Transition to Green Technology along the Supply Chain*

Philippe Aghion
Collège de France, INSEAD, LSE, and CEPR

Lint Barrage
ETH Zurich and CEPR

Eric Donald
University of Pittsburgh

David Hémous
U Zurich and CEPR

Ernest Liu
Princeton and NBER

June 2025

Abstract

We analyze a model of green technological transition along a supply chain. The model generates a unique equilibrium for given initial conditions but multiple steady-states. We show that: (i) even in the presence of Pigouvian environmental taxation, targeted sectoral subsidies are generally necessary to implement the social optimum; (ii) small, targeted industrial policy may bring large welfare gains; (iii) a government which is unable to subsidize greenification in more than one sector or price carbon at its true social cost should primarily target downstream sectors; (iv) overinvesting in greenification in the wrong upstream branch may derail the overall transition towards greenification. Finally, we calibrate our model to decarbonization of heavy duty transportation (trucking, aviation, etc.) via hydrogen. We find that, absent industrial policy, the economy can get stuck in the "wrong" steady-state with CO2 emissions vastly above the social optimum even with a Pigouvian carbon price in place.

*We thank Ufuk Akcigit, Adrien Bilal, Johannes Boehm, John Hassler, Xavier Jaravel, Per Krusell, Sjak Smulders, Jean Tirole, and Fabrizio Zilibotti as well as seminar participants at Harvard, Bocconi, the San Francisco Fed, the Conference on the Economics of Innovation at College de France, the Yrjö Jahnsson Foundation on Climate Change and Inequality, the NBER Summer Institute, INSEAD, Stockholm University, UCL, Cornell, University of Toronto, University Carlos III, CUHK, and HKU for very helpful comments and suggestions. Philippe Aghion acknowledges the support he received from the Kurt Björklund Chair in Innovation and Growth at INSEAD and from the SCOR Foundation for Science. David Hémous gratefully acknowledges the financial support of the European Commission (DG ECFIN).

1 Introduction

There is a growing consensus worldwide for the need to speed up the transition away from fossil fuels to slow down and eventually curb global warming. However, there is no unanimity among policy leaders when it comes to the best choice of climate policy instrument. In Europe, carbon pricing appears to hold the upper hand, whereas in the US or China, priority has been given to industrial policy as illustrated by the Inflation Reduction Act. Until recently the climate economics literature has emphasized carbon pricing as the main policy instrument to reduce emissions without paying as much attention to the industrial policy leg.

In this paper, we develop a dynamic model of technological transition along the supply chain to show that sector-specific industrial policy can be required on top on carbon pricing in order to optimally solve the energy transition problem. Indeed, the green transition requires the development of new supply chains, which themselves need to become greener over time. For instance, electric vehicles can replace fossil fuel vehicles, but they rely on an upstream input: batteries. The production of batteries themselves is CO_2-intensive and further innovation will be needed to make batteries cleaner. And, clean batteries may require new inputs, that again will have to become greener over time, etc. In that context, we argue that the coordination of innovation incentives along the supply chain provides a new rationale for the use of industrial policies.

We lay out the baseline model in Section 2. Each layer in the supply chain produces a "good" which is a Cobb-Douglas aggregate of a mass of industrial processes or varieties—the most downstream good being the consumption good. Each variety of that good can be produced either in a dirty way using labor only or, if that process has been "greenified", in a clean way using labor and the immediate upstream good. To move from dirty to clean, a variety in a given sector—i.e. a given layer in the supply chain—needs to undergo "greenification".[1] We assume heterogeneous fixed costs of greenification across varieties within a sector, so that some varieties may be greenified at one point in time while others still have to greenify. We also assume that once the social cost of carbon has been taken into account, producing using the clean technology is cheaper than producing using the dirty technology, so that a variety producer will always choose the former once the variety

1. Greenification is a stand-in here for a process that allows replacing fossil fuels as a production inputs with other inputs that do not directly generate emissions. In practice, this often, but not always, involves replacing fossil fuels with electricity or hydrogen.

has been greenified. A producer may choose to incur the fixed greenification cost in exchange for a one-period exclusive right of using the clean technology and Bertrand-compete with competitive producers that use the dirty technology. One period after the greenification of a variety, the clean technology becomes available to all producers, and the production of that variety becomes competitive.

Under these assumptions, the incentive to greenify, for any variety producer in any sector on the supply chain, depends on the degree of greenification downstream (more greenification downstream, which uses the variety as inputs, increases the demand for that variety, and thereby revenues for the innovator) and on greenification upstream (more greenification upstream, which supplies inputs to the variety, reduces the cost of producing the variety once it has been greenified, and thereby allows the innovator to charge a higher mark-up). The resulting cross-sectoral strategic complementarities in greenification generate a coordination problem: insufficient greenification in other sectors reduces the private incentives for variety producers in a given sector to greenify. While our model is set in the context of the green transition, its insights generalize to situations where the economy may switch from one technology to another superior one that requires the development of its own supply chain: cross-sectoral strategic complementarities may lead to insufficiently low adoption without policy intervention.[2]

In Section 3, we characterize the equilibrium in the absence of industrial policy. We show that the equilibrium is unique for given initial conditions, but cross-sectoral strategic complementarities in greenification generally lead to a multiplicity of steady-states.

In Section 4, we characterize the social optimum, and show that it generally differs from the decentralized solution even once emissions are optimally priced through a Pigouvian tax. Abstracting from the difference in the time horizon of the Social Planner and private agents, the key source of inefficiency is that the decentralized economy may be stuck in a local steady-state which is not globally optimal. As a result, the social optimum may differ from the equilibrium with Pigouvian emission taxes even though the private incentives to greenify are locally in line with the social incentives. However, we show that the socially optimal steady-state can be uniquely implemented through the combination of a Pigouvian tax with a set of temporary sector specific greenification subsidies.

2. For instance, one could re-cast our model from being about the green transition to being about the adoption of a modern technology that relies on complex inputs versus a traditional technology that uses a simpler production process. In that sense, our model introduces supply chains in Zeira (1998).

Even though we have an environment that displays complementarities as in other models, e.g., the Big Push model of Murphy et al. (1989), the fact that our complementarities occur along the supply chain leads to drastically different insights.

First, our model generates the possibility that a small and temporary subsidy to greenification that targets key sectors can be sufficient to achieve large welfare gains by moving the decentralized equilibrium just a little out of an inefficient steady-state and then relying on market forces to complete the transition towards the socially efficient steady-state: then, large, sustained interventions across all sectors are not needed.

Second, our framework has implications for how to prioritize public intervention between upstream and downstream, which we analyze in Section 5. When a government is constrained to subsidize a single sector in a vertical supply chain, the structure of the network suggests that it should prioritize downstream sectors. Intuitively, greenification propagates proportionally upstream through the demand channel but less than proportionally downstream through the cost channel—because the upstream sector is only one of the inputs used by a clean variety downstream. We generalize these insights to networks more complex than vertical supply chains.

Furthermore, in Section 6 we show that a second realistic policy restriction, where carbon prices are set below the true social cost of carbon, provides additional justification for targeting downstream greenification. With suboptimal carbon price, industrial policy should incorporate the effect of greenification on emissions. This emission-reduction motive for greenification favors downstream greenification when initial greenification is low: as in that case, greenifying upstream sectors has little effect on equilibrium emissions. Thus, our model provides a robust set of reasons for prioritizing downstream greenification in the face of restricted policy instruments.

Third, we rationalize the possibility of horizontally misdirected clean industrial policies, with resulting welfare losses in Section 7. We illustrate this point in an extended version of our model with two layers, where, on top of using labor, the dirty technology in the downstream sector also uses inputs from another upstream sector that can itself be greenified. In that context, greenification in the two upstream sectors—for the dirty and clean inputs to downstream varieties—are strategic substitutes. Over-investing in greenification of the upstream sector associated with the dirty technology may derail the overall transition towards greenification downstream.

Finally, we provide a quantitative application of our model to the decarbonization of long-range and heavy-duty (LRHD) transportation in the United States. We consider "greenification" via a switch to hydrogen, a key technology to help decarbonize LRHD transportation (IPCC 2024) with emerging commercial developments in trucking, rail, and passenger aircraft.[3] Importantly, hydrogen can itself be produced using fossil fuels—currently the dominant method—or from green electricity, highlighting the critical importance of full value chain greenification in this context (Rapson and Muehlegger 2023). First, we find that the multiplicity of steady-states is empirically relevant: even a carbon tax set equal to a high social cost of carbon (\$300/tCO_2 in \$2022) is consistent with multiple steady-states, including the current low greenification state and a high greenification state. Second, very high carbon prices (around \$440/tCO_2) would be required to eliminate the multiplicity of steady-states. Third, complementing emissions taxes with a temporary downstream subsidy toward the development of LRHD transportation technologies (i) is sufficient to induce high levels of greenification and (ii) yields large welfare gains estimated at \$3.2 trillion in present value.

Our paper relates to several strands of literature. First, on the macroeconomics of climate change, starting with the so-called "integrated assessment models" literature (IAMs) with early contributions such as by Nordhaus (1994), and pursued more recently, e.g. by Golosov, Hassler, Krusell, and Tsyvinski (2014). However, this literature often takes technological change as given, and its emphasis is on the optimal design of carbon tax policies. More closely related to our analysis is the literature on directed technical change and the environment, in particular Acemoglu, Aghion, Bursztyn, and Hémous (2012), henceforth AABH, who show that optimal climate policy in the presence of endogenous directed innovation requires combining a carbon tax and green research subsidy. The model in AABH relies on knowledge externalities, not on cross-sectoral strategic complementarities and coordination, as it does not model clean innovations along the supply chain.[4]

Second, a set of recent contributions study the positive properties of environmental policies in static production networks with carbon emissions, a setting where the optimal intervention remains a uniform carbon tax (e.g. see King, Tarbush, and Teytelboym 2019; Devulder and Lisack 2020; Martin, Muuls, and Stoerk, 2023; Mahen 2025). We depart from

3. While rail transportation is greenified in some parts of the world, in the United States (our area of focus), diesel fuels still account for over 95% of railway energy consumption (Davis and Boundy 2020).

4. Acemoglu et al. (2016) present another model of directed technical change in the environmental context. In their model as in ours, intermediates are aggregated in a Cobb-Douglas way and can be produced with a clean or a dirty input, but there is no supply chain or coordination issue.

these papers by introducing endogenous technological change and by analyzing dynamic strategic complementarities in technology adoption along the supply chain.

Third, we relate to papers on strategic complementarities in technology adoption, including the seminal work of Murphy et al. (1989), recent work by Sturm (2023), and in an environmental setting, Greaker and Midttømme (2016), Dugoua and Dumas (2021, 2024), and Smulders and Zhou (2024). While these papers feature multiple equilibria, our dynamic model has multiple steady-states but a unique equilibrium for given initial conditions; hence generating unambiguous predictions on the impact of industrial policy.[5] This feature enables our model to maintain tractability—despite rich strategic interactions along the supply chain—and generate new insights on how industrial policy should target key sectors to aid the technological transition.

A fourth related strand is the literature on industrial policy, in particular Greenwald and Stiglitz (2006), who model the infant industry argument based on "learning by doing" externalities, and Murphy et al (1989) who model Rosenstein-Rodan (1943)'s Big Push idea (see Juhasz, Lane, and Rodrik, 2023, for a recent survey). However, none of these papers considers supply chains and the associated motives for sectoral policies.[6]

2 Model

We now present our baseline model of a vertical supply chain in a green transition.

Preferences and Production Technology. Time is discrete and denoted by t. The consumer demand side of the economy consists of a continuum of mass one of agents with the same intertemporal utility

$$\sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \ell_t - a_t\right), \tag{1}$$

5. Also see Crouzet, Gupta, and Mezzanotti (2023) for a dynamic model of strategic complementarity in the context of adopting electronic payments.

6. Closest to our paper is Liu (2019), who analyzes sector-specific policy in a production network with market imperfections, showing that targeting upstream sectors—as done in South Korea and China—can boost aggregate welfare. Liu and Ma (2023) examine optimal cross-sector R&D allocation within an innovation network featuring knowledge spillovers. Donald (2023) incorporates Liu and Ma's innovation network into AABH's directed innovation framework, highlighting spillovers as critical for clean innovation policy. Buera and Trachter (2024) contemporaneously study industrial policy in a static network with endogenous technology adoption.

where c_t denotes the consumption flow, ℓ_t denotes the representative individual's labor supply, a_t denotes the disutility of pollution, and β is the discount factor.

The production side is a vertical supply chain consisting of N layers or "sectors" which we rank from the most upstream, namely i =1, to the most downstream, $i = N$, which corresponds to the consumption good. Production y_{it} in each sector i at time t is the aggregate output of a continuum of mass one of sector-specific varieties (ν), according to:

$$\ln y_{it} = \int_0^1 \ln y_{it}(\nu) \, \mathrm{d}\nu.$$

In turn each variety ν in any sector i can be produced using either a dirty or clean technology. Three features distinguish clean and dirty technologies. First, the dirty technology is associated with pollution while the clean one is not. Second, the dirty technology is always available, while the clean technology is only available for a variety ν in sector i that is "greenified"—greenification occurs through a process described below. And third, the dirty technology only uses labor (one for one), while the clean technology uses good $i-1$ as an intermediate input together with labor in a Cobb-Douglas fashion. More specifically, we assume that the most upstream input is produced according to:

$$y_{1t}(\nu) = \ell_{d1t}(\nu) + \gamma_{1t}(\nu) e^z \ell_{c1t}(\nu),$$

and that for all $i > 1$:

$$y_{it}(\nu) = \ell_{dit}(\nu) + \gamma_{it}(\nu) \left(\frac{e^z \ell_{cit}(\nu)}{\alpha_i}\right)^{\alpha_i} \left(\frac{m_{it}(\nu)}{1-\alpha_i}\right)^{1-\alpha_i}, \tag{2}$$

where: (i) $\gamma(\nu)$ is an indicator function, equal to 1 for greenified varieties and to 0 for non-greenified varieties; (ii) $\alpha_i \in (0,1)$ for all $i > 1$; (iii) for all i, $\ell_{dit}(\nu)$ denotes the labor input used by variety ν in sector i using the dirty technology; (iv) for all i, $\ell_{cit}(\nu)$ and $m_{it}(\nu)$ denote respectively the labor input and the amount of intermediate input from sector $i-1$ used for producing variety ν with the clean technology; (v) e^z is the relative (labor-augmenting) productivity of using the clean, rather than dirty, technology.

We assume that producing each unit of output using the dirty technology generates ξ units of disutility due to pollution. The total disutility of pollution is thus $a_t = \xi \ell_{dt}$, where ℓ_{dt} is the total labor input used for dirty production in the economy, and ξ is the social unit cost of pollution.[7]

7. This setting is equivalent to one where production with the dirty technology uses a free and inexhaustible fossil fuel resource together with labor in a Leontief way. Alternatively, the model can accommo-

Our initial focus is therefore on a vertical supply chain with strategic complementarities in greenification across sectors: this case is both theoretically interesting and empirically relevant, as switching toward green technologies requires thinking specifically about how clean the inputs of clean technologies really are (as illustrated by the debate around the high emission intensity of EV batteries). Yet, as we shall see below, our analysis and results apply beyond vertical supply chains. First, we can relax the simplifying assumption that the relative productivities z and emission rates ξ are the same for all sectors as shown in our numerical exercise of Section 8. Second, our insights carry over to more complex networks for the green production process as analyzed in Sections 5.3 and B.9. Third, our assumption that only the clean technology uses a supply chain is less stringent than it may appear at first sight: our model is isomorphic to one where there is a separate complex network for the dirty production process without greenification, and in fact, our qualitative insights still carry through if the dirty production process shares some inputs with the clean production process but benefits less from upstream greenification.[8] Instead, greenification across some sectors becomes strategic substitutes if the greenification of some inputs benefits dirty production more than clean production, a case we analyze in Section 7.

Market Clearing. Labor market clearing requires that, at any time t:

$$\ell_{ct} = \sum_{i=1}^{N} \ell_{cit} \equiv \sum_{i=1}^{N} \int_0^1 \ell_{cit}(\nu)\, \mathrm{d}\nu, \quad \ell_{dt} = \sum_{i=1}^{N} \ell_{dit} \equiv \sum_{i=1}^{N} \int_0^1 \ell_{dit}(\nu)\, \mathrm{d}\nu, \quad \ell_t = \ell_{ct} + \ell_{dt} + \ell_{et},$$

where ℓ_{cit} and ℓ_{dit} are the labor inputs used for clean and dirty production in sector i (integrating across varieties within each sector), ℓ_{ct} and ℓ_{dt} are the total labor used for clean and dirty production (summing across all sectors), and ℓ_{et} is the total labor employed

date extraction costs for the resource if ξ includes both environmental damages and the extraction costs and the carbon tax τ introduced below includes both the tax itself and the extraction cost. For simplicity, we assume a constant ξ but the analysis can be straightforwardly generalized to a time-varying ξ. We do not model a carbon cycle but note that the damages generated at time t, a_t, need not be felt by the household solely at t—so that our model also covers the case where emissions generate long-lasting damages.

8. With a separate dirty network that features no greenification (i.e. $\gamma(v)$ is constant for all sectors that are input in the dirty production process), the different dirty sectors can simply be aggregated along the value chain and this economy is de facto identical to one where only labor is used for the dirty technology (potentially with heterogeneous z and ξ). Greenification across sectors remains strategic complement, delivering similar insights to the ones we develop here when the clean and the dirty production processes share some inputs that can be greenified but the dirty production process relies less on them.

for greenification (we specify the greenification technology below).

Market clearing in the upstream sectors $i = 1, \ldots, N-1$ equalizes total output of sector i with its use as an intermediate input across varieties in sector $i+1$:

$$y_{i,t} = \int_0^1 m_{i+1,t}(\nu)\ \mathrm{d}\nu,$$

whereas market clearing for the downstream consumption good N simply boils down to:

$$c_{Nt} = y_{Nt}.$$

Greenification and Market Power. To operate the clean technology, a variety must be "greenified". A producer of variety ν in sector i must incur a one-time sunk cost which is sector-variety specific to greenify production. We order varieties in each sector i by increasing cost of greenification and let $\phi_i(s)$ denote the cost of greenification associated with the variety quantile s in sector i, where that cost is expressed in labor units.[9]

We let $F_i(\cdot)$ denote the CDF cost distribution, that is, for any cost ϕ, $F_i(\phi)$ is the measure of the set of variety quantiles s in sector i with costs $\phi_i(s)$ less than ϕ. Let χ_{it} denote the fraction of greenified varieties at time t in sector i, which also corresponds to the cut-off quantile s beyond which varieties cease to be greenified. The sum of all greenification fixed costs up to χ_{it} is given by $\mathcal{F}_i(\chi_{it}) \equiv \int_0^{\chi_{it}} \phi_i(s)\ \mathrm{d}s$.[10] The collection of χ_{it}'s across sectors form the key state variables of the economy. Starting from an initial condition $\{\chi_{i0}\}_{i=1}^N$ at $t = 0$, greenification raises χ_{it}'s monotonically over time, until χ_{it}'s converge to a steady-state.

We assume that the dirty technology is operated competitively, but that, if she greenifies her variety, the producer of that variety earns one period of monopoly profit upon greenification, with fringe producers operating the dirty technology as competitors.[11] Starting from the subsequent period, the clean technology becomes freely available to all producers so that the variety is again competitively produced.

9. Our model of greenification is directly inspired by the automation literature where after an innovation or payment of a fixed cost, labor can be replaced by capital in the production process (see e.g. Zeira 1998). Colmer et al. (2025) provide empirical backing for such a model of greenification.

10. There is an obvious relationship between F_i and $\mathcal{F}_i$. Namely, for any $s : F_i(\phi_i(s)) = s$ or equivalently $\phi_i(s) = F_i^{-1}(s)$. This directly leads to $\mathcal{F}_i(\chi_{it}) = \int_0^{\chi_{it}} F_i^{-1}(s)\,\mathrm{d}s$.

11. With this market structure, it is never profitable for more than one producer of the same variety to spend the sunk cost to greenify.

Policy Instruments. We assume that the government can: (i) impose a carbon tax τ on any variety which uses the dirty technology; (ii) or impose a cap-and-trade limit $\bar{\ell}_d$ on the amount of dirty input used (leading to a carbon price τ); and (iii) implement an industrial policy that is a set of sector-specific, time-varying greenification subsidies q_i. In what follows, we take the carbon tax τ as constant for simplicity but the analysis generalizes straightforwardly to a time-varying τ.

3 Equilibrium

We now characterize the equilibrium of the economy if it only resorts to a carbon tax.

Equilibrium Price. In each period, the representative consumer solves

$$\max_{c_t,\ell_t} \ln c_t - \ell_t - a_t \qquad \text{s.t. } p_t c_t = w_t \ell_t + \pi_t + T_t,$$

where p_t is the price index of the consumption good, π_t are profits, T_t is a lump-sum transfer from the government, and w_t is the wage rate which we normalize to one. Consumer optimization implies that the total expenditure on the consumption good is equal to one:

$$p_t c_t = 1. \tag{3}$$

Given that dirty production uses only labor one-for-one, then, absent policy interventions the marginal cost of the dirty technology is equal to the wage, namely 1. If instead the government taxes pollution (or imposes a binding cap-and-trade limit), the marginal cost of dirty production is $1 + \tau$, with $\tau > 0$. We assume that τ and/or z are large enough to ensure that $1 + \tau > e^{-z}$. This condition ensures that a producer always uses the clean technology once her variety has been greenified. For notational simplicity, we define $Z \equiv \ln(1 + \tau) + z$, as the tax-adjusted relative productivity of clean versus dirty technology, so we assume that $Z > 0$.[12]

The equilibrium price index p_{it} for good i satisfies

$$\ln p_{it} \equiv \int_0^1 \ln p_{it}(\nu)\, \mathrm{d}\nu, \tag{4}$$

where, for each variety ν in sector i, $p_{it}(\nu)$ is determined as follows. If variety ν has been greenified by the previous period, then it is priced at the marginal cost of the clean

12. $Z > 0$ implies that the labor portion of the cost of the clean technology is lower than the cost of the dirty technology. This is a necessary and sufficient condition to ensure that the clean technology is cheaper than the dirty one (regardless of the share of greenified varieties) because the intermediate inputs parts of the cost is a weighted average between clean and dirty labor costs.

technology. If variety ν has not yet been greenified in the previous period, then it is priced at the marginal cost of the dirty technology. This is trivial for producers using the dirty technology, but it is also true for a newly greenified variety. The producer of a newly greenified variety is a monopolist who faces a fringe that uses the dirty technology. Given the unit demand elasticity, she will charge a price equal to the marginal cost of the fringe.

The market structure implies that in the most upstream sector (sector 1), the price of a variety that has been greenified in the previous period is equal to $p_{1t}(\nu) = e^{-z}$, whereas the price of a non greenified or of a newly greenified variety is $p_{1t}(\nu) = 1 + \tau$. Next, in sectors $i > 1$, we have:

$$p_{it}(\nu) = \begin{cases} e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i} & \text{if the variety has been greenified by time } t-1, \\ 1+\tau & \text{otherwise,} \end{cases} \tag{5}$$

as the marginal cost of dirty production is $1+\tau$, while that of clean production is $e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i}$.

To get more explicit expressions for these equilibrium prices, it will be helpful to consider, for all i, the network adjusted share μ_{it} of greenified content in the production of any greenified variety in sector i. In the most upstream sector 1, we obviously have $\mu_{1t} = 1$. In more downstream sectors $i > 1$, μ_{it} is recursively determined by:

$$\mu_{it} = \alpha_i + (1 - \alpha_i)\,\chi_{i-1,t}\mu_{i-1,t}. \tag{6}$$

In words, the greenified content of a greenified variety in sector i, is equal to the direct share of clean labor input α_i plus $(1 - \alpha_i)$ times the aggregate greenified content of inputs from $i-1$, which in turn is equal to the greenified content $\mu_{i-1,t}$ of each greenified variety in sector $i-1$ times the fraction $\chi_{i-1,t}$ of greenified varieties.

We can now solve for the price index in each sector. For the most upstream sector 1, we use (4) and (5) and solve for downstream prices by induction:

$$p_{1t} = (1+\tau)\,e^{-\chi_{1,t-1}Z} \text{ and } p_{it} = (1+\tau)\,e^{-\chi_{i,t-1}\mu_{it-1}Z}. \tag{7}$$

Therefore higher greenification in sector i or upstream of sector i reduces the price of sector i but only with a one-period delay. This is because the cost reduction achieved by greenification is only passed down to consumers once the monopoly position of the innovator expires.

Equilibrium Profits. The incentive to greenify depends on the profits that a newly greenified variety obtains. We now derive these profits. The producer of a newly greenified variety charges a price $1+\tau$ but faces marginal costs equal to $e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i}$, therefore she charges a mark-up $\theta_{i,t}$ given by

$$\theta_{i,t} = \frac{1+\tau}{e^{-\alpha_i z} p_{i-1,t}^{1-\alpha_i},} = e^{Z\mu_{it-1}},$$

where the second equality uses Equations (7) and (6) to substitute for p_{i-1t}. Higher greenification upstream (a higher μ_{it-1}) enables the innovator to charge a higher mark-up but again only with a 1-period delay. Once a clean input is produced competitively, it becomes cheaper than the dirty input, resulting in a lower marginal cost for the downstream clean producer. The profits share of revenues is $1-\theta_{i,t}^{-1}$, and the input costs share is $\theta_{i,t}^{-1}$.

We next derive the equilibrium revenue r_{it} of a producer in sector i at time t. In the most downstream sector N, which produces the final consumption good, we know that

$$r_{Nt} = p_t c_t = 1.$$

From there we move upstream, as revenues trickle up from downstream to upstream sectors. Take as given the revenues $r_{i+1,t}$ of a producer in sector $i+1$ at time t. Then, sector i's good is only used as an input by the greenified varieties in sector $i+1$. For any variety in sector i,the revenue r_{it} includes both sales to previously greenified varieties and sales to newly greenified varieties. There is a mass $\chi_{i+1,t-1}$ of previously greenified varieties. These are produced competitively, so that a share $1-\alpha_{i+1}$ of their revenues go to sector i. There is a mass $\chi_{i+1,t} - \chi_{i+1,t-1}$ of newly greenified varieties, where only a share $\theta_{i+1,t}^{-1} = e^{-Z\mu_{i+1.t-1}}$ of their revenues go to the payment of inputs, out of which a share $1-\alpha_{i+1}$ go to sector i. We then obtain:

$$\begin{aligned} r_{it} &= \underbrace{\chi_{i+1,t-1} r_{i+1,t} \left(1-\alpha_{i+1}\right)}_{\text{sales to previously greenified varieties}} + \underbrace{\left(\chi_{i+1,t} - \chi_{i+1,t-1}\right) r_{i+1,t} e^{-Z\mu_{i+1,t-1}} \left(1-\alpha_{i+1}\right)}_{\text{sales to newly greenified varieties}} \\ &= \widetilde{\chi}_{i+1,t} r_{i+1,t} \left(1-\alpha_{i+1}\right), \end{aligned}$$

where we define the sector $i+1$'s revenue share spent on clean inputs as

$$\widetilde{\chi}_{i+1,t} \equiv \chi_{i+1,t-1} + \left(\chi_{i+1,t} - \chi_{i+1,t-1}\right) e^{-Z\mu_{i+1,t-1}}. \tag{8}$$

This in turn immediately yields the following expression for the equilibrium revenue accruing from downstream to good i production:

$$r_{it} = \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{jt}(1-\alpha_j)\right). \tag{9}$$

Higher greenification downstream (higher $\widetilde{\chi}_{jt}$ for $j > i$) increases demand for upstream producers and therefore their revenues.

The corresponding profit from greenification for any variety producer in sector i, is then simply equal to the profit share times revenues:

$$\pi_{it} = (1 - e^{-Z\mu_{it-1}})r_{it}. \tag{10}$$

Greenification. A variety producer in sector i greenifies at time t if and only if π_{it} is bigger than the cost of greenification of that variety. It then immediately follows that the equilibrium share of greenifying varieties χ_{it} in sector i at time t satisfies

$$\chi_{it} = F_i(\pi_{it}) = F_i\left(\left(1 - e^{-Z\mu_{it-1}}\right)\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{jt}(1-\alpha_j)\right)\right) \tag{11}$$

whenever this is greater than χ_{it-1} and to χ_{it-1} otherwise, as there is no dis-greenification. Equation (11) shows that the incentive of a variety producer in sector i to greenify depends positively upon both upstream and downstream greenification. On the one hand, (past and contemporaneous) downstream greenification (captured by $\Pi_{j=i+1}^{N}\widetilde{\chi}_{jt}$) increases total revenue accruing to good i production r_{it} (a demand effect), and on the other hand, (past) upstream greenification increases $\mu_{i,t-1}$ and therefore the profit share $\left[1 - e^{-Z\mu_{it-1}}\right]$ of a newly greenified producer in sector i (an input cost effect).[13]

Two further remarks can be made at this point. First, there are complementarities in greenification across sectors which will be a source of multiplicity of steady-states. As we shall see below, absent adequate industrial policy, nothing guarantees that the economy will converge to the equilibrium with maximum greenification. Second, the incentives to greenify travel upstream contemporaneously whereas they travel downstream with a one period lag. This features ensures that, given any initial condition $\{\chi_{i0}\}$, the model features a unique equilibrium path $\{\chi_{it}\}_{t\geq 0}$.

Equilibrium Equations. Our analysis leads to the following simple characterization of an equilibrium. Given initial greenification shares $\{\chi_{i0}\}$ at $t = 0$, an equilibrium with

13. Upstream greenification also decreases revenues as it reduces the sales to newly greenified varieties (see 8). This effect is temporary and its impact on profits is generally dominated by the effect of upstream greenification on the profit share. $Z < \ln 2$ is a sufficient condition to get π_{it} always decreasing in μ_{it}.

a carbon tax $\{\tau\}$ is a sequence of greenification shares $\{\chi_{it}\}_{t>0}$ such that

$$\chi_{it} = \max\left\{\chi_{i,t-1}, F_i\left(\left(1-e^{-Z\mu_{i,t-1}}\right)\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{jt}(1-\alpha_j)\right)\right)\right\}, \tag{12}$$

where the sequences $\{\mu_{it}\}$ and $\{\tilde{\chi}_{it}\}$are defined iteratively through $\mu_{1t}=1$, (6) and (8). Despite the complementarities in greenification, the market structure ensures that the equilibrium is unique. In Appendix B.1, we show:

Proposition 1. *Given initial condition $\{\chi_{i0}\}$, the economy with carbon taxes $\{\tau_t\}$ features a unique equilibrium path $\{\chi_{it}\}_{t>0}$.*

Multiplicity of Steady-States. We next characterize the steady-states of the economy. In a steady-state, there are no newly greenified varieties, so $\widetilde{\chi}_i = \chi_i$ for all i. It follows that a steady-state with a carbon tax τ is a set of greenification shares $\{\chi_i\}$ such that:

$$\mu_1 = 1, \quad \mu_i = \alpha_i + \chi_{i-1}\mu_{i-1}(1-\alpha_i) \text{ and} \tag{13}$$

$$\chi_i \geq F_i\left(\left(1-e^{-Z\mu_i}\right)\prod_{j=i+1}^{N}\chi_j\left(1-\alpha_j\right)\right). \tag{14}$$

Condition (14) is an inequality for the same reason as the "max" notation in (12): technically, because greenification cannot decrease, any $\{\chi_i^{ss}\}$ for which (14) holds as a strict inequality is also a steady-state provided that the economy starts with $\chi_{i0} = \chi_i^{ss}$. These are, however, not very interesting steady-states since, starting from $\chi_{i0} < \chi_i^{ss}$, there is no path that the economy can follow to reach these steady-states (without direct government intervention). We therefore generally ignore them in our analysis, and we focus only on steady-states in which Condition (14) holds as an equality:

$$\chi_i = F_i\left(\left(1-\left[e^z\left(1+\tau\right)\right]^{-\mu_i}\right)\prod_{j=i+1}^{N}\chi_j\left(1-\alpha_j\right)\right). \tag{15}$$

While the equilibrium is unique, complementarities in greenification ensure that the economy generally features more than one steady-state. In Appendix B.2, we show:

Proposition 2. *For a given carbon tax τ, there may exist multiple steady-states over a non-empty open set of parameters whenever $N \geq 2$. There is a unique steady-state when $N = 1$.*

Naturally, the same logic extends to the case where the government implements a cap-and-trade system instead of a carbon tax. In particular, multiple steady-states remain possible: for instance, a steady-state with a high level of greenification and a low price of carbon can co-exist with a steady-state with a low level of greenification and a high price for carbon. Both steady-states may achieve the same level of emissions (if the cap binds), but the low greenification steady-state does it with lower output (see Appendix B.3).[14]

4 Social Optimum

We now solve for the Social Planner problem and compare the optimum with the decentralized economy. Starting from initial conditions $\{\chi_{i0}\}$, the Social Planner seeks to maximize the intertemporal utility of consumption minus the labor costs (inclusive of labor hired for greenification) and minus the pollution costs due to the dirty technology. Greenification, once the fixed costs are paid, weakly pushes out the production possibility frontier. With linear labor disutility, the social costs of greenifying a given varieties are independent of the share of already greenified varieties, so that all greenification happens immediately in the optimum. The optimum features instantaneous greenification of (a fraction of) varieties in all sectors in the initial period and no further greenification in subsequent periods. With constant technology levels after the initial greenification, labor allocation to clean and dirty production, and output are constant. Therefore, keeping in mind that $c_t = y_{Nt}$, that ξ is the social cost of pollution, and using $\sum_{t=0}^{\infty} \beta^t = 1/(1-\beta)$, the Social Planner solves:

$$\max_{\{\ell_{di}, \ell_{ci}, \chi_i\}} \frac{1}{1-\beta} \left(\ln y_N - (1+\xi) \sum_{i=1}^{N} \ell_{di} - \sum_{i=1}^{N} \ell_{ci} \right) - \sum_{i=1}^{N} \left(\mathcal{F}_i(\chi_i) - \mathcal{F}_i(\chi_{i,0}) \right). \tag{16}$$

We assume that $(1+\xi) e^z > 1$, which ensures that the Social Planner uses the clean technology whenever it is available. Then, the Planner treats all varieties of a given status (greenified or not) in a given sector symmetrically, which ensures that $\ell_{it}(\nu) = \ell_{ci}(\nu) = \ell_{ci}/\chi_i$ and $m_{ci}(\nu) = y_{i-1}/\chi_i$ if the variety is greenified and $\ell_{it}(\nu) = \ell_{di}(\nu) = \ell_{di}/(1-\chi_i)$ if the variety is not greenified. We can then write output in sector i as

$$\ln y_i = \chi_i \ln \left[\left(\frac{e^z \ell_{ci}}{\chi_i \alpha_i} \right)^{\alpha_i} \left(\frac{y_{i-1}}{\chi_i (1-\alpha_i)} \right)^{1-\alpha_i} \right] + (1-\chi_i) \ln \frac{\ell_{di}}{1-\chi_i}. \tag{17}$$

Thus, we can characterize the Social Planner's problem as (proof in Appendix A):

14. A cap at zero, however, eliminates the multiplicity since it forces full greenification in all sectors, but it will generally not be optimal to force emissions to zero.

Proposition 3. *The Social Planner's problem can be rewritten as*

$$\max_{\{\chi_i \geq \chi_{i0}\}} \ln\left(\left(1+\xi\right)e^z\right)\sum_{i=1}^{N}\chi_i\alpha_i\prod_{j=i+1}^{N}\left[\chi_j\left(1-\alpha_j\right)\right]-\left(1-\beta\right)\sum_i\mathcal{F}_i\left(\chi_i\right). \tag{18}$$

If the solution $\{\chi_i\}$ is interior, it must satisfy the first-order conditions:

$$\chi_i = F_i\left(\frac{\ln\left(\left(1+\xi\right)e^z\right)}{1-\beta}\mu_i\prod_{j=i+1}^{N}\left[\chi_j\left(1-\alpha_j\right)\right]\right). \tag{19}$$

Steady-States versus the Social Optimum. At this stage, it is worth comparing the steady-state decentralized equilibrium with the social optimum. Obviously, for given degrees of greenification $\chi's$, the decentralized economy in steady-state allocates production resources (labor) efficiently provided that the carbon tax is at the Pigouvian level: $\tau = \xi$.

More interestingly, we can compare greenification in the two economies by comparing Equation (15) and the First-Order Condition for the social optimum (19). There are two differences. First, the factor $\frac{1}{1-\beta}$ on the RHS of (19) is absent from the RHS of (15). This captures an intertemporal spillover effect whereby the social gain from greenification carries over to the whole future, whereas private producers benefit from greenification for one period only, given our assumption that patents expire after one period. This inefficiency could be corrected with a uniform subsidy to greenification at rate β.

A second difference comes from the term $1-e^{-Z\mu_i}$ on the RHS of (15) versus $\mu_i \ln\left(\left(1+\xi\right)e^z\right) = \mu_i Z$ on the RHS of (19)—assuming that the Social Planner implements a Pigouvian tax $\tau = \xi$, which enables us to use the same notation $Z = z + \ln\left(1+\xi\right)$ for both problems. This difference reflects the fact that the Social Planner maximizes total surplus, including consumer surplus, whereas producers only maximize their private rent from greenification.[15] The social surplus is larger than the private surplus, so that ceteris paribus, there is too little greenification in equilibrium (for $Z > 0$, we get that $1 - e^{-Z\mu_i} < \mu_i Z$). Nevertheless, if Z is close to zero, this difference is negligible: $1 - e^{-Z\mu_i} \approx \mu_i Z$, with the difference being second order in Z. Therefore, abstracting from $\frac{1}{1-\beta}$ factor on the RHS of (19), the two Equations (15) and (19) become nearly identical.

15. To see this, suppose there is just one sector, and that the carbon tax is set equal to the social cost of pollution. The model is then equivalent to having a (dirty) technology with productivity $\frac{1}{1+\xi}$ and a clean technology with productivity e^z. Under Cobb-Douglas preferences, the demand curve is $p = 1/y$. With competitive production, the consumer surplus of improving the productivity of a variety from $\frac{1}{1+\xi}$ to e^z is $\int_{\frac{1}{1+\xi}}^{e^z} \frac{1}{y}\,\mathrm{d}y = \ln\left(1+\xi\right)e^z$, which differs from the private rent from such a productivity improvement.

Is it the case, then, that a uniform subsidy β to greenification coupled with a Pigouvian carbon tax is enough to decentralize the optimum in steady-state when the overall advantage of the clean technology is small (i.e. $(1+\xi)\,e^{z}$ is close to 1)? The answer is no, which reveals the fundamental reason why industrial policy is warranted in our set-up: cross-sectoral strategic complementarities in greenification. That is, insufficient greenification in sectors downstream and/or upstream to sector i, reduces private incentives to greenify in sector i (see (19)), typically leading to multiple steady-state equilibria as shown above.[16] Then, starting from initially low levels of greenification in all sectors, the economy may get stuck in a steady-state which also features low levels of greenification, even though the optimum might involve a high level of greenification in all sectors. The following example with Pigouvian taxation on emissions illustrates this point.[17]

Example 1. We consider a two sector supply chain, so $N = 2$. We set $z = 0$ for simplicity and assume that ξ is small. In addition, we assume that greenification is uniformly subsidized at rate β , so that firms only face the greenification cost $(1-\beta)\,\phi$. Under these conditions, both the steady-state equilibrium equations and the first order conditions for the social optimum can be written as:

$$\chi_1 = F_1\left(\frac{\xi\chi_2\,(1-\alpha_2)}{1-\beta}\right) \text{ and } \chi_2 = F_2\left(\frac{\xi\,(\alpha_2+\chi_1\,(1-\alpha_2))}{1-\beta}\right). \tag{20}$$

where F_1 and F_2 are chosen so that the two curves in Figure 1a intersect three times—at A, B, and C—all of which correspond to a steady-state of the decentralized economy. As Figure 1a shows, the two steady-states A and C are both stable, whereas B is unstable. For β sufficiently large, the social optimum will correspond to point C, whereas a decentralized economy starting from initially low or no greenification will end up being stuck at the low greenification steady-state A.

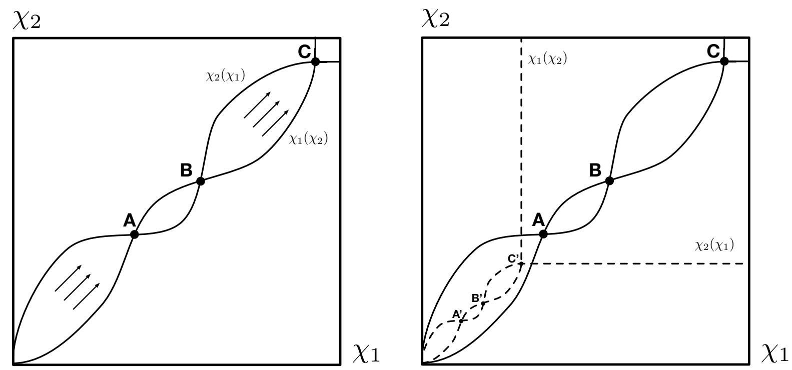
Without the uniform subsidy β, the decentralized steady-states satisfy

$$\chi_1 = F_1\left(\xi\chi_2\,(1-\alpha_2)\right) \text{and} \chi_2 = F_2\left(\xi\,(\alpha_2+\chi_1\,(1-\alpha_2))\right).$$

These two equations correspond to the two dash lines in Figure 1b, which also intersect three times at the steady-state equilibria A', B', and C'. Then an economy with low

16. This also implies that the First-Order Conditions (19) are generally not sufficient for the global optimum.

17. Of course, a sufficiently high carbon tax can remove the multiplicity of steady-states, but implementing such a tax above its Pigouvian value ξ is suboptimal.



(a) Decentralized economy with a subsidy β

(b) Decentralized economy without a subsidy β

Figure 1. Multiple steady-states: First-order conditions of the Social Planner's problem and decentralized economy with and without a subsidy β.

or no initial greenification will end up being stuck at the—even lower greenification—steady-state A'. The uniform subsidy β allows an economy with initially very low or no greenification to converge to A instead of A', but sector specific subsidies are required on top of the uniform subsidy to make the economy converge to C.

Implementing the Social Optimum. Starting from the same initial conditions of low levels of greenification in all sectors, the Social Planner can achieve the optimum through a set of temporary subsidies to greenification—given that the equilibrium, for any given policy and initial conditions, is unique as shown in the previous section. More precisely, we establish (proof in Appendix B.4):

Proposition 4. *The optimal steady-state can be implemented through a carbon price together with a whole set of time-varying sector specific greenification subsidies.*

Note that Proposition 4 is about decentralizing the optimal steady-state but not the full optimal allocation.[18] In the full optimum, users of intermediate inputs should buy newly greenified inputs at marginal costs, which requires the implementation of subsidies for the use of intermediates. However, with such instruments (a Pigouvian carbon tax, sector specific subsidies for greenification, and subsidies to the use of intermediates), the

18. In the proof of Proposition 4, at $t = 1$, the optimal environmental tax and greenification levels are implemented, but newly greenified varieties are priced monopolistically and thus inefficiently under-supplied.

equilibrium will generally not be unique and a Social Planner would have to use the type of instruments analyzed by Sturm (2023) to ensure uniqueness.[19]

Small Subsidies can make a Big Difference. Are large subsidies—big-push policies—always necessary to move the economy away from a low greenification steady-state? Figure 2 describes an example with three steady-state equilibria: no greenification, full greenification, and an interior, unstable steady-state, which is close to the no greenification steady-state. Starting from no greenification, small sector-specific subsidies are enough to move the economy a little beyond the unstable steady-state from which point the economy converges on its own toward the high greenification steady-state. Provided that consumers are sufficiently patient, the full greenification steady-state dominates the other two and corresponds to the optimum. We develop a formal example along these lines in Appendix B.5.

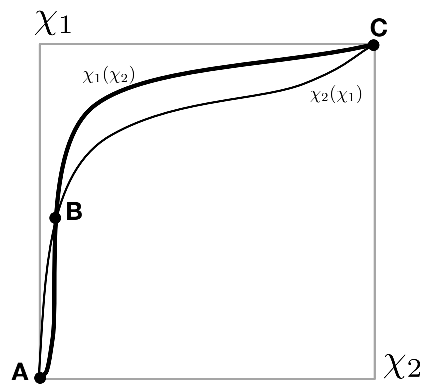


Figure 2. Small subsidies are sufficient to escape no-greenification steady-state

5 Propagation and Appropriate Industrial Policy

Our analysis so far has considered a government that could intervene in several sectors at the same time, since implementing the optimum generically requires such a multi-faceted intervention. In practice, however, it may be that a government is constrained to focus on one or a few key sectors at a time. What can be achieved in that case? We first analyze how a targeted policy may or may not get the economy out of a no-greenification trap.

19. The logic extends to the case where carbon pricing is enforced through a cap-and-trade system instead of a tax. As long as there are multiple steady-states, sector specific subsidies to greenification remain necessary, on top of cap-and-trade, to ensure that the economy converges toward the optimal steady-state.

Second, we focus on marginal changes in greenification around a steady-state and argue that a marginal increase in greenification in one sector affects greenification incentives in other sectors more significantly the more downstream the targeted sector. Finally, we extend that analysis to more general networks.

5.1 Escaping the No Greenification Trap

If an economy is trapped in a no-greenification steady-state, which sector should a government prioritize? To answer this question, we consider a special case of a supply chain with $N \geq 3$ sectors in Appendix B.6, and suppose that the basic parameters and the F_i's are such that no greenification in all sectors is a steady-state. We assume that initially the economy is stuck in this no greenification steady-state and that the government can directly greenify a positive mass of varieties in one sector only. Then, we show that, provided that greenification costs are not too high, greenification starts propagating upstream immediately if the government targets the most downstream sector N as the demand channel operates contemporaneously. If instead the government targets sector $N-1$, then greenification start propagating with a one-period delay: through the input cost channel it propagates next period to the most downstream sector and then through the demand channel it propagates immediately from that sector to all upstream sectors (again for sufficiently low greenification costs). However, should the government target sectors that are more upstream than $N-1$, then the input cost channel never reaches the most downstream sector so that greenification never propagates.

5.2 Targeted Marginal Interventions around a Steady-State

We now focus on a stable steady-state equilibrium and consider the effect of a marginal intervention. In presence of a (now permanent) subsidy to greenification q_i in sector i, a steady-state equilibrium satisfies:

$$\chi_i = F_i\left(\frac{\pi_i}{1-q_i}\right) \text{ with } \pi_i = \underbrace{\left(1-e^{-\mu_i Z}\right)}_{\text{input cost reduction}} \underbrace{\prod_{j=i+1}^{N} \chi_j\left(1-\alpha_j\right)}_{\text{demand from downstream}},$$

$$\text{with } \mu_1 = 1, \quad \mu_i = \alpha_i + \chi_{i-1}\mu_{i-1}(1-\alpha_i) \text{ for } i \in \{2, ...N\}.$$

While greenification incentives can propagate in both directions (downstream through the input cost channel and upstream through a demand effect), the effect is asymmetric:

the same perturbation in the greenification rate χ_k in sector k tends to generate more greenification incentives in upstream sectors $i < k$ through the demand channel than in downstream sectors ($i > k$) through the input cost channel—in fact, the latter is close to zero in a steady-state with low greenification rates. In other words, starting from a steady-state with low greenification across all sectors, the bang-for-the-buck is generically higher for policy that induces greenification first in more downstream sectors.

To see this more formally, note that the elasticity of greenification in sector i to a subsidy in sector k can be decomposed into

$$\frac{\partial \ln \chi_i}{\partial \ln q_k} = \frac{\partial \ln \chi_i}{\partial \ln \pi_i} \frac{\partial \ln \pi_i}{\partial \ln \chi_k} \frac{\partial \ln \chi_k}{\partial \ln q_k}.$$

The first and third terms on the right depend on the cost distribution of sectors i and k respectively. In contrast, the middle term, representing the elasticity of greenification incentives in i (i.e. profits) with respect to greenification in k, depends only on network structure and is therefore the object of our attention. In Appendix B.7, we prove:

Proposition 5. *i) An increase in greenification downstream raises greenification incentive one-for-one:* $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = 1$ *if* $k > i$. *ii) An increase in greenification upstream raises incentive less-than-one-for-one*

$$\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = \frac{\mu_i Z e^{-\mu_i Z}}{1 - e^{-\mu_i Z}} \frac{\mu_k \prod_{j=1}^{i-k} \chi_{i-j} \left(1 - \alpha_{i-j+1}\right)}{\mu_i} < 1 \text{ if } k < i. \tag{21}$$

iii) Incentives propagated from upstream relies on greenification along the chain: $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} \to 0$ *if* $\chi_j \to 0$ *for any* $j \in [k, i)$.

Part i) establishes that exogenously more greenification in downstream sectors generates proportional gains in greenification incentives in all upstream sectors. This is because greenification downstream increases the market for greenified varieties upstream 1 for 1.. In contrast, Part ii) establishes that exogenously more greenification upstream generates less than proportional gains in greenification incentives in downstream sectors, and the incentives gets smaller and smaller the further downstream we go. This is because both $\frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} < 1$ and $\frac{\mu_k \prod_{j=1}^{i-k} \chi_{i-j}(1-\alpha_{i-j+1})}{\mu_i} < 1$. The latter expression is the share of greenified content coming from sector k in the greenified content of sector i. It decreases as k is further upstream from i (i.e for smaller k) and the presence of the term $\prod_{j=1}^{i-k} \chi_{i-j}$ shows that the downstream propagation of incentives can be subject to bottlenecks and weak links. Intuitively, greenification in upstream sector k reduces the cost of greenified varieties in downstream sector $i > k$ but less than proportionately because at each stage of the production process, the cost of producing a given intermediate depends on the costs of the more upstream intermediates—in proportion to how green the supply chain from k to i is—as well as on labor costs.

The contrast in how the incentives propagate is particularly stark when at least one sector features low greenification, as the additional downstream incentives resulting from upstream greenification can become arbitrarily small. To see this, note that μ_i (the network-adjusted share of greenified content for producing a greenified variety in sector i) is bounded away from 0 so that for any sector k upstream to i ($k < i$), the additional incentive in sector i generated by greenification in sector k, $\frac{\partial \ln \pi_i}{\partial \ln \chi_k}$, goes to 0 whenever one of the χ_j tends to 0 for any j in between k and i. In other words, the downstream propagation of greenification incentives from k to i is only as strong as the weakest link along the chain between k and i. Therefore, starting from a steady-state with low greenification in at least some sectors ($\chi_i \approx 0$ for some i), the downstream propagation of incentives from targeting upstream sectors is close to zero, and industrial policy should first target downstream sectors.[20]

Our intuitive reasoning in this subsection is incomplete in two respects. First, it considers the change in incentives in sector i from perturbing χ_k, holding all other χ_j's constant. However, in equilibrium, χ_j would respond for all j, generating further changes in incentives. Second, it focuses on perturbations in a steady-state, while ignoring the sequence of changes in incentives along the transition. But these intuitions continue to hold when the path of χ_{jt}'s respond to the exogenous shock in χ_k as well. We shall now focus on a special case where a more formal statement can be made.[21]

5.3 Generalizations

How general is the intuition that a Social Planner should prioritize the greenification of downstream sectors? In this subsection, we extend our setup beyond a vertical supply chain, but still assume that greenification innovations can only occur in sectors providing inputs for the clean production process.

20. This argument requires the mild condition that the relative marginal costs of greenification across sectors are bounded when the levels of greenification is low.

21. Here, we are examining the propagation of greenification, but given that the end goal of green industrial policy is to reduce carbon emissions, we consider in Section 6 how changes in greenification in different sectors affects equilibrium emissions.

Two Downstream Sectors. One argument to target upstream instead of downstream sectors is that a single upstream sector may produce inputs for several downstream sectors. To allow for this, we now consider a simple network with 2 layers ($N = 2$) but two downstream sectors $2a$ and $2b$. For simplicity, we assume that these two sectors have the same labor share α. We assume that consumption is Cobb-Douglas between the two sectors with consumption shares denoted λ_a and λ_b. The revenues of the two downstream sectors are then given by $r_{2a} = \lambda_a$ and $r_{2b} = \lambda_b$, while the revenues of the upstream sector in steady-state are given by:

$$r_1 = (1-\alpha)\left(\lambda_a \chi_{2a} + \lambda_b \chi_{2b}\right).$$

Following the same steps as in the baseline model, we obtain that a steady-state is characterized by $\chi_i = F_i(\pi_i)$ for $i \in \{1, 2a, 2b\}$, where steady-state profits are given by:

$$\pi_1 = \left(1 - e^{-Z}\right) r_1, \ \pi_{2k} = \left(1 - e^{-Z\mu_2}\right) \lambda_k \text{ for } k \in \{a, b\} \text{ with } \mu_2 = \alpha + (1-\alpha)\chi_1.$$

We can then immediately derive how (in steady-state) greenification incentives in one sector directly depend on the level of greenification in other sectors as:

$$\frac{\partial \ln \pi_1}{\partial \ln \chi_{2k}} = \frac{\chi_{2k}\lambda_k}{\chi_{2a}\lambda_a + \chi_{2b}\lambda_b} \text{ for k} \in \{a, b\} \text{ and } \frac{\partial \ln \pi_{2k}}{\partial \ln \chi_1} = \frac{\mu_2 Z e^{-\mu_2 Z}}{1 - e^{-\mu_2 Z}} \frac{(1-\alpha)\chi_1}{\alpha + (1-\alpha)\chi_1}. \tag{22}$$

Upstream propagation of greenification incentives from sectors $2a$ and $2b$ to sector 1 now occurs in proportion to the importance of each of the two downstream sectors for sector 1's output, whereas the expression for downstream propagation of greenification remains the same as with a single downstream sector.

Even though the elasticities for downstream propagation are no longer always smaller than those for upstream propagation, it is still true that for low levels of greenification, it is better to target downstream sectors. To see this, note that as $\chi_1 \to 0$ we have that $\frac{\partial \ln \pi_{2k}}{\partial \ln \chi_1} \to 0$, so it is always strictly better to first target at least one of the two downstream sectors. In contrast, even if $\chi_{2a}, \chi_{2b} \to 0$, it is still the case that $\frac{\partial \ln \pi_1}{\partial \ln \chi_{2k}} \nrightarrow 0$ for at least one of $k = a, b$—and it is true for both $k = a, b$ if the ratio χ_{2a}/χ_{2b} stays bounded and above zero in the limit—so greenification still propagates upstream.

Empirically, electricity is one example where one might expect stronger potential for downstream propagation. Electricity is an input in just about everything, its greenification share is non-negligible, and many of its downstream applications have long since replaced their fossil fuel analogs. For example, the economy has already completely transitioned from oil lanterns to electric lighting, which maps to a high downstream χ_j in our model. To explore this issue further, Appendix C.3 presents two stylized quantifications of Equation (22) for the US economy with electricity generation as the upstream sector and downstream sectors based on either broad sectoral categories (industry, commercial, residential, transportation) or the manufacturing sector (durable, nondurable). While we find that the downstream elasticity *can* indeed be larger in this setting, this is generally not the case despite the high greenification shares observed in this setting.

General Network Structure. We now analyze the propagation of greenification incentives under a general network structure. Consider production functions for varieties in sectors $i, j = 1, \ldots, N$:

$$y_{it}(\nu) = \ell_{dit}(\nu) + \gamma_{it}(\nu)\left(\frac{e^z \ell_{cit}(\nu)}{\alpha_i}\right)^{\alpha_i} \prod_j \left(\frac{m_{ijt}(\nu)}{\sigma_{ij}}\right)^{\sigma_{ij}},$$

where $\sigma_{ij} \in [0, 1)$ is the elasticity of input j in the production of clean varieties of good i, so $\sum_j \sigma_{ij} + \alpha_i = 1$. The network structure encapsulated in these elasticities may take a general form. For expositional simplicity, we assume the last sector N is a final goods sector that supplies only to the consumer ($\sigma_{iN} = 0\ \forall i$) and that the consumer purchases only from sector N. The market clearing condition is

$$y_{jt} = \sum_{i=1}^{N} \int_0^1 m_{ijt}(\nu)\, \mathrm{d}\nu \qquad \text{for } j < N,$$

whereas market clearing for the final good N is $c_t = y_{Nt}$.[22]

We now derive, in a steady-state of this environment, the effect of marginal greenification in each sector j on the profits of a newly greenified variety in sector i. The price of a variety in sector i is

$$p_{it}(\nu) = \begin{cases} e^{-\alpha_i z} \prod_j p_{jt}^{\sigma_{ij}} & \text{if the variety has been greenified by time } t-1 \\ 1+\tau & \text{otherwise} \end{cases}$$

where p_{jt} (without the variety index) is the price of the sectoral bundle. As before we denote by μ_{it} the greenified content for the production of a clean variety of good i. We

22. The equilibrium remains unique if the network is acyclic (meaning that if inputs flow from sector i to j, directly or indirectly, then inputs do not flow from j to i). Liu and Tsyvinski (2024), show that the US input-output matrix is largely acyclical.

then get the updated recursive formulation

$$\mu_{it} = \alpha_i + \sum_{j=1}^{N} \sigma_{ij} \chi_{jt} \mu_{jt}. \tag{23}$$

With this updated expression for μ_{it}, sectoral prices are still given by equation (7).

Following the same logic as in the baseline model, sectoral revenues are given by

$$r_{jt} = \sum_{i=1}^{N} \sigma_{ij} \tilde{\chi}_{it} r_{it}, \tag{24}$$

with $\tilde{\chi}_{it}$ still defined as in Equation (8). The profit of a newly greenified variety is the product of profit margins and revenues:

$$\pi_{it} = \left(1 - \frac{e^{-\alpha_i z}}{1+\tau} \prod_j p_{jt}^{\sigma_{ij}}\right) r_{it} = \left(1 - e^{-\mu_{i,t-1} Z}\right) r_{it},$$

with the revenue in the final sector being $r_N = 1$, exactly as in the baseline case.

We can now focus on a steady-state. We can derive the marginal impact of additional greenification in sector j on the profit of a newly greenified variety in sector i as

$$\frac{\partial \ln \pi_i}{\partial \ln \chi_j} = \frac{Z e^{-\mu_i Z}}{1 - e^{-\mu_i Z}} \frac{\mathrm{d}\mu_i}{\mathrm{d} \ln \chi_j} + \frac{\mathrm{d} \ln r_i}{\mathrm{d} \ln \chi_j}.$$

The first term captures the effect of marginal greenification in sector j on the profit margin, while the second term captures the impact on revenue.

We now solve for the two channels as a function of the production network. Define $\hat{\mathbf{\Sigma}} \equiv [\sigma_{ij} \chi_j]$, and let $Diag(\boldsymbol{\mu})$ denote the diagonal matrix with entries $\{\mu_i\}$. In matrix notation, Equation (23) becomes

$$\boldsymbol{\mu} = \left(\boldsymbol{I} - \hat{\mathbf{\Sigma}}\right)^{-1} \boldsymbol{\alpha}.$$

Totally differentiating with respect to χ_j, we can solve for the matrix $\mathbf{\Gamma} \equiv \left[\frac{\mathrm{d}\mu_i}{\mathrm{d} \ln \chi_j}\right]$ as

$$\mathbf{\Gamma} = \hat{\mathbf{\Sigma}} \left(\boldsymbol{I} - \hat{\mathbf{\Sigma}}\right)^{-1} Diag(\boldsymbol{\mu}).$$

To analyze the network impact of greenification on revenue, let $\mathbf{\Omega}$ be the matrix whose ji-th entry $\frac{\sigma_{ij} r_i \chi_i}{r_j}$ describes the fraction of revenue sector j derives from selling to sector i in steady-state. Totally differentiating (24), we have

$$\mathrm{d} \ln r_j = \sum_{i=1}^{N} \frac{\sigma_{ij} \chi_i r_i}{r_j} \left(\mathrm{d} \ln \chi_i + \mathrm{d} \ln r_i \right),$$

or in vector form,

$$\mathrm{d}\ln \boldsymbol{r} \;=\; (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\,\boldsymbol{\Omega}\,\mathrm{d}\ln \boldsymbol{\chi}.$$

We are now ready to derive the network impact of greenification on profits. The matrix $\boldsymbol{\Psi} \equiv \left[\frac{\partial \ln \pi_i}{\partial \ln \chi_j}\right]$ is given by

$$\boldsymbol{\Psi} \;=\; \underbrace{Diag\left(\frac{Ze^{-\mu_i Z}}{1-e^{-\mu_i Z}}\right)\left(\hat{\boldsymbol{\Sigma}}+\hat{\boldsymbol{\Sigma}}^2+\cdots\right)Diag\left(\boldsymbol{\mu}\right)}_{\text{input cost channel}}+\underbrace{\left(\boldsymbol{\Omega}+\boldsymbol{\Omega}^2+\cdots\right)}_{\text{market size channel}}.$$

The matrix power series $\hat{\boldsymbol{\Sigma}}+\hat{\boldsymbol{\Sigma}}^2+\cdots$ and $\boldsymbol{\Omega}+\boldsymbol{\Omega}^2+\cdots$ capture the network transmission of greenification in one sector j on the profits, and thus the incentives to greenify, in another sector i, respectively through the input cost channel and the market size channel. The first power is the direct effect between sector pairs i and j; each successive term captures indirect transmission through an additional layer of input-output linkages.

We make three observations. First, as in our baseline model, the input cost channel transmits *forward*, from input supplier to input user. This is evident as the effect transmits through the power series of the matrix $\hat{\boldsymbol{\Sigma}}$, the ij-th entry of which scales with σ_{ij}, the cost share sector i's clean production on input j. The diagonal matrix $Diag\left(\boldsymbol{\mu}\right)$ to the right of the matrix power series demonstrates that the effect is stronger if the clean share of value-added is higher in the input supplying sector j. By contrast, the market size channel transmits *backwards*, from input user to input supplier. This is evident as the effect transmits through the power series of the matrix $\boldsymbol{\Omega}$, the ij-th entry of which is $\frac{\sigma_{ji}\chi_j r_j}{r_i}$, capturing the equilibrium fraction of output from sector i that is sent to sector j as inputs.

Second, for a focal intermediate producer i, the sum of input cost effects from all of its suppliers is always less than one ($\sum_j \sigma_{ij}\chi_j \leq 1-\alpha_i < 1$), whereas by the market clearing condition (24), the sum of the market size effects is always equal to one ($\sum_i \frac{\sigma_{ij}\chi_i r_i}{r_j} = 1$). Moreover, as the input cost effects scale with the degree of greenification in the input-supplying sector χ_j, in an economy where there is little greenification across all sectors, the total network effect of the input cost channel coming from marginal greenification in all of sector i's suppliers can become arbitrarily small relative to even the first round of direct market size effects coming from each of the buyers of sector i's inputs. Formally, denote the degree of greenification in an original steady-state as $\chi_i = \epsilon x_i$ for all sectors

i, with $x_i > 0$. In the limit as $\epsilon \to 0$, for each sector i and j such that $\sigma_{ij} > 0$, we get

$$\lim_{\epsilon \to 0} [\boldsymbol{\Omega}]_{ij} > \lim_{\epsilon \to 0} \sum_{k=1}^{N} \left[\hat{\boldsymbol{\Sigma}} \left(\boldsymbol{I} - \hat{\boldsymbol{\Sigma}} \right)^{-1} \right]_{ik} = 0.$$

Our third observation is a corollary of the second. Continue to denote the degree of greenification in an original steady-state as $\chi_i = \epsilon x_i$. If the network is acyclic, then for a sector j upstream to i (i.e. $\left[\Sigma^k\right]_{ij} > 0$ for some $k \in \mathbb{N}^*$), as $\epsilon \to 0$, the asymmetric propagation of incentives to greenify imply that $\frac{\mathrm{d} \ln \pi_i}{\mathrm{d} \ln \chi_j} \leq \frac{\mathrm{d} \ln \pi_j}{\mathrm{d} \ln \chi_i}$.

We conclude that our insights from the baseline model of a vertical supply chain continue to hold in this general network environment. Greenification is strategically complementary across sectors, through an input cost channel which propagates from input-suppliers to input-buyers as well as a market size channel which propagates from input-buyers to input-suppliers. Moreover, when greenification is low across the economy, the market size channel dominates the input cost channel, so that in an acyclic network, targeting downstream sectors creates more incentives across the economy to greenify.

6 Industrial Policy with Incomplete Carbon Prices

We have so far assumed Pigouvian carbon prices. Unfortunately, global carbon prices are far below the true social cost of carbon, and some countries, notably the US, are explicitly trying to use industrial policy as a substitute for a carbon price, rather than as a supplement. For this reason, we now relax the assumption of Pigouvian carbon prices and consider optimal industrial policy in the face of suboptimal carbon prices.

6.1 Planner's Problem

First, we consider a version of the Planner's problem where the carbon price is restricted. That is, the Planner has a complete set of greenification subsidies, but due to some political restriction, the carbon price τ is set to an exogenous level below the true social cost ξ.[23] In this case, the policy problem requires selecting a level of greenification knowing that production decisions will be made in a distorted equilibrium. To keep the analysis otherwise comparable to that of Section 4, we allow the Planner to remove the monopoly distortion, and as before, all greenification will happen in the initial period, allowing us to drop time subscripts from the problem. To consider the equilibrium relationship between the allo-

23. We retain the assumption that $1+\tau > e^{-z}$ so that there is an incentive to greenify in the equilibrium.

cation of labor and the level of greenification, we define functions $\{\ell_{di}(\{\chi_j\}), \ell_{ci}(\{\chi_j\})\}$:

$$\big\{\ell_{di}\big(\{\chi_j\}\big), \ell_{ci}\big(\{\chi_j\}\big)\big\} = \text{argmax}\Big[p_N y_N - \sum_{i=1}^{N}\Big((1+\tau)\ell_{di} + \ell_{ci}\Big)\Big], \tag{25}$$

where the final good price p_N is set according to the household condition (3). Equation (25) stipulates that the allocation of labor must be consistent with profit maximization, and because the economy is production efficient, it is as if there is a single, price-taking firm solving the above problem. Note that this firm must pay the numeraire wage for labor as well as the carbon price of τ for dirty labor.

The Planner then maximizes household utility with industrial policy that anticipates this equilibrium relationship. They solve

$$\max_{\{\chi_j\}} \frac{1}{1-\beta}\Big(\ln(y_N) - (1+\xi)\sum_{i=1}^{N}\ell_{di} - \sum_{i=1}^{N}\ell_{ci}\Big) - \sum_{i=1}^{N}\mathcal{F}_i(\chi_i). \tag{26}$$

Optimal industrial policy in this second-best setting is characterized as follows (proof in Appendix B.8):

Proposition 6. *In the absence of a Pigouvian carbon price, optimal greenification satisfies*

$$\chi_i = F_i\left(\frac{\ln((1+\tau)e^z)}{1-\beta}\mu_i \prod_{j=i+1}^{N}\left[\chi_j(1-\alpha_j)\right] + \frac{\xi-\tau}{1-\beta}\Big(-\frac{\partial \ell_d}{\partial \chi_i}\Big)\right). \tag{27}$$

Proposition 6 shows how to adjust industrial policy in the face of suboptimal carbon prices. The first term mirrors that of Equation (19) in reflecting the direct, perpetual benefit of greenification through expanding production. The only difference is that equilibrium output is determined by the carbon price τ, rather than ξ. The second term reflects the wedge between the carbon price and its true social cost, and it pushes industrial policy to consider how greenification influences the equilibrium level of emissions. Now, greenification in a given sector should be higher in so far as it generates a larger reduction in equilibrium emissions (i.e. a more negative $\frac{\partial \ell_d}{\partial \chi_i}$), and the emphasis on emission reductions should scale with the carbon price wedge. Naturally, we recover the first-best whenever carbon prices are Pigouvian.

6.2 Equilibrium Emissions

Proposition 6 raises a natural question similar to that considered in Section 5: where along the supply chain will greenification most effectively reduce emissions? Interestingly, it is again downstream greenification that generates the greatest equilibrium response when the level of greenification elsewhere is low.[24]

To derive equilibrium emissions, recall that sector i's spending on labor in dirty production is equal to the share of non-greenified intermediates $1 - \chi_i$. Summing across sectors and taking into account that the overall price of labor in dirty production is $1 + \tau$, we get that in equilibrium emissions and labor in dirty production are both given by:

$$\ell_d = \frac{1}{1+\tau} \sum_{i=1}^{N} r_i (1 - \chi_i).$$

Differentiating, we have that emission reductions follow

$$-\frac{\partial \ell_d}{\partial \chi_i} = \frac{1}{1+\tau} r_i \left[1 - (1 - \alpha_i) \sum_{j=1}^{i-1} \prod_{k=j+1}^{i-1} \chi_k (1 - \alpha_k)(1 - \chi_j) \right]. \tag{28}$$

Therefore, the impact of greenification on emissions depends on the level of greenification both downstream and upstream. The market size of a sector, r_i, is determined by downstream demand, so upstream greenification will be less effective at reducing emissions when downstream sectors are less greenified. The term in the square brackets represents the direct displacement of dirty production—the one—minus the offsetting increase in emissions coming from increased demand for dirty inputs in all upstream sectors. Since $\alpha_j > 0 \ \forall j$, this offsetting term must be less than one, guaranteeing that emissions decrease with greenification (as long as $r_i \neq 0$).[25]

In the previous sections, we have argued that there is strategic complementarity in the adoption of greenification, and now we can see that there is a complementarity in emission reductions: greenification in each sector is more effective when the others are

24. Downstream's greater emission reductions hold in addition to downstream's greater propagation of greenification because in this section we will hold greenification elsewhere constant.

25. To see this, note that $(1 - \alpha_i) \sum_{j=1}^{i-1} \prod_{k=j+1}^{i-1} \chi_k (1 - \alpha_k)(1 - \chi_j) < \sum_{j=1}^{i-1} \prod_{k=j+1}^{i-1} \chi_k (1 - \chi_j) \leq \sum_{j=1}^{i-1} \prod_{k=j+1}^{i-1} \chi_k (1 - \chi_j) + \prod_{j=1}^{i-1} \chi_j = 1$. The final equality is because one can interpret the preceding sum as a sum of probabilities representing the number of coin flips before achieving a tails, when one is flipping $i - 1$ many coins with heads probability χ_j.

more greenified: formally, $-\frac{\partial^2 \ell_d}{\partial \chi_i \partial \chi_k} \geq 0$.[26] As before, this complementarity implies that, for low levels of greenification, targeting the downstream sector will be more effective. To see this, consider again the case where $\chi_j = \epsilon x_j$. Then for any $i < j$, we have that

$$\lim_{\epsilon \to 0} \frac{-\partial \ell_d / \partial \chi_i}{-\partial \ell_d / \partial \chi_j} = \lim_{\epsilon \to 0} \prod_{k=i+1}^{j} \chi_k (1-\alpha_k) \left[\frac{1-(1-\alpha_i)}{1-(1-\alpha_j)} \right] = 0.$$

When greenification goes to zero, the more downstream sector j achieves greater emission reductions. Intuitively, emission reductions for a given sector go to zero as greenification in its downstream sectors goes to zero, and the rate of reduction is of the same order as the number of downstream sectors. However, input shares less than one imply that emission reductions will not go to zero as greenification in upstream sectors goes to zero. Thus, the effectiveness of greenification at reducing emissions is more "vulnerable" to low greenification downstream, rather than upstream. Hence, for low levels of greenification, policymakers should focus on downstