L = (\mathbf{u}_q^L)^\top \mathbf{u}_m^R = \mathbf{1}_{q=m}$ (the same biorthonormality, with the roles of the two vectors in the inner product exchanged) collapses the sum to $q = m$:

$$\Sigma_a = \sum_m L(\lambda_m)^2 \sigma_{\theta,m}^2 \mathbf{u}_m^R (\mathbf{u}_m^R)^\top.$$

Finally, $\Sigma_a^\Pi = \Pi^{1/2} \Sigma_a \Pi^{1/2} = \sum_m L(\lambda_m)^2 \sigma_{\theta,m}^2 (\Pi^{1/2} \mathbf{u}_m^R) (\Pi^{1/2} \mathbf{u}_m^R)^\top = \sum_m L(\lambda_m)^2 \sigma_{\theta,m}^2 \mathbf{v}_m \mathbf{v}_m^\top$, using $\Pi^{1/2} \mathbf{u}_m^R = \Pi^{1/2} \Pi^{-1/2} \mathbf{v}_m = \mathbf{v}_m$. $\square$

We now prove Proposition 8 in the paper.

Proposition. (Proposition 8 in the paper) Under Assumption 1, given α , κ , and Π , the eigenvalue λ_m and eigenvector $\mathbf{u}_m^R$ are identified whenever $\sigma_{\theta,m}^2 > 0$, via:

$$\lambda_m = L^{-1} \left(\sqrt{\frac{\sigma_{a,m}^2}{\sigma_{\theta,m}^2}} \right),$$

where $\tilde{\Sigma}_\theta \equiv \mathbf{U}_L^\top \Sigma_\theta \mathbf{U}_L$ and $\tilde{\Sigma}_a \equiv \mathbf{U}_L^\top \Sigma_a \mathbf{U}_L$ are the covariance matrices of the adjusted domestic interests and actions, with $\mathbf{U}_L \equiv [\mathbf{u}_1^L, \dots, \mathbf{u}_N^L]$ the matrix of left eigenvectors, so that $\sigma_{\theta,m}^2$ and $\sigma_{a,m}^2$ are the variances of $\tilde{\theta}_m$ and $\tilde{a}_m$.

Proof. Recovering $\mathbf{v}_m$. By Assumption 1, $\Sigma_\theta^\Pi = \Pi^{1/2} \Sigma_\theta \Pi^{1/2}$ is, as shown in the proof of Lemma A.1, diagonalized by $\{\mathbf{v}_m\}$ with diagonal entries $\sigma_{\theta,m}^2$. Given Π (equivalently $\boldsymbol{\pi}$, the vector of structural importance), Σ_θ^Π is directly computable from the observable matrix Σ_θ , and by the spectral theorem its eigenvectors $\{\mathbf{v}_m\}$ —and hence $\mathbf{u}_m^R = \Pi^{-1/2} \mathbf{v}_m$ —are recoverable directly from data, up to the same sign/rotation ambiguity as in the symmetric case (Proposition 6), which does not affect recovery of $\tilde{\mathbf{H}} = \sum_m \lambda_m \mathbf{v}_m \mathbf{v}_m^\top$ or $\mathbf{G} = \Pi^{-1/2} \tilde{\mathbf{H}} \Pi^{1/2}$ since both depend on $\mathbf{v}_m$ only through the

sign-invariant outer product $\mathbf{v}_m \mathbf{v}_m^\top$.

Recovering λ_m . By Lemma A.1, projecting $\Sigma_a^\Pi = \Pi^{1/2} \Sigma_a \Pi^{1/2}$ onto the eigenbasis $\{\mathbf{v}_m\}$ gives diagonal entries

$$\sigma_{a,m}^2 = \mathbf{v}_m^\top \Sigma_a^\Pi \mathbf{v}_m = L(\lambda_m)^2 \sigma_{\theta,m}^2,$$

using orthonormality of $\{\mathbf{v}_m\}$ to isolate the m -th term of the sum in Lemma A.1. If $\sigma_{\theta,m}^2 \neq 0$, divide by $\sigma_{\theta,m}^2 = \sigma_{\theta,m}^2$ and take the nonnegative square root (justified since $L(\lambda) \geq 0$ for every $\lambda \in [-(1-\alpha), 1-\alpha]$ by Proposition 4, so $|L(\lambda_m)| = L(\lambda_m)$):

$$L(\lambda_m) = \sqrt{\frac{\sigma_{a,m}^2}{\sigma_{\theta,m}^2}}.$$

Since $L(\cdot)$ is strictly increasing on this range (Proposition 4), it is invertible on its image, so

$$\lambda_m = L^{-1}\left(\sqrt{\frac{\sigma_{a,m}^2}{\sigma_{\theta,m}^2}}\right),$$

uniquely determined by the observable (given Π) quantities $\sigma_{a,m}^2$ and $\sigma_{\theta,m}^2$. Together with the recovered $\mathbf{v}_m$, this identifies $(\lambda_m, \mathbf{v}_m)$ and hence the term $\lambda_m \mathbf{v}_m \mathbf{v}_m^\top$ of $\tilde{\mathbf{H}} = \sum_m \lambda_m \mathbf{v}_m \mathbf{v}_m^\top$, and thus of $\mathbf{G} = \Pi^{-1/2} \tilde{\mathbf{H}} \Pi^{1/2}$, whenever $\sigma_{\theta,m}^2 \neq 0$.

Failure of identification when $\sigma_{\theta,m}^2 = 0$. If $\sigma_{\theta,m}^2 = 0$, Lemma A.1 gives $\sigma_{a,m}^2 = L(\lambda_m)^2 \cdot 0 = 0$ for every value of λ_m : the observed moment carries no information about λ_m , exactly as in the symmetric case (Proposition 6). Consequently $\tilde{\mathbf{H}}$ (and $\mathbf{G}$) is identified only on the subspace spanned by $\{\mathbf{v}_m : \sigma_{\theta,m}^2 > 0\}$. $\square$

J Additional Theoretical Results

In this section of the Online Appendix, we report additional theoretical results, as discussed in the paper. In Subsection J.1, we introduce a contestability index, which measures the share of the foreign influence directed at each country that is offset by influence in the opposing direction. In Subsection J.2, we use an example geopolitical network of four countries to illustrate our approach. In Subsection J.3, we show that with stochastic international issues each country's *ex ante* welfare can be measured as its expected payoff over the distribution of the draws of domestic interests. In Subsection J.4, we state the condition on the cost of influence under which the equilibrium is well posed when the geopolitical network has negative entries.

J.1 Contestability Index

Equilibrium geopolitical actions ($\mathbf{a}$) in our model are shaped by the strategic influence ($\Omega(\boldsymbol{\theta})$) induced by domestic interests ($\boldsymbol{\theta}$) given the geopolitical network ($\mathbf{G}$). To systematically characterize the role of strategic influence, we introduce a *contestability index*, which measures the share of the foreign influence directed at each country that is offset by influence in the opposing direction. Formally,

we define the contestability index ($\chi_i(\boldsymbol{\theta}) \in [0, 1]$) for each country i as one minus the absolute value of the net foreign influence that i receives, relative to the gross foreign influence it receives:

$$\chi_i(\boldsymbol{\theta}) \equiv 1 - \frac{\left| \sum_{m \neq i} \omega_{im}(\boldsymbol{\theta}) \right|}{\sum_{m \neq i} |\omega_{im}(\boldsymbol{\theta})|},$$

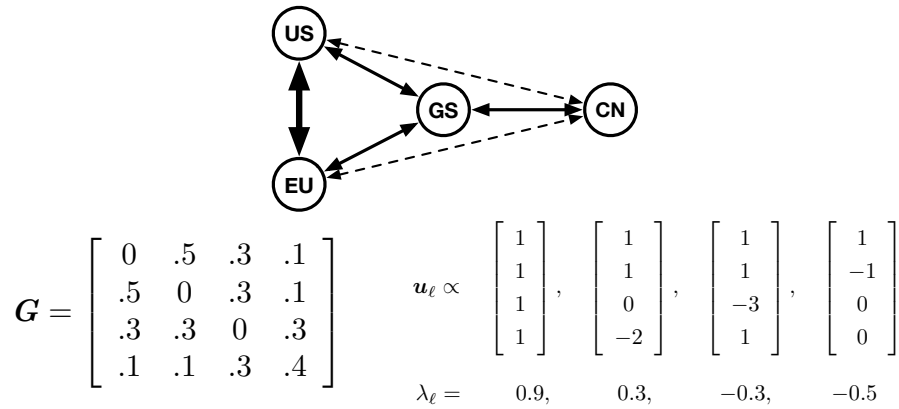
where we leave out self-influence to focus on contestability from foreign influence. We set the index to zero when gross foreign influence is zero. The index is the share of the gross foreign influence on a country that is offset by influence in the opposing direction. It depends only on total influence in each direction, not on how those totals are distributed across foreign countries.

At the lower bound of the contestability index ($\chi_i = 0$), all nonzero foreign influences point in the same direction. Therefore, a low value for the contestability index (χ_i) indicates a strong consensus of foreign influence, which pushes country i 's action (a_i) in that direction. In contrast, at the upper bound of the contestability index ($\chi_i = 1$), the net influence on country i sums exactly to zero despite active foreign influence. Hence, a high value for the contestability index (χ_i) reflects a geopolitical tug-of-war, in which country i is a highly contested swing state or a battleground state for foreign influence. In this case, foreign actors exert competing, contradictory pressures, which partially cancel each other out, and hence have less effect on country i 's action (a_i).

J.2 Example Network

To illustrate these analytical results and provide further intuition, we consider an example network (G) of four countries, as illustrated in the top panel of Figure A1. Countries 1 and 2 (we refer to them as the US and the EU, respectively) are strongly connected; country 3 is equally connected to everyone (the Global South); and country 4 (China) is only weakly connected to countries 1 and 2.

Figure A1: Example of a Geopolitical Network



Note: Countries 1 and 2 (the US and the EU, respectively) are strongly connected; country 3 is equally connected to everyone (the Global South (GS)); and country 4 (China (CN)) is only weakly connected to countries 1 and 2. Top panel shows a network graph with thick solid arrows denoting the strongest connections; medium solid arrows denoting intermediate connections; and medium dashed arrows showing the weakest connections. Lower panel shows the network matrix G , with elements G_{ij} denoting the strength of network connections between countries i and j , and the implied eigenvalues (λ_ℓ) and eigenvectors (u_ℓ).

We report the assumed strength of the network connections (G_{ij}) and the implied eigenvalues (λ_ℓ) and eigenvectors (u_ℓ) in the lower panel of Figure A1. The first eigenvector u_1 is a constant vec-

tor. The second eigenvector $\mathbf{u}_2$ captures the leading geopolitical cleavage: international issues where the US and the EU take one side; China takes the polar opposite; and the Global South occupies the middle. When domestic interests align with this axis, the Global South's contestability index reaches its upper bound ($\chi = 1$). Because the Western bloc and China attempt to pull the Global South in exactly opposite directions, their competing strategic influence cancels out, thereby rendering it a highly contestable swing state for such international issues.

The third eigenvector $\mathbf{u}_3$ captures a different cleavage—one where China aligns with the US and the EU, but all three disagree with the Global South. In this scenario, the Global South's contestability index drops to its lower bound ($\chi = 0$). Rather than facing a tug-of-war, the Global South is subjected to monolithic foreign influence, as all major global powers exert influence to shift its geopolitical action in the exact same direction.

Finally, the eigenvector associated with the smallest eigenvalue captures localized disagreements, such as a sharp policy fracture strictly between the US and the EU. Because the US and the EU exert pressure to pull the rest of the network in diametrically opposite directions, their efforts effectively neutralize each other abroad. As a result, both China and the Global South absorb perfectly offsetting foreign influence, pushing their contestability indices to exactly one ($\chi = 1$), and turning the rest of the globe into a battleground for an intra-Western schism.

J.3 *Ex Ante Welfare*

With stochastic international issues, each country's *ex ante* welfare can be measured as its expected payoff ($\mathbb{E}_\theta [U_i]$) over the distribution of the draws of domestic interests (θ). We now show that the *ex ante* welfare of each country depends on the network structure as well as the set of international issues that arise. We express each country i 's *ex ante* welfare as a function of the eigenvectors and eigenvalues of G , as well as of the matrix Σ_θ capturing the covariance of domestic interests.

Proposition A.1. (*Ex Ante Welfare*) *Under graph stationarity, ex ante welfare for country i can be written as:*

$$\mathbb{E}_\theta [U_i] = \sum_{\ell} \sigma_{\theta, \ell}^2 \times \Gamma_{i\ell}(\{u_k\}, \{\lambda_k\}),$$

where $\Gamma_{i\ell}(\{u_k\}, \{\lambda_k\})$ is country i 's payoff along geopolitical cleavage ℓ , defined as:

$$\begin{aligned} \Gamma_{i\ell} = & (L(\lambda_\ell) - 1) \left[2 \frac{\alpha^2 - (1 - \lambda_\ell)^2}{\kappa(1 - \lambda_\ell)^2 - \alpha} - \alpha(L(\lambda_\ell) - 1) \right] u_{i\ell}^2 \\ & - L(\lambda_\ell)^2 \sum_q C_{\ell\ell q} (1 - \alpha - 2\lambda_\ell + \lambda_q) u_{iq} \\ & - \frac{1}{\kappa} \sum_p \left(\sum_q C_{\ell pq} \left[\frac{L(\lambda_\ell)(1 - \lambda_q)}{1 - \lambda_p} - 1 \right] u_{iq} \right)^2, \end{aligned}$$

where $L(\cdot)$ is as defined in Proposition 4, and $C_{\ell pq} \equiv \sum_{k=1}^N u_{k\ell} u_{kp} u_{kq} = \langle \mathbf{u}_\ell \circ \mathbf{u}_p, \mathbf{u}_q \rangle$ is the spectral interaction tensor that captures how eigenmodes interact through the nonlinearity of the country-level products of the eigenvectors $u_{k\ell} u_{kp} u_{kq}$.

Proof. See Online Appendix K. □

Proposition A.1 shows that the *ex ante* welfare of country i depends on the network structure (as encoded in the eigenvectors $\{u_k\}$ and eigenvalues $\{\lambda_k\}$ of $\mathbf{G}$), the equilibrium interactions between countries through coordination and influence (as captured by the function $\Gamma_{i\ell}(\cdot)$), and the distribution of international issues (as captured by the variance $\sigma_{\theta,\ell}^2$ with which each latent cleavage is drawn).

Ex ante welfare is additive across each of the geopolitical cleavages, because of Assumption 1 and the quadratic form of the payoff. Each of these geopolitical cleavages ℓ captures a latent axis of disagreement among countries. The function $\Gamma_{i\ell}$ measures country i 's equilibrium payoff exposure to cleavage ℓ —how much country i stands to gain or lose when an international issue loads on that cleavage. This exposure depends on i 's position in the network: a country can be highly exposed to one cleavage but insulated from another. For instance, the US may be centrally positioned along the axis of US–Soviet rivalry, but peripheral to a localized conflict between India and Pakistan. How influential a country is for other nations' policy choices, how contested its policy choices are through strategic influence, and how much policy coordination it engages in are all contingent on the specific cleavage.

The variance $\sigma_{\theta,\ell}^2$ determines how prominently geopolitical cleavage ℓ features in the distribution of international issues. A large variance $\sigma_{\theta,\ell}^2$ implies that international issues are more likely to load on cleavage ℓ , increasing its weight in every country's *ex ante* welfare. The additive structure for *ex ante* welfare implies that one can evaluate the welfare consequences of each cleavage independently, and then aggregate across the full spectrum of cleavages.

J.4 Well-Posedness with Negative Network Entries

The proofs of Propositions 1 and 2 use the nonnegativity of $\mathbf{G}$ in two ways: to show that $(\mathbf{I} - \mathbf{G})^{-1}$ has nonnegative entries, which gives the convex-combination property, and to show that $\kappa > 1/\alpha$ makes each country's influence problem concave. In contrast, the closed-form solutions and the spectral decoupling of Proposition 4 require only symmetry and constant row sums, provided that no eigenvalue of $\mathbf{G}$ equals one and $\kappa(1 - \lambda_m)^2 \neq \alpha$ for all m . The remaining conclusions require conditions on the spectrum of $\mathbf{G}$: concavity of each country's coordination problem requires $G_{ii} < 1$ for every i , since its second derivative is $-2(1 - G_{ii})$, which holds automatically when $\mathbf{G}$ is nonnegative. Existence and uniqueness of the equilibrium require $\kappa > \kappa^*(\mathbf{G})$, where $\kappa^*(\mathbf{G})$ is defined below. The positivity and monotonicity of the scaling factor L , on which the identification argument

of Proposition 6 rests, require $\max_m \lambda_m < 1 - \sqrt{\alpha/\kappa}$. The bound $L \leq 1$ requires that the consensus eigenvalue $1 - \alpha$ remain the largest, which our estimation imposes as a normalization (Section 3.2). Since our estimated networks are reconstructed from five eigenvectors, they can contain negative entries. Therefore, we now state the condition under which our equilibrium characterization applies to a general symmetric $\mathbf{G}$ with row sums $1 - \alpha$ that can contain negative entries.

Consider a deviation in which country i changes the influence it exerts on each country by $\mathbf{x} = (x_1, \dots, x_N)$. By the coordination-stage best response, the action profile changes by $\mathbf{y} = (\mathbf{I} - \mathbf{G})^{-1} \mathbf{x}$, and the second-order change in country i 's payoff is

$$\frac{1}{2} d^2 U_i = -\kappa \|\mathbf{x}\|^2 + y_i^2 - \sum_j G_{ij} y_j^2.$$

Country i pays the convex cost of the additional influence, gains from moving its own action, and is penalized through the coordination term for moving its neighbors' actions. With $G_{ij} \geq 0$ the last term is a penalty; on a link with $G_{ij} < 0$ it becomes a reward, which is why negative entries threaten concavity. Since U_i is quadratic in country i 's controls, its influence problem is concave if and only if this form is nonpositive for every $\mathbf{x}$, that is, if and only if

$$\kappa \geq \kappa_i^*(\mathbf{G}) \equiv \lambda_{\max} \left((\mathbf{I} - \mathbf{G})^{-1} [\mathbf{e}_i \mathbf{e}_i^\top - \text{Diag}(\mathbf{G}_{i\cdot})] (\mathbf{I} - \mathbf{G})^{-1} \right),$$

where $\mathbf{e}_i$ is the i -th unit vector and $\text{Diag}(\mathbf{G}_{i\cdot})$ is the diagonal matrix formed from the i -th row of $\mathbf{G}$. Let $\kappa^*(\mathbf{G}) \equiv \max_i \kappa_i^*(\mathbf{G})$.

Proposition. (Well-Posedness) *Let $\mathbf{G}$ be symmetric with row sums $1 - \alpha$, with no eigenvalue equal to one and $\kappa(1 - \lambda_m)^2 \neq \alpha$ for all m , and with $G_{ii} < 1$ for every i so that each country's coordination problem is concave. If $\kappa > \kappa^*(\mathbf{G})$, every country's influence problem is strictly concave and the closed forms in Proposition 1 are the unique equilibrium. If $\kappa < \kappa^*(\mathbf{G})$, some country's payoff is unbounded above at every strategy profile, and no pure-strategy equilibrium exists. If $\mathbf{G}$ is nonnegative, then $\kappa^*(\mathbf{G}) \leq 1/\alpha$, so $\kappa > 1/\alpha$ suffices.*

Proof. Sufficiency and necessity follow from the quadratic form, whose Hessian does not depend on $\boldsymbol{\theta}$ or on other countries' strategies. For the last claim, fix $\mathbf{x}$, let $\mathbf{y} = (\mathbf{I} - \mathbf{G})^{-1} \mathbf{x}$ and $t \equiv (\mathbf{G}\mathbf{y})_i$, so that $x_i = y_i - t$. Because the weights G_{ij} are nonnegative and sum to $1 - \alpha$, the Cauchy–Schwarz inequality gives $\sum_j G_{ij} y_j^2 \geq t^2 / (1 - \alpha)$. Hence

$$y_i^2 - \sum_j G_{ij} y_j^2 \leq y_i^2 - \frac{t^2}{1 - \alpha} \leq \frac{(y_i - t)^2}{\alpha} = \frac{x_i^2}{\alpha} \leq \frac{\|\mathbf{x}\|^2}{\alpha},$$

where the middle inequality is $(\sqrt{1 - \alpha} y_i - t / \sqrt{1 - \alpha})^2 \geq 0$. Thus $\kappa_i^* \leq 1/\alpha$ for every i . The bound is sharp: it is approached by networks in which a country is nearly disconnected from all others. $\square$

We now make remarks about this proposition. First, when $\mathbf{G}$ has negative entries but the consensus mode remains dominant, the convex-combination property in Proposition 2 can fail, since $(\mathbf{I} - \mathbf{G})^{-1}$ can have negative entries; actions are then affine rather than convex combinations of domestic interests. Second, the network disagreement measure $\mathcal{S}(\boldsymbol{\theta})$ can be negative on pairs with $G_{ij} < 0$; the identity $\mathcal{S}(\mathbf{u}_k) = 1 - \alpha - \lambda_k$ in Proposition 3 still holds. Third, in the directed extension the same argument applies with the cost $\kappa(\pi_k/\pi_i)\omega_{ki}^2$, provided every π_i is strictly positive, giving the condition $\kappa > \kappa_i^*(\mathbf{G}) \equiv \pi_i \lambda_{\max} \left(\Pi^{-1/2} (\mathbf{I} - \mathbf{G})^{-\top} [\mathbf{e}_i \mathbf{e}_i^\top - \text{Diag}(\mathbf{G}_i)] (\mathbf{I} - \mathbf{G})^{-1} \Pi^{-1/2} \right)$ for every i , where $(\mathbf{I} - \mathbf{G})^{-\top}$ denotes the inverse of the transpose, which differs from $(\mathbf{I} - \mathbf{G})^{-1}$ when $\mathbf{G}$ is asymmetric. The inequality $y_i^2 - \sum_j G_{ij} y_j^2 \leq x_i^2/\alpha$ in the proof above uses only $G_{ij} \geq 0$ and the row sums, and $x_i^2 \leq (1/\pi_i) \sum_k \pi_k x_k^2$, so the bound $\kappa^*(\mathbf{G}) \leq 1/\alpha$ continues to hold whenever $\mathbf{H}$ is nonnegative; if some π_i were negative, that country's cost would be a subsidy and no value of κ would restore concavity.

K Proof of Proposition A.1

Proposition. (*Proposition A.1 in Section J.3*) Under graph stationarity, ex ante welfare for country i can be written as:

$$\mathbb{E}_\theta [U_i] = \sum_\ell \sigma_{\theta, \ell}^2 \times \Gamma_{i\ell}(\{u_k\}, \{\lambda_k\}),$$

where $\Gamma_{i\ell}(\{u_k\}, \{\lambda_k\})$ is country i 's payoff along geopolitical cleavage ℓ , defined as:

$$\begin{aligned} \Gamma_{i\ell} = & (L(\lambda_\ell) - 1) \left[2 \frac{\alpha^2 - (1 - \lambda_\ell)^2}{\kappa(1 - \lambda_\ell)^2 - \alpha} - \alpha(L(\lambda_\ell) - 1) \right] u_{i\ell}^2 \\ & - L(\lambda_\ell)^2 \sum_q C_{\ell\ell q} (1 - \alpha - 2\lambda_\ell + \lambda_q) u_{iq} \\ & - \frac{1}{\kappa} \sum_p \left(\sum_q C_{\ell pq} \left[\frac{L(\lambda_\ell)(1 - \lambda_q)}{1 - \lambda_p} - 1 \right] u_{iq} \right)^2, \end{aligned}$$

where $L(\cdot)$ is as defined in Proposition 4, and $C_{\ell pq} \equiv \sum_{k=1}^N u_{k\ell} u_{kp} u_{kq} = \langle \mathbf{u}_\ell \circ \mathbf{u}_p, \mathbf{u}_q \rangle$ is the spectral interaction tensor that captures how eigenmodes interact through the nonlinearity of the country-level products of the eigenvectors $u_{k\ell} u_{kp} u_{kq}$.

Proof. We derive the expected payoff $\mathbb{E}[U_i]$ when $\boldsymbol{\theta}$ is drawn from a distribution whose covariance Σ_θ is diagonal in $\mathbf{G}$'s eigenbasis. The payoff of country i is

$$U_i = - \underbrace{\left[\alpha (a_i - \theta_i)^2 + \sum_j G_{ij} (a_i - a_j)^2 \right]}_{\text{First term (beauty contest)}} + 2 \underbrace{\left(\sum_j \omega_{ij} \right) (a_i - \theta_i)}_{\text{Second term (influence received)}} - \underbrace{\kappa \sum_k \omega_{ki}^2}_{\text{Third term (influence exerted)}}. \quad (\text{A.26})$$

Preliminaries. Let $\mathbf{G} = \sum_m \lambda_m \mathbf{u}_m \mathbf{u}_m^\top$ with $\{\mathbf{u}_m\}$ orthonormal. Define $\tilde{\theta}_m = \mathbf{u}_m^\top \boldsymbol{\theta}$ and $\tilde{\Sigma}_\theta =$

Diag $(\sigma_{\theta,1}^2, \dots, \sigma_{\theta,N}^2)$, so that $\mathbb{E} [\tilde{\theta}_m \tilde{\theta}_p] = \sigma_{\theta,m}^2 \mathbf{1}_{m=p}$. The equilibrium action in mode m is $\tilde{a}_m = L_m \tilde{\theta}_m$ where

$$L(\lambda) \equiv \frac{(\kappa\alpha - 1)(1 - \lambda)}{\kappa(1 - \lambda)^2 - \alpha}. \quad (\text{A.27})$$

The equilibrium net influence received, $\sum_j \omega_{ij}$, has mode- m projection $\Phi_m \tilde{\theta}_m$ where

$$\Phi(\lambda) \equiv \frac{\alpha^2 - (1 - \lambda)^2}{\kappa(1 - \lambda)^2 - \alpha} = (1 - \lambda) L(\lambda) - \alpha, \quad (\text{A.28})$$

the second equality following from $\sum_j \omega_{ij} = [(\mathbf{I} - \mathbf{G}) \mathbf{a} - \alpha \boldsymbol{\theta}]_i$. In country coordinates:

$$a_i - \theta_i = \sum_m (L_m - 1) \tilde{\theta}_m u_{im}, \quad (\text{A.29})$$

$$\sum_j \omega_{ij} = \sum_m \Phi_m \tilde{\theta}_m u_{im}. \quad (\text{A.30})$$

Overlap tensor. Define

$$C_{mpq} \equiv \sum_{k=1}^N u_{km} u_{kp} u_{kq}. \quad (\text{A.31})$$

The expansion identity $u_{im} u_{ip} = \sum_q C_{mpq} u_{iq}$ and the $(\mathbf{I} - \mathbf{G})$ -identity

$$u_{im} u_{ip} - \sum_j G_{ij} u_{jm} u_{jp} = \sum_q C_{mpq} (1 - \lambda_q) u_{iq} \quad (\text{A.32})$$

will be used throughout.

First term (beauty contest). From (A.29):

$$\mathbb{E} [\alpha (a_i - \theta_i)^2] = \alpha \sum_m \sigma_{\theta,m}^2 (L_m - 1)^2 u_{im}^2.$$

For the coordination loss, $a_i - a_j = \sum_m L_m \tilde{\theta}_m (u_{im} - u_{jm})$, so

$$\mathbb{E} \left[\sum_j G_{ij} (a_i - a_j)^2 \right] = \sum_m \sigma_{\theta,m}^2 L_m^2 \sum_j G_{ij} (u_{im} - u_{jm})^2.$$

Expanding $\sum_j G_{ij} (u_{im} - u_{jm})^2$ and applying $\sum_j G_{ij} u_{jm}^2 = \sum_q C_{mmq} \lambda_q u_{iq}$:

$$\sum_j G_{ij} (u_{im} - u_{jm})^2 = (1 - \alpha - 2\lambda_m) u_{im}^2 + \sum_q C_{mmq} \lambda_q u_{iq}. \quad (\text{A.33})$$

Second term (influence received). From (A.29) and (A.30), using $\mathbb{E} [\tilde{\theta}_m \tilde{\theta}_p] = \sigma_{\theta,m}^2 \mathbf{1}_{m=p}$:

$$\mathbb{E} \left[2 \left(\sum_j \omega_{ij} \right) (a_i - \theta_i) \right] = 2 \sum_m \sigma_{\theta,m}^2 \Phi_m (L_m - 1) u_{im}^2. \quad (\text{A.34})$$

Third term (influence exerted). The bilateral first-order condition for the optimal influence ω_{ki}

exerted by i on k is:

$$\kappa \omega_{ki} = \sum_j G_{ij} (a_i - a_j) [(\mathbf{I} - \mathbf{G})^{-1}]_{jk} + \mathbf{1}_{k=i} (a_i - \theta_i).$$

Define $B_{ip} \equiv \kappa \boldsymbol{\omega}_i^\top \mathbf{u}_p$. By Parseval, $\kappa^2 \sum_k \omega_{ki}^2 = \sum_p B_{ip}^2$. Projecting the first-order condition onto $\mathbf{u}_p$, substituting the spectral expansions, and applying (A.32):

$$B_{ip} = \sum_m \tilde{\theta}_m \sum_q C_{mpq} \left[\frac{L_m (1 - \lambda_q)}{1 - \lambda_p} - 1 \right] u_{iq}. \quad (\text{A.35})$$

Taking expectations:

$$\mathbb{E} \left[\kappa \sum_k \omega_{ki}^2 \right] = \frac{1}{\kappa} \sum_m \sigma_{\theta,m}^2 \sum_p \left(\sum_q C_{mpq} \left[\frac{L_m (1 - \lambda_q)}{1 - \lambda_p} - 1 \right] u_{iq} \right)^2. \quad (\text{A.36})$$

Combining. $\mathbb{E}[U_i] = \sum_m \sigma_{\theta,m}^2 \Gamma_{im}$ where

$$\begin{aligned} \Gamma_{im} = & \underbrace{-\alpha (L_m - 1)^2 u_{im}^2 - L_m^2 \left[(1 - \alpha - 2\lambda_m) u_{im}^2 + \sum_q C_{mmq} \lambda_q u_{iq} \right]}_{\text{First term (beauty contest)}} \\ & + \underbrace{2 \Phi_m (L_m - 1) u_{im}^2}_{\text{Second term (influence received)}} \\ & - \underbrace{\frac{1}{\kappa} \sum_p \left(\sum_q C_{mpq} \left[\frac{L_m (1 - \lambda_q)}{1 - \lambda_p} - 1 \right] u_{iq} \right)^2}_{\text{Third term (influence exerted)}}. \end{aligned} \quad (\text{A.37})$$

Interpretation. The first term captures the beauty-contest payoff: both the cost of deviating from national interest ($\alpha (L_m - 1)^2$) and the coordination loss with neighbors. The coordination loss couples eigenmodes through the overlap tensor C_{mmq} , reflecting how country i 's squared loading on cleavage m relates to its neighbors' loadings on other cleavages. The second term captures the benefit of influence received: since $\Phi_m \leq 0$ and $L_m \leq 1$, this term is nonnegative, so influence receipt partially offsets the beauty-contest distortion. The third term is the cost of influence exerted, which couples all eigenmodes through the products C_{mpq} —even though equilibrium actions decouple mode by mode, the quadratic cost of bilateral influence introduces cross-mode interactions. $\square$

L Additional Empirical Results

In this section of the Online Appendix, we report additional empirical results, as discussed in the paper. In Subsection L.1, we provide further details on our selection of variables from the Varieties of Democracy (V-Dem) data. In Subsection L.2, we report descriptive statistics on country coverage in the Varieties of Democracy (V-Dem), United Nations General Assembly (UNGA), and proxies for

geopolitical linkages datasets. In Subsection L.3, we provide evidence on the fit of the joint principal components of the cross-country covariance matrices for geopolitical actions and domestic interests, further validation of our inferred geopolitical network, and robustness tests for alternative values of our three calibrated parameters. In Subsection L.4, we assess the relative importance of economic versus security dependence for bilateral network connections over time. In Subsection L.5, we provide evidence on the properties of the geopolitical networks of China and the US. In Subsection L.6, we document the heterogeneity in network connections across all countries. In Subsection L.7, we examine the stability of the geopolitical network over time. Finally, in Subsection L.8, we report additional results for our counterfactuals in Section 5 of the paper.

L.1 Varieties of Democracy (V-Dem) Data

We measure countries' domestic interests (θ) using the Varieties of Democracy (V-Dem) dataset, which aggregates expert evaluations to score countries across hundreds of disaggregated domestic political and social dimensions, such as freedom of the press, gender equality, civil liberties, and judicial independence. The V-Dem dataset contains over one thousand variables that record features of different dimensions of political institutions. These variables are aggregated into 48 separate index variables, each of which summarizes a country's political institutions relating to a given topic, such as freedom of expression or electoral democracy. We retain the 37 indices whose cross-country differences are related to differences in UNGA voting positions, screened by a penalized regression of ideal-point differences on index differences, as listed in Table A1 below. The 11 omitted indices are composites or near-duplicates of retained ones, such as the civil liberties index, whose three component indices are retained, and the accountability index, whose vertical, diagonal, and horizontal components are retained.

L.2 Sample Descriptive Statistics

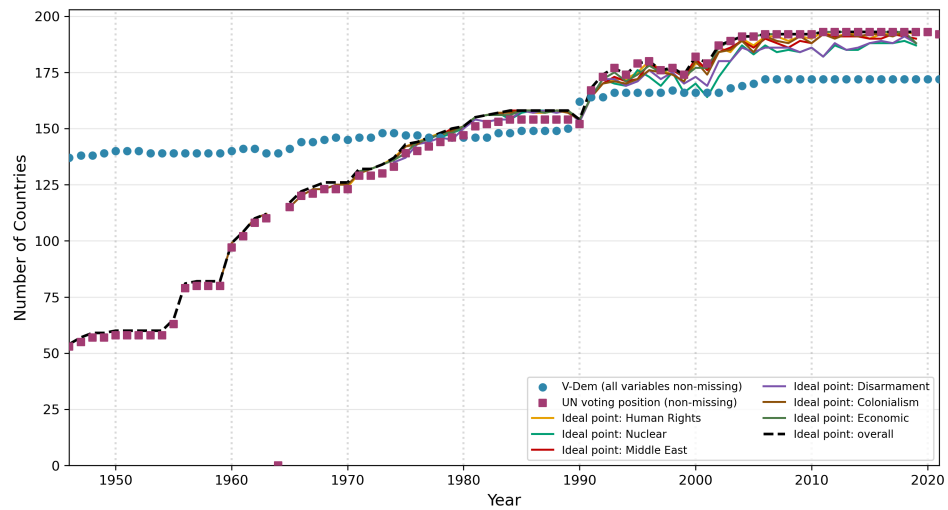
In this section of the Online Appendix, we report descriptive statistics on country coverage in the datasets used in our empirical analysis. Figure A2 shows country coverage in the Varieties of Democracy (V-Dem) and United Nations General Assembly (UNGA) datasets, together with the coverage of the overall ideal-point estimates and of each of the six issue-specific ideal-point estimates. Figure A3 shows country coverage in the data on bilateral proxies for geopolitical linkages, including the Formal Bilateral Influence Capacity (FBIC) dataset, the CEPII Gravity dataset (bilateral trade, distance and indicators for cultural and historical linkages from Conte et al. 2022), and the Polity Project (the difference in polity scores between countries).

Table A1: Varieties of Democracy Variables Used for Domestic Interests

Variable	Description
v2x_EDcomp_thick	Electoral component index
v2x_api	Additive polyarchy index
v2x_clphy	Physical violence index
v2x_clpol	Political civil liberties index
v2x_clpriv	Private civil liberties index
v2x_corr	Political corruption index
v2x_cspart	Civil society participation index
v2x_delibdem	Deliberative democracy index
v2x_diagacc	Diagonal accountability index
v2x_divparctrl	Divided party control index
v2x_egaldem	Egalitarian democracy index
v2x_elecoff	Elected officials index
v2x_elecreg	Electoral regime index
v2x_ex_confidence	Confidence dimension index
v2x_ex_direlect	Direct election dimension index
v2x_ex_hereditary	Hereditary dimension index
v2x_ex_military	Military dimension index
v2x_ex_party	Ruling party dimension index
v2x_feduni	Division of power index
v2x_freexp_altinf	Freedom of Expression and Alternative Sources of Information index
v2x_gencl	Women civil liberties index
v2x_gender	Women political empowerment index
v2x_genpp	Women political participation index
v2x_horacc	Horizontal accountability index
v2x_hosabort	Presidential election aborted
v2x_hosinter	Chief executive no longer elected
v2x_jucon	Judicial constraints on the executive index
v2x_libdem	Liberal democracy index
v2x_liberal	Liberal component index
v2x_partip	Participatory component index
v2x_partipdem	Participatory democracy index
v2x_polyarchy	Electoral democracy index
v2x_pubcorr	Public sector corruption index
v2x_regime	Regimes of the world – the RoW measure
v2x_regime_amb	Regimes of the world – the RoW measure with categories for ambiguous cases
v2x_suffr	Share of population with suffrage
v2x_veracc	Vertical accountability index

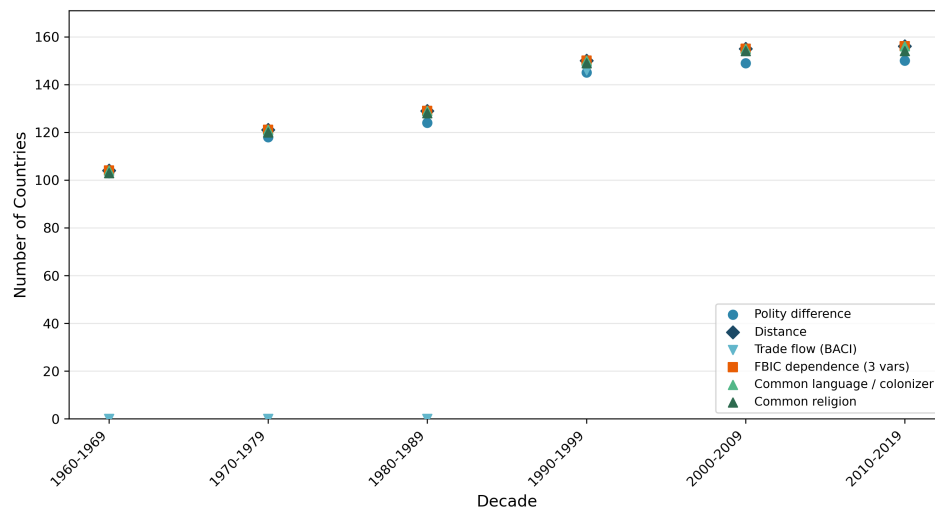
Note: 37 variables from the Varieties of Democracy (V-Dem) dataset retained after screening for relevance to differences in the UNGA actions as estimated by Bailey et al. (2017).

Figure A2: Country Coverage in the Varieties of Democracy (V-Dem) and UNGA Datasets and in the Ideal-Point Estimates



Note: Number of countries with non-missing data in the Varieties of Democracy (V-Dem) and UNGA datasets, together with the number of countries with a non-missing estimate of the overall ideal point and of each of the six issue-specific ideal points (human rights, nuclear, Middle East, disarmament, colonialism, and economic issues).

Figure A3: Country Coverage for Bilateral Proxies for Geopolitical Links



Note: Number of countries with non-missing data for the bilateral proxies for geopolitical links.

L.3 Joint Principal Components, Validation, and Robustness Tests

In subsection L.3.1, we provide evidence on the fit of the joint principal components of the cross-country covariance matrices of geopolitical actions and domestic interests, as discussed in Section 3.2 of the paper. Subsection L.3.3 provides further validation of our inferred geopolitical network using a permutation placebo and a residual bootstrap, as discussed in Section 3.4 of the paper. Subsection L.3.4 demonstrates the robustness of our results across alternative values of our three calibrated parameters.

L.3.1 Fit and Alignment of the Joint Principal Components

Table A2 reports, by decade and for each dataset separately, the explained variance of the joint principal components. The four cleavages account for 79 to 84 percent of the variance in domestic interests and 94 to 98 percent of that in geopolitical actions, against 85 to 88 and 96 to 99 percent for each dataset’s own components.

Table A3 reports the alignment of the leading cleavages. In every decade the leading cleavage of UN voting is aligned with the leading cleavage of domestic interests, with a cosine similarity of 0.73 to 0.85, against 0.07 for unrelated directions and 0.92 to 1.00 for the same dataset across the two halves of a decade, and with no other institutional component: the next-largest cosine similarity is 0.27 to 0.38 and belongs to a component carrying 4 to 9 percent of institutional variance. Table A4 reports the corresponding statistics for cleavages 2 to 5. Cleavages estimated on the first half of a decade account for 73 to 81 percent of the institutional variance in the second half, against 81 to 87 percent for the first half’s own leading institutional components.

Table A2: Fit of the Joint Principal Components by Decade

Decade	Consensus retained (five components)		Consensus removed (four cleavages)			
	Explained variance		Explained variance		Own four components	
	Interests	Actions	Interests	Actions	Interests	Actions
1960s	0.95	0.98	0.81	0.98	0.85	0.99
1970s	0.94	0.96	0.79	0.94	0.85	0.96
1980s	0.96	0.98	0.84	0.98	0.87	0.99
1990s	0.96	0.97	0.79	0.96	0.86	0.97
2000s	0.97	0.97	0.82	0.97	0.88	0.98
2010s	0.96	0.97	0.81	0.97	0.88	0.98

Note: Explained variance is the share of the trace of each $\Pi_d^{1/2}$ -weighted matrix along the retained columns of U_d . Columns 1–2 apply the five retained components to the covariance matrices $\Sigma_{a,d}$ and $\Sigma_{\theta,d}$, which retain the consensus component; columns 3–6 apply the four cleavages to the same matrices with the consensus component removed. Columns 5–6 report the explained variance achieved by each dataset’s own four leading principal components.

Table A3: Alignment of the Leading Cleavages of UN Voting and Domestic Interests

Decade	Cosine	Next-largest cosine with any other component	Unrelated directions	Split-half ceiling Votes	Interests
1960s	0.75	0.34	0.07	0.92	0.98
1970s	0.85	0.27	0.07	0.92	0.96
1980s	0.84	0.30	0.07	0.99	0.97
1990s	0.77	0.30	0.07	0.99	0.98
2000s	0.73	0.38	0.06	0.99	0.99
2010s	0.74	0.32	0.07	1.00	0.98

Note: Cosine between the leading principal component of the $\Pi_d^{1/2}$ -weighted covariance matrix of geopolitical actions and that of domestic interests, consensus removed (column 1); the largest cosine between the leading component of actions and any other component of domestic interests (column 2); the mean cosine between the leading component of actions and a random direction (column 3); and the cosine between the leading components of the same dataset estimated on the first and second halves of the decade (columns 4–5).

Table A4: Cleavages 2–5: Variance Shares and Split-Half Replication

Decade	Variance share of cleavages 2–5		Split-half replication
	Interests	Actions	
1960s	0.81	0.98	0.73 (0.83)
1970s	0.79	0.95	0.76 (0.81)
1980s	0.84	0.98	0.80 (0.84)
1990s	0.79	0.96	0.78 (0.85)
2000s	0.82	0.97	0.81 (0.87)
2010s	0.81	0.97	0.80 (0.85)

Note: Columns 1–2 report the share of the cross-country variance in each dataset (consensus removed) along cleavages 2–5. The last column reports the share of the second half-decade’s variance in domestic interests accounted for by cleavages estimated on the first half-decade, with the share achieved by the first half-decade’s own four leading components in parentheses.

L.3.2 Estimation of GDP Exponent β

We estimate the GDP exponent β_d separately for each decade by matching the relationship between GDP and the ratio of influence exerted to influence received in the FBIC data. Table A5 reports the decade-specific regression coefficients and Table A6 the estimated exponents. We use the median of these six exponents as the common GDP exponent β for all decades.

Table A5: Empirical FBIC and GDP Regressions by Decade

	Outcome: $\ln \left(\frac{\text{influence exerted}}{\text{influence received}} \right)_{id}$					
	1960–1969	1970–1979	1980–1989	1990–1999	2000–2009	2010–2019
$\hat{b}_d$	0.762*** (0.036)	0.755*** (0.030)	0.777*** (0.035)	0.759*** (0.027)	0.725*** (0.026)	0.708*** (0.023)
Observations	104	121	129	150	155	156

Note: Each column reports a separate cross-country OLS regression $\ln \left(\frac{\text{influence exerted}}{\text{influence received}} \right)_{id} = a_d + b_d \ln \text{GDP}_{id} + \epsilon_{id}$. These regressions use the decade-average values of influence exerted, influence received, and GDP. Each regression has one observation per country. These regressions use heteroskedasticity-robust standard errors, with significance levels denoted * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A6: Estimated GDP Exponents by Decade

	1960–1969	1970–1979	1980–1989	1990–1999	2000–2009	2010–2019
β_d	0.814	0.779	0.805	0.719	0.687	0.706

Note: Each decade's exponent solves $b_d^{\text{model}}(\beta_d) = \hat{b}_d$, using that decade's empirical FBIC slope as the target. The network and model-implied influence are recalculated at each candidate exponent.

L.3.3 Permutation Placebo and Residual Bootstrap

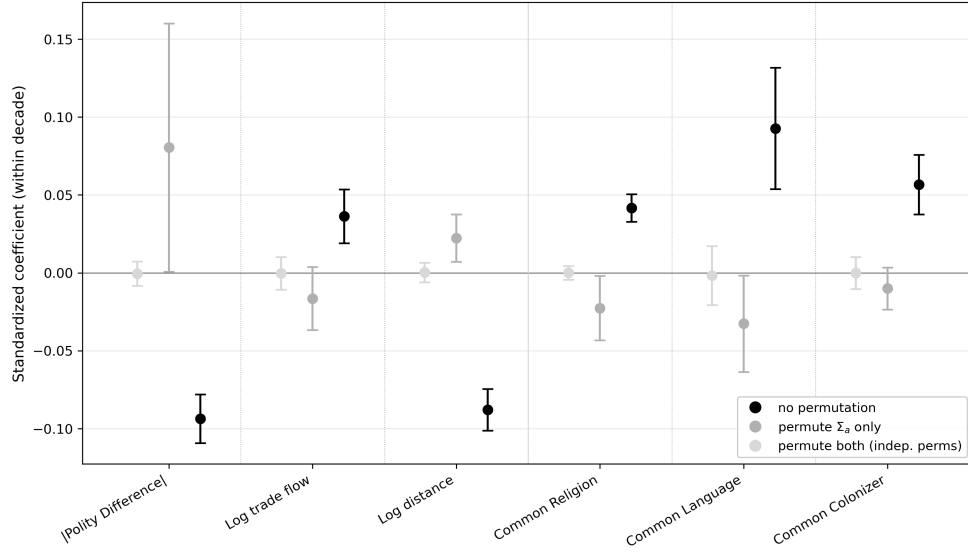
In order to further demonstrate that the recovered networks $\mathbf{G}_d$ reflect systematic underlying geopolitical ties, we undertake two robustness checks, as reported in Figure A4. The first exercise (panel a) is a permutation placebo: we randomly relabel the countries in the action covariance matrix (or both the action and interests covariance matrices) before recovering $\mathbf{G}_d$. This severs the true correspondence between a country's actions and its interests, so that any structure that survives is mechanical rather than substantive. This exercise shows that the regression coefficients shrink toward zero and most become statistically insignificant.

The second check (panel b) is a residual bootstrap exercise that examines the sensitivity of the validation regressions to the variation discarded by the rank-five reconstruction. Because our estimation procedure estimates a common basis for the two covariance matrices, imperfect fit in the joint basis means that the original covariance matrices have some portion that is explained by the top-five joint principal components, and some part that is a residual. Holding the retained components fixed, we repeatedly replace the residual with a random matrix that keeps its eigenvalues but replaces its eigenvectors with a random orthonormal set, so that the residual has the same overall magnitude but its weight is spread across arbitrary country pairs. We then re-estimate $\mathbf{G}_d$ and re-run our validation regressions.

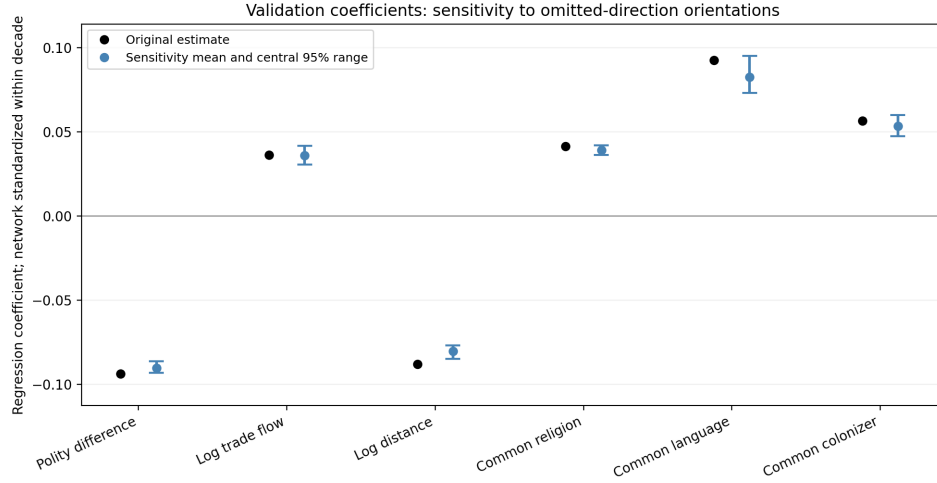
These two robustness exercises behave as they should if G_d captures real geopolitical ties: the validation coefficients are not statistically distinguishable from zero when the inputs are scrambled in the first exercise, but the coefficients remain stable in sign and magnitude when we bootstrap the residuals in the second exercise. Together these two exercises indicate that the estimated G_d networks do not simply reflect some mechanical consequences of our estimation procedure, but instead recover geopolitical structure whose relation to external measures like distance or trade behaves as expected.

A third check asks whether the results depend on how the common basis is estimated. We re-estimate the network with an exact joint diagonalization of the two matrices in place of the eigenvectors of their average, and separately with the two demeaned covariance matrices normalized to equal weight. The leading cleavage is unchanged (cosine at least 0.995 with the baseline under the exact joint diagonalization and at least 0.973 under equal weights, in every decade). The US remains the most autonomous country and China the country with the most control, the signs and significance of the coefficients in Table 1 are unchanged.

Figure A4: Robustness of G_d



(a) Permutation of matrices



(b) Bootstrap of residuals

Note: Panel (a) is a permutation placebo. Recall that each decade's network G_d is recovered from the joint principal components of two cross-country covariance matrices—of preferences ($\Sigma_{\theta,d}$) and of actions ($\Sigma_{a,d}$)—weighted by economic size Π_d . To test whether the validation results require the true country-to-country correspondence among these inputs, we apply a random permutation to the countries of one covariance matrix before recovery, leaving the other matrix and Π_d at their true labels. The coefficients from two variants are shown—permuting $\Sigma_{a,d}$ only and permuting both $\Sigma_{a,d}$ and $\Sigma_{\theta,d}$ independently—alongside the unpermuted network. For each draw we re-recover G_d , and re-run the specification of Table 1. Black points and lines mark the unpermuted (true) coefficient/CI; the gray markers plot the mean of the validation coefficient across permutations, and its bar spans ± 1.96 times the standard deviation of those point estimates across permutations. The true coefficients are large and precisely estimated for every covariate, whereas the permuted coefficients are generally statistically indistinguishable from zero. Panel (b) is a residual bootstrap. The network is recovered from only the retained joint principal components of the preference and action covariance matrices; the remaining variation is unused. To test whether the validation results depend on this unused component, we split each Π_d -weighted covariance into its retained low-rank signal plus a residual, and form bootstrap draws that hold the signal fixed while replacing the residual with a random matrix of identical eigenvalue spectrum but randomly rotated eigenvectors—that is, noise of the same overall magnitude but arbitrary country loadings. For each of 100 draws we add the resampled residual back, re-recover G_d , and re-run the validation regressions as in Table 1. The bootstrap coefficients remain close to the true value, with the same sign and comparable magnitude, indicating that the findings are driven by the retained low-rank signal rather than the discarded residual. Covariance-preserving rotations conditional on the country sample and beta. Ranges are not sampling confidence intervals.

L.3.4 Robustness to Parameters α , κ , and ϵ

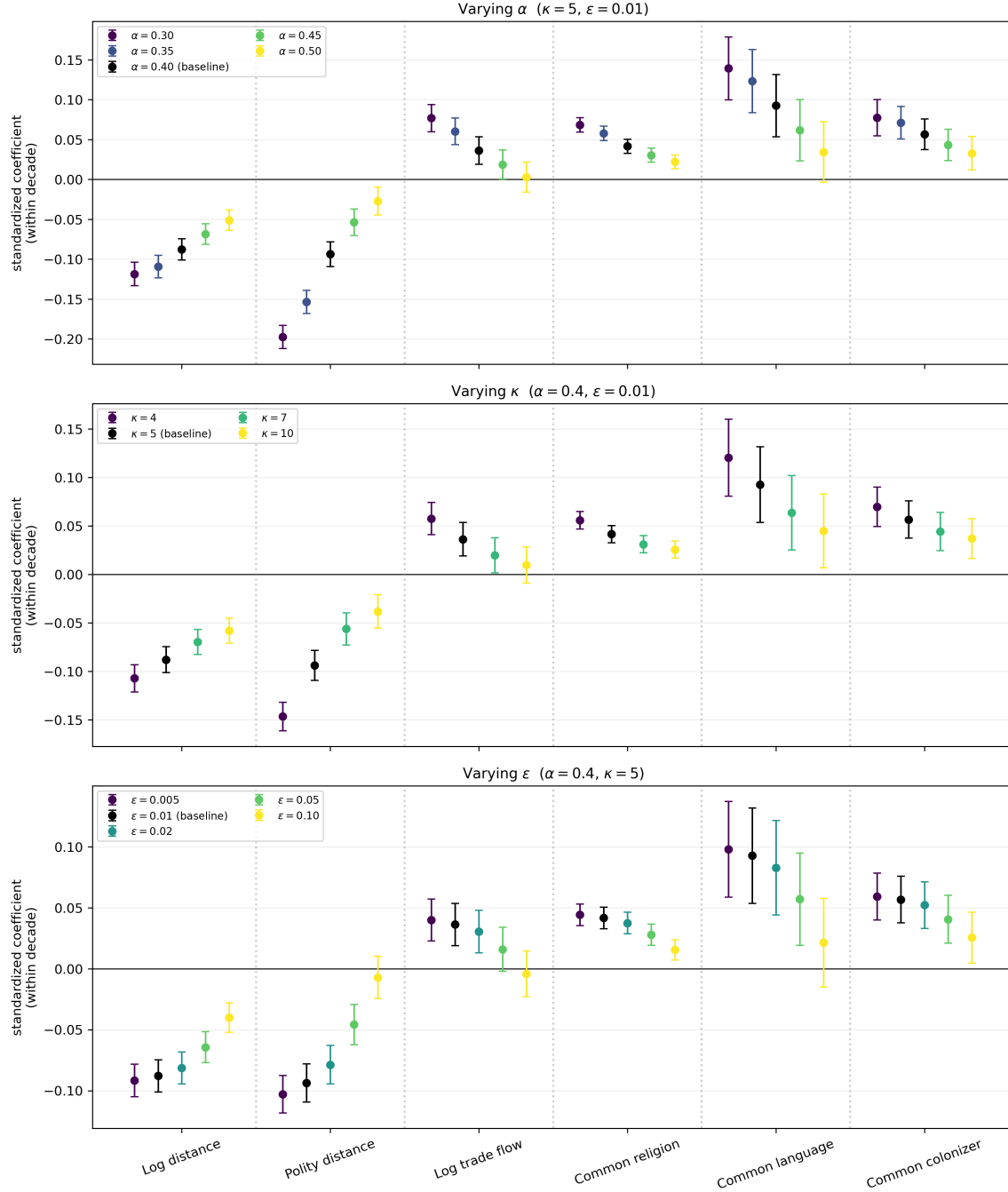
In this section, we examine the sensitivity of our results to our three calibrated parameters: the weight on domestic interests α , the cost of influence κ , and the eigenvalue gap ϵ . We re-estimate the geopolitical network under alternative values of each parameter, varying one parameter at a time with the other two at their baseline values, and holding the structural importance exponent fixed at its baseline estimate ($\beta = 0.75$). Figure A5 repeats the validation regressions of Table 1 with each re-estimated network as the dependent variable. Table A7 reports the Spearman rank correlation between the country ordering of each of the three notions of power under each set of parameter values and the ordering under the baseline.

Table A7: Rank Stability of the Three Notions of Power Across Parameter Values

Variant	Autonomy	Control	Removal	Minimum
$\alpha = 0.30$	0.998	0.999	0.992	0.992
$\alpha = 0.35$	0.999	1.000	0.998	0.998
$\alpha = 0.45$	0.999	1.000	0.997	0.997
$\alpha = 0.50$	0.961	0.999	0.989	0.961
$\kappa = 4$	0.999	1.000	0.998	0.998
$\kappa = 7$	0.999	1.000	0.997	0.997
$\kappa = 10$	0.993	0.999	0.992	0.992
$\epsilon = 0.005$	1.000	1.000	1.000	1.000
$\epsilon = 0.02$	1.000	1.000	0.999	0.999
$\epsilon = 0.05$	0.999	0.998	0.994	0.994
$\epsilon = 0.10$	0.994	0.995	0.981	0.981

Note: Spearman rank correlation between the full ordering under the indicated parameter values and the ordering under the baseline ($\alpha = 0.4$, $\kappa = 5$, $\epsilon = 0.01$), for each of the three notions of power: autonomy, $|\mathcal{L}(\mathbf{G})_{ii}|$; control, $\frac{1}{N-1} \sum_{j \neq i} |\mathcal{L}(\mathbf{G})_{ji}| / |\mathcal{L}(\mathbf{G})_{ii}|$; and the change in world actions from removing a country, computed on the 2010–2019 network with identical issue draws under every set of parameter values. The final column is the minimum across the three notions. Only parameter values satisfying the model’s regularity conditions are shown: (i) $\kappa\alpha > 1$; (ii) $\kappa > \kappa^*(\mathbf{G})$, the well-posedness condition of the influence game evaluated on the re-estimated network; and (iii) a recovered eigenvalue spectrum strictly below the operator singularity $1 - \sqrt{\alpha/\kappa}$.

Figure A5: Validation-Regression Coefficients Across Parameter Values



Note: Each marker is the coefficient b from the validation regression $G_{ijd}^{\text{std}} = b \times x_{ijd} + \alpha_{id} + \gamma_{jd} + e_{ijd}$, where G_{ijd}^{std} is the ij -th entry of the network re-estimated under the indicated parameter values and standardized within decade, x_{ijd} is one dyadic covariate at a time (standardized within decade for the four continuous covariates, binary for the two indicators), and α_{id} and γ_{jd} are receiver-decade and sender-decade fixed effects. Bars are ± 1.96 standard errors, clustered by unordered country pair. The baseline ($\alpha = 0.4, \kappa = 5, \epsilon = 0.01$) is shown in black in every panel. Only parameter values satisfying the model's regularity conditions are shown.

L.4 Economic Versus Security Ties

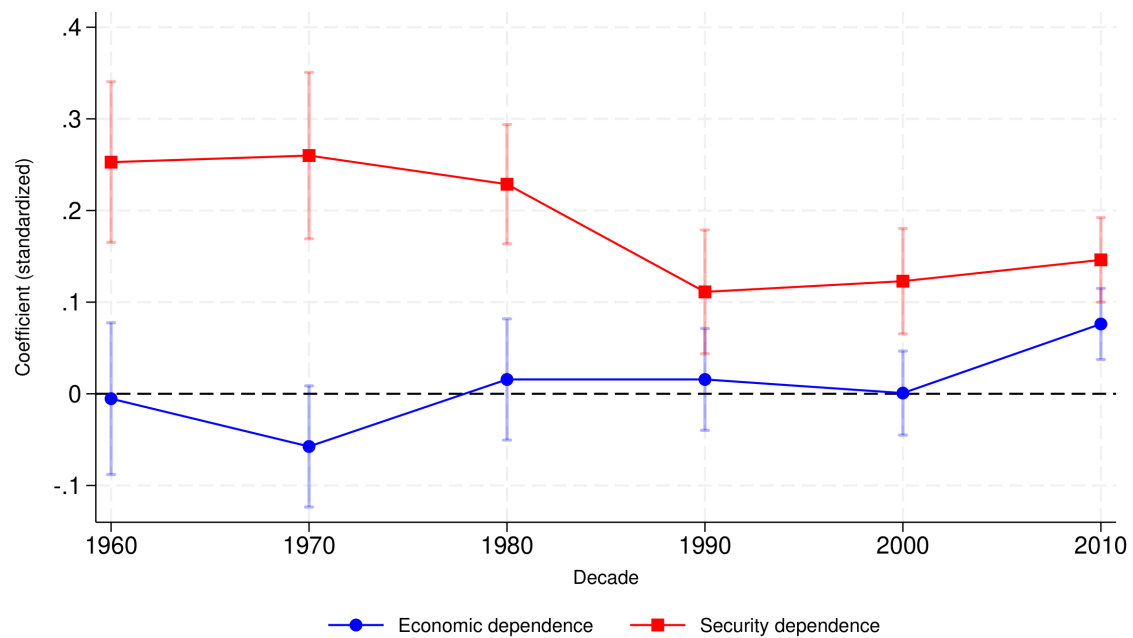
In this section of the Online Appendix, we provide additional empirical evidence on the relative importance of economic and security dependence for geopolitical network connections over time. The Formal Bilateral Influence Capacity (FBIC) dataset that we used to estimate the structural importance of countries ($\pi_{id} = \text{GDP}_{id}^\beta$) disaggregates overall dependence into separate measures of economic and security dependence. Using these disaggregated measures, we estimate the relationship between our geopolitical network connections (G_{ijd}), economic dependence (E_{ijd}) and security dependence (S_{ijd}) for each decade separately:

$$G_{ijd} = b_{1d}E_{ijd} + b_{2d}S_{ijd} + \alpha_{id} + \gamma_{jd} + u_{ijd},$$

where α_{id} and γ_{jd} are receiver-decade and sender-decade fixed effects, respectively, and u_{ijd} is a stochastic error.

In Figure A6, we display the estimated coefficients on economic dependence (b_{1d}) and security dependence (b_{2d}) over time. Again the inclusion of the receiver-decade and sender-decade fixed effects implies that these estimated coefficients are identified from bilateral variation. In the early decades of our sample period, we find that security dependence dominates economic dependence, which is consistent with the heightened geopolitical tension in these peak years of the Cold War. Over the course of our sample period, economic dependence becomes more important relative to security dependence, which is consistent with the globalization of the world economy increasing the relative importance of economic relationships between countries.

Figure A6: Economic Versus Security Dependence and Geopolitical Ties Over Time



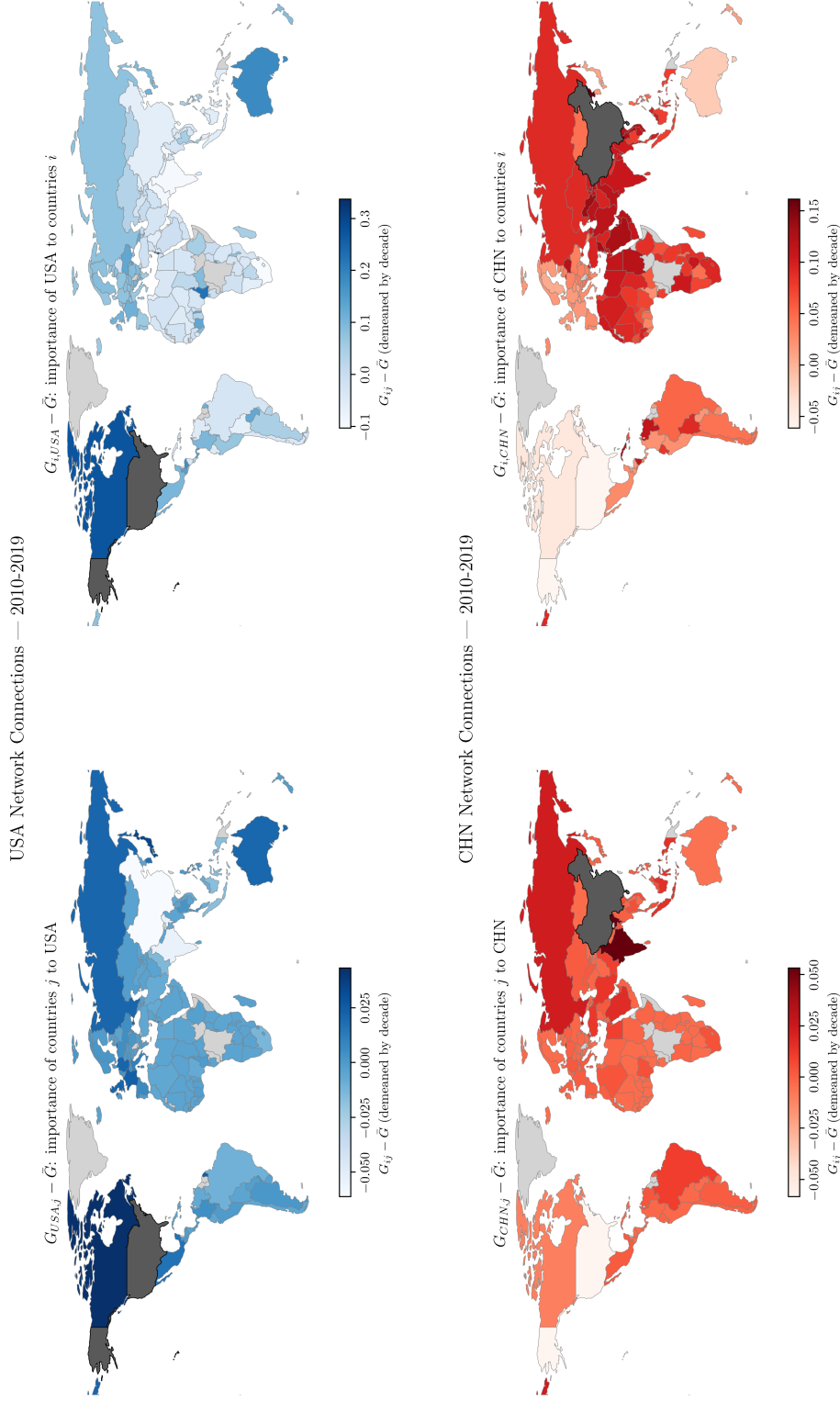
Note: Estimated coefficients from a regression of inferred geopolitical connections on both economic dependence and security dependence; regression estimated separately by decade; colored markers correspond to the point estimates; vertical bars correspond to the 95 percent confidence intervals.

L.5 Geopolitical Networks of China and the US

In this section of the Online Appendix, we provide further evidence on the empirical properties of our geopolitical network. We illustrate the geopolitical network of the US and China as the two leading powers at the end of our sample period. The top panel of Figure A7 maps the geopolitical network for the US during 2010–2019. The top-left map displays the importance of other countries j to the US (the US outward network). The top-right map shows the importance of the US to other countries j (the US inward network). We find an intuitive pattern with many similarities between the outward and inward networks. For example, the US and the UK have a mutual interest in one another. But there are other more one-sided relationships. For example, the US cares relatively more about China than China cares about the US, while New Zealand places a relatively higher weight on the US than the US places on New Zealand. Perhaps the most striking difference between the two panels is the upper and lower limits of the color scales. The range for other countries' weights on the US is roughly four times larger than that for the US weights on other countries. This difference reflects the asymmetry of the geopolitical network, in which the US is more structurally important than other countries. Therefore, other countries are more influenced by their geopolitical connections to the US than the US is influenced by its geopolitical connections to them.

Following a similar structure, the bottom panel of Figure A7 maps the geopolitical network for China during 2010–2019. The bottom-left map displays the importance of other countries j to China (China's outward network). The bottom-right map shows the importance of China to other countries j (China's inward network). We find that China has a very different geopolitical network than the US, with strong connections to countries such as Russia, North Korea, and India with which the US has relatively weak links. More broadly, the US has the strongest connections to Canada and Western Europe, whereas China has some of its strongest connections in Asia. China's inward and outward networks also exhibit some similarities. For example, Russia and China have a mutual interest in one another. But there are again differences between these inward and outward networks. Besides the US-China example discussed above, many African countries care relatively more about China than China cares about them. As before, the upper and lower limits of the color scales differ markedly between the two panels, because of the asymmetries in the geopolitical network. Given China's structural importance in the network, other countries are more influenced by their geopolitical connections to China than China is influenced by its geopolitical connections to them.

Figure A7: US and China Outward and Inward Connections

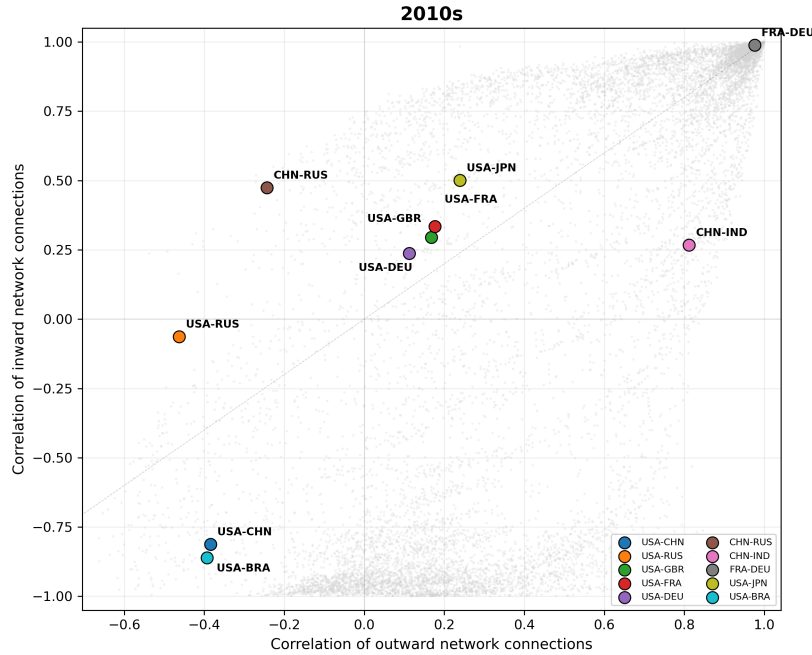


Note: Top panel displays the US geopolitical network connections; top-left map shows the US outward connections (the importance of other countries to the US); top-right map presents the US inward connections (the importance of the US to other countries); bottom panel displays China's geopolitical network connections; bottom-left map shows China's outward connections (the importance of other countries to China); bottom-right panel presents China's inward connections (the importance of China to other countries); in the top panel, darker blue denotes a more positive weight; in the bottom panel, darker red indicates a more positive weight; scales vary across all four maps. All network connections are shown relative to the 2010s G network's mean edge for ease of interpretation. All maps use Natural Earth's country-level dataset (version 5.1.1, released in May 2022), with the same geographic boundaries used across all decades. The country boundaries follow the source dataset and do not imply any position by the authors regarding sovereignty, territorial claims, or political status.

L.6 Network Heterogeneity

Our finding that China and the US have different geopolitical networks in the previous section of this Online Appendix raises the question of whether this simply reflects geopolitical rivalry between these two countries or whether these cross-country differences are a more general feature of the data. In this section of the Online Appendix, we explore the correlations between countries' geopolitical networks more broadly, as illustrated in Figure A8. On the vertical axis, we display the correlation of countries' inward geopolitical networks (the similarity with which other countries care about them). On the horizontal axis, we display the correlation of countries' outward geopolitical networks (the similarity with which they care about other countries). Each dot in the figure corresponds to a bilateral pair of countries.

Figure A8: Correlation of Inward and Outward Connections for Each Country Pair in the 2010s



Note: For each pair of countries in our sample (e.g., USA–CHN) for the decade 2010–2019, the figure displays the correlation between the two countries' inward connections (vertical axis) against the correlation between the two countries' outward connections (horizontal axis); each gray dot shows a different country pair; selected country pairs highlighted using color as noted in the legend; a perfectly symmetric network would feature all country pairs along the diagonal; inward connections (vertical axis) measure how much other countries care about a country; outward connections (horizontal axis) measure how much a country cares about others.

Perhaps the most remarkable feature of this figure is the heterogeneity in the structure of countries' geopolitical networks. The US and China have negatively correlated inward and outward connections, whereas France and Germany have highly correlated connections in both directions. China and India also have positively correlated connections in both directions, with a higher correlation for outward connections. China and Russia, by contrast, have negatively correlated outward connections but positively correlated inward connections. This heterogeneity highlights the insights from our approach of being able to recover unobserved geopolitical networks from observed domestic interests and geopolitical actions. Based on the regressions reported in Table 1 of the paper, this

heterogeneity in geopolitical networks is associated with differences in geography, economic ties, and cultural and historical relationships.

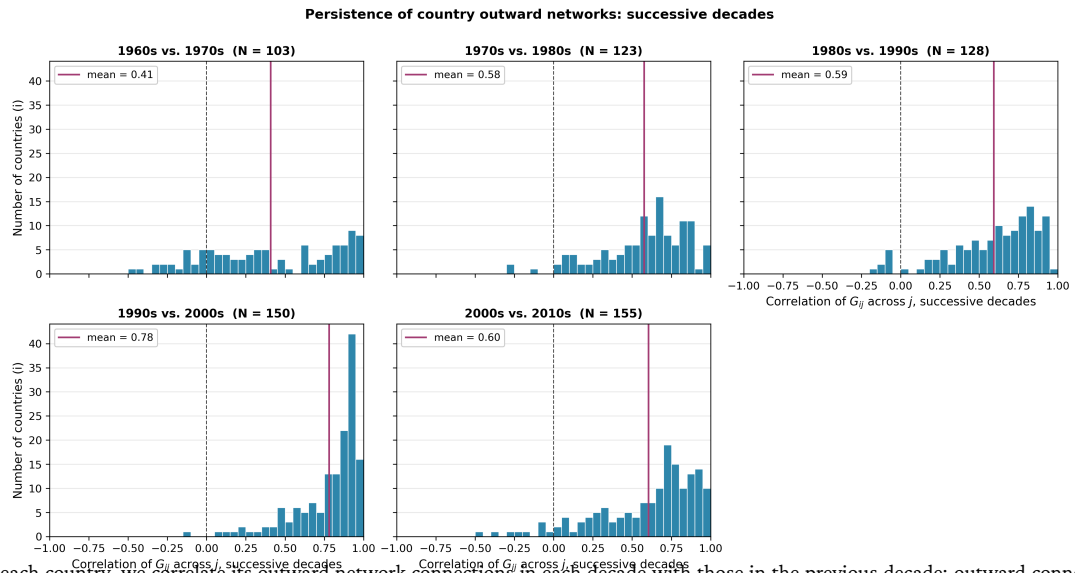
L.7 Temporal Stability of the Geopolitical Network

Our findings for the changing importance of economic and security dependence over time in Online Appendix L.4, and our results for the change in the Cold War cleavage over time in Subsection 4.1 of the paper, raise the more general question of the stability of the geopolitical network over time. To provide evidence on this question, we correlate each country's outward network connections in each decade with those in the previous decade. Outward connections for each country capture how much it cares about other countries. Positive correlations indicate persistence of geopolitical ties over time; negative correlations represent reversals in geopolitical links over time.

In Figure A9, we display the distribution of these correlation coefficients across countries as histograms for each successive pair of decades. In most cases, we find strong persistence in geopolitical connection over time. The average correlation coefficient between decades is typically 0.50 or higher. There is a mass of countries with correlation coefficients concentrated at values of above 0.50.

This pattern of results provides evidence that geopolitical network connections are typically stable over time, which is consistent with persistent effects of geography, history and alliance structures on international relations between countries. Nevertheless, large-scale geopolitical realignments can occasionally occur, such as the transformation of geopolitical networks that occurred in the aftermath of the fall of the Iron Curtain. More broadly, there is substantial heterogeneity across countries in the persistence of geopolitical networks. This heterogeneity is perhaps not surprising given the degree to which many nations' political institutions and roles in international politics have shifted in the last half-century. Again our approach explicitly allows for this heterogeneity by recovering the evolving geopolitical network from changes in the cross-country covariance structure of countries' geopolitical actions and domestic interests without imposing it on the data.

Figure A9: Stability of Geopolitical Networks over Time



Note: For each country, we correlate its outward network connections in each decade with those in the previous decade; outward connections for each country correspond to how much it cares about other countries; we display the distribution of these correlation coefficients across countries as histograms for each successive pair of decades.

L.8 Counterfactuals

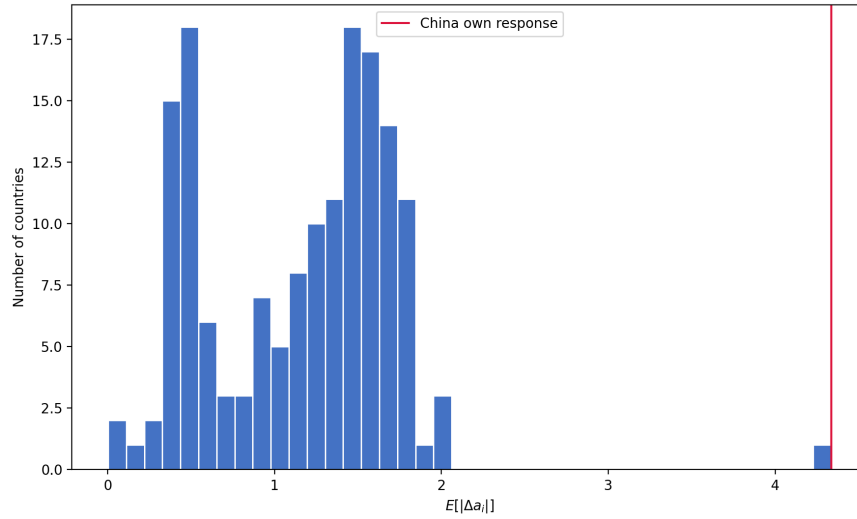
In this section of the Online Appendix, we report additional results for our counterfactuals in Section 5 of the paper.

L.8.1 Counterfactuals for Changes in Domestic Interests

We begin with our counterfactuals for changes in domestic interests from Section 5.1 of the paper. We first report some additional results for the counterfactual for China with US interests, as discussed in the paper. We next report the opposite counterfactual for the US with Chinese preferences.

China with US Interests We first report additional results for our counterfactual from Section 5.1.1 of the paper in which we replace the domestic interests of China with those of the US.

Figure A10: Changes in Each Country’s Action in Response to Setting China’s Domestic Interest Equal to the US Domestic Interest (Histogram)



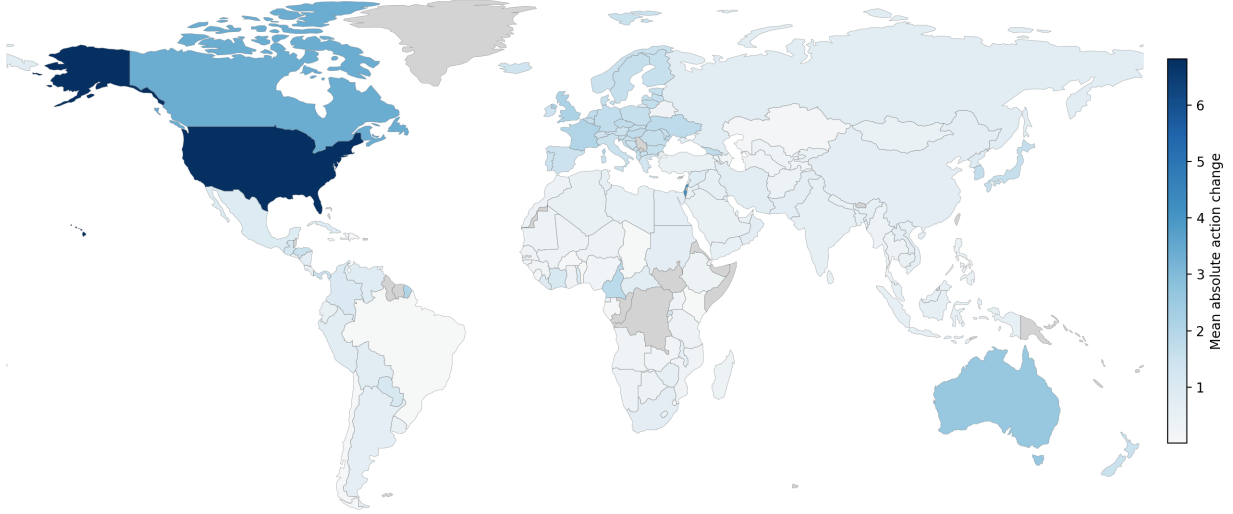
Note: Histogram of the mean absolute change $\mathbb{E}[|\Delta a_i|]$ in each country’s equilibrium action when China’s interest is set equal to that of the US. Mean taken over the estimated distribution of international issues.

US with Chinese Interests In Section 5.1.1 of the paper, we report the results of a counterfactual in which we replace the domestic interests of China with those of the US. In this section of the Online Appendix, we instead report the results of a counterfactual in which we replace the domestic interests of the US with those of China (Figure A11). We find the same qualitative pattern of results: changes in the domestic interests of a major power propagate throughout the geopolitical network, affecting the equilibrium actions of many countries whose own interests remain unchanged.

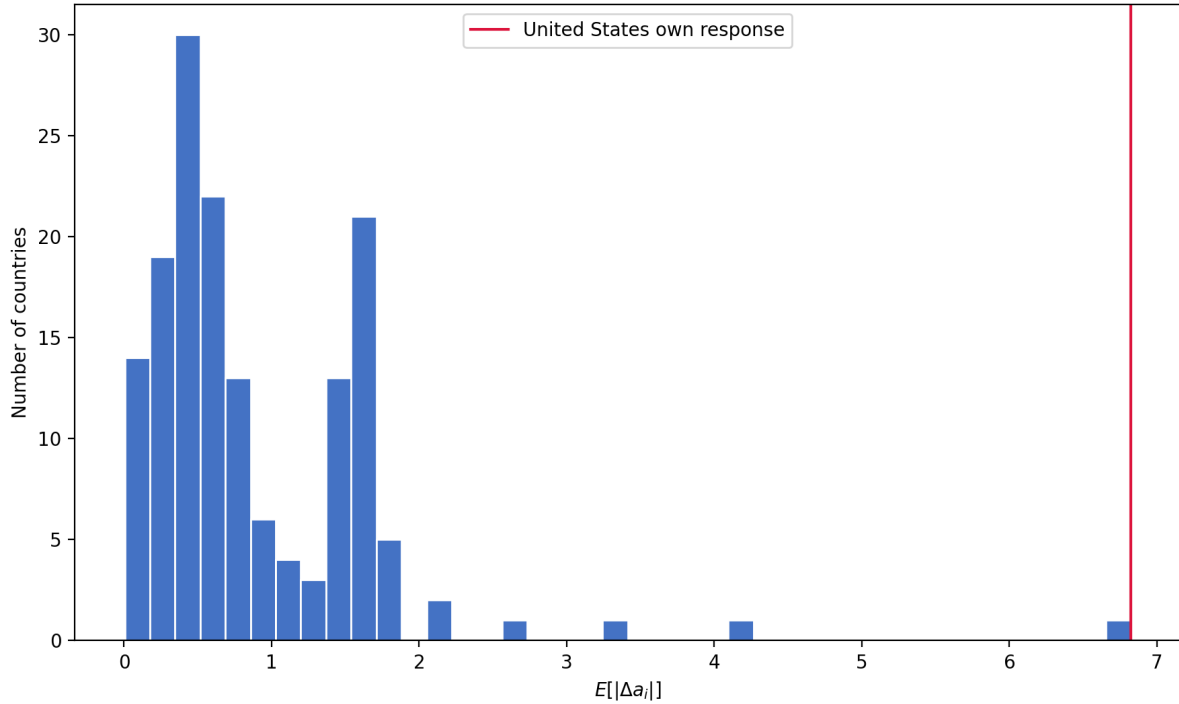
Autonomy and Control We next report additional results for our counterfactual in which we change the domestic interests of each country in turn from Section 5.1.2 of the paper. We use these counterfactuals to compute our first two notions of geopolitical power: autonomy and control. A country’s autonomy captures the responsiveness of its actions to its own interests, which reflects

Figure A11: Equilibrium Responses to the US with Chinese Interests

(a) Map of action changes



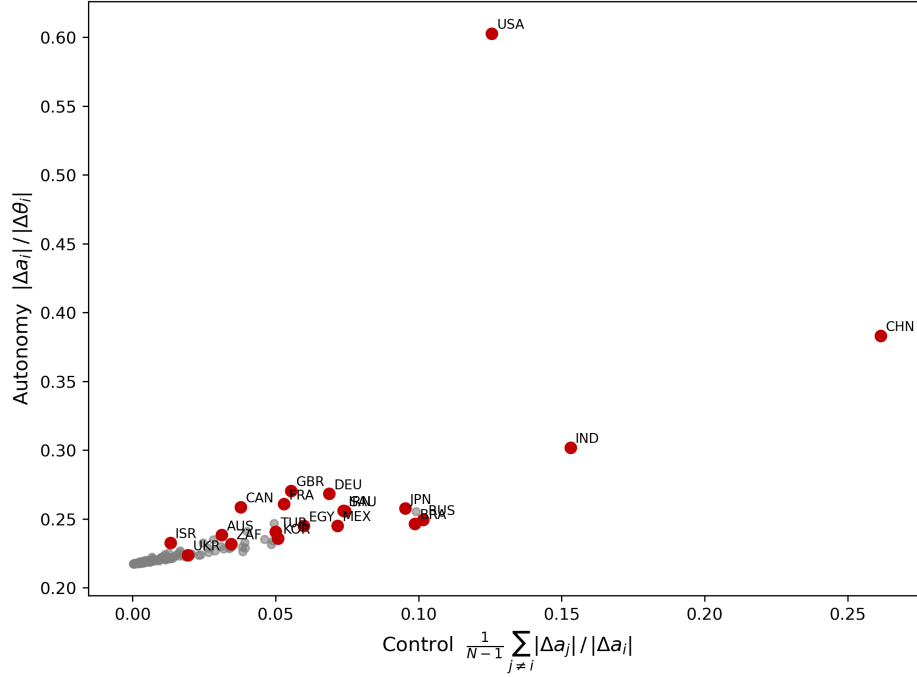
(b) Histogram of action changes



Note: Panel (a): map of the mean absolute change $\mathbb{E}[|\Delta a_i|]$ in each country's equilibrium action when the USA's interest is set equal to that of China, equal to the absolute entry $|\mathcal{L}(\mathbf{G})_{i,USA}|$ of the operator times the expected absolute gap between USA and Chinese interests. Panel (b): histogram of the same mean absolute changes. Mean taken over the estimated distribution of international issues. All maps use Natural Earth's country-level dataset (version 5.1.1, released in May 2022), with the same geographic boundaries used across all decades. The country boundaries follow the source dataset and do not imply any position by the authors regarding sovereignty, territorial claims, or political status.

the extent to which the country has independence to act regardless of its network connections. A country's control captures the responsiveness of other countries' actions to its own actions, which reflects the extent to which the country can use its own actions to regulate the actions of other countries.

Figure A12: Response of Own Actions to Own Domestic Interests Versus Response of Others' Actions to Own Actions



Note: Vertical axis shows a country's autonomy, measured as the size of its own action change relative to the size of its own interest change ($|\Delta a_i|/|\Delta \theta_i|$), which equals the diagonal entry $|\mathcal{L}(\mathbf{G})_{ii}|$ of the operator mapping interests to actions; complete pass-through of interests into actions equals one. Horizontal axis shows a country's control, measured as the average across the other countries of the size of the change in their actions relative to the size of the change in the country's own actions in response to the change in its interests ($\frac{1}{N-1} \sum_{j \neq i} |\Delta a_j|/|\Delta a_i|$), which equals $\frac{1}{N-1} \sum_{j \neq i} |\mathcal{L}(\mathbf{G})_{ji}|/|\mathcal{L}(\mathbf{G})_{ii}|$; one-for-one movement of every other country equals one. Both are ratios of entries of the operator and do not depend on the issue or on the size of the change in interests. Each observation in the figure corresponds to a different country.

L.8.2 Counterfactuals for Changes in Network Connections

We next turn to our counterfactuals for changes in network connections from Section 5.2 of the paper.

Power and the Structure of the Game In Section 5.2.2 in the paper, we report the results of counterfactuals that exclude each country i in turn from the network and recompute equilibrium actions among the remaining countries. We evaluate the change in the equilibrium configuration of actions among the remaining countries, as measured by the expected (over issues) Euclidean distance between their action vectors with and without the excluded country: $\mathbb{E}[\|\mathbf{a}_{-i} - \mathbf{a}(\mathbf{G})_{-i}\|]$, with the notation of Section 5.2.2 of the paper. This change in the equilibrium action vector in response to the exclusion of each country reflects the impact of both its interests and its network connections, where its network connections depend on its GDP-based structural importance weight (π_i).

In Figure 6 in the paper, we display the expected Euclidean distance between the equilibrium

action vectors of the remaining countries against the structural importance of each excluded country. We show that the superpowers dominate: excluding China or the US produces a mean Euclidean action change of about 10.2, approximately twice the effect for India of 5.0, and about 2.4 times the effects for Japan and Russia of 4.3 each.

L.8.3 Which Shocks Matter Most?

Our counterfactuals differ in kind from one another, because some are shocks to domestic interests, whereas others are shocks to network connections. Even among counterfactuals of the same kind, the magnitude of the shocks differs, which makes it hard to directly compare their impact on actions.

In Table A8, we report two approaches to comparing the results of these counterfactuals on a common footing. First, we scale the change in equilibrium actions by the size of the shock (s), and compute an elasticity of actions with respect to the shock (ε_a), as reported in the third column of the table. Second, we evaluate the implied changes in UN voting patterns, such that the counterfactuals can be compared in terms of their impact on votes. Using the observed data on actions from an individual vote (a), our estimated geopolitical network for each decade (G_d), and our model's first-order conditions in Proposition 7, we recover the implied domestic interests for that vote (θ).¹ Starting from this observed equilibrium, we undertake a counterfactual in which we change either domestic interests or geopolitical connections, and solve for counterfactual actions (a'), where we use a prime to denote a counterfactual value.²

Measured in elasticity terms, shocks to great power interests are the most potent: the counterfactual assigning US interests to China, and the counterfactuals moving Chinese or US interests between the 25th and 75th percentiles have elasticities of around 2.3. In comparison, the network change with the largest elasticity, which involves a weakening of links between the US and its European NATO allies, has an elasticity of 0.75. All other changes in network connections have elasticities of below one.

The columns for discrete changes in UN voting outcomes tell a similar story. Assigning to China the domestic interests of the US changes 9,726 individual UN votes, which is more than any other exercise in the table. In the counterfactuals excluding countries from the network, we find that excluding China changes more votes than excluding the US (1,090 against 600), but the changes from China's exclusion are concentrated on only 65 resolutions. In contrast, excluding the US changes votes for more than five times as many resolutions.

¹Countries that were absent for a UN vote are filled in as abstaining (vote = 0). This way of dealing with missing values is used for this UN vote counterfactual exercise and for the state visits and model-implied influence exercise (results in Table 2).

²Actual actions correspond to votes that take integer values: $a_i \in \{1, 0, -1\}$ for yes (1), abstain (0) or no (-1). We round counterfactual actions (a'_i) to the nearest integer to obtain counterfactual votes. We compute the total number of foreign votes changed and the number of resolutions with at least one changed vote.

Table A8: Summary of the Counterfactuals for Changes in Domestic Interests and Changes in Network Connections

Counterfactual exercise	s	ε_a	UN votes, 2010s	
			Votes	Resolutions
<i>Changes in Domestic Interests</i>				
China with US interests	0.209	2.35	9,726	289
China IQR preference change	0.190	2.35	6,088	253
US IQR preference change	0.143	2.20	3,408	533
India IQR preference change	0.101	1.12	249	34
<i>Changes in Network Connections</i>				
Retain 25% of US–European NATO links	0.177	0.75	284	199
India strengthens US–European NATO links	0.177	0.41	0	0
India strengthens China–Russia links	0.177	0.17	0	0
Retain 25% of China–Russia links	0.040	0.28	0	0
Exclude China from the network	0.543	0.54	1,090	65
Exclude the US from the network	0.540	0.56	600	352

Note: This table measures the consequences of different shocks relevant to each counterfactual exercise. IQR denotes the interquartile range. For domestic-interest counterfactuals, the shock size is $s = \|\Delta\Theta\|_F / \|\Theta\|_F$. For network counterfactuals, it is $s = \|\text{off}(G') - \text{off}(G)\|_F / \|\text{off}(G)\|_F$, where off sets the diagonal to zero. When measuring the shock size for the third notion of power, the removed row and column are represented by zeros. The action elasticity is $\varepsilon_a = (\|\Delta A\|_F / \|A\|_F) / s$. The columns of Θ and A contain the same 1,000 simulated issues; country removals compare actions on the same remaining countries' rows. The two network link-reduction exercises retain 25% of each selected link and apply the diagonal compensation rule described in Footnote 18. The final two columns use 766 retained UN General Assembly roll calls in 2010–2019, containing 113,154 originally observed country votes. For each roll call, we invert the action equilibrium to recover interests, apply the counterfactual, and discretize the resulting actions into yes, abstain, or no. Missing votes (which are, at most, 25% of a vote's observations) are treated as abstentions for this inversion. Counts of vote changes include only countries whose original vote is observed and exclude changes in the votes of the country or countries being shocked. We therefore report foreign vote changes. The final column counts roll calls with at least one foreign vote change.

References

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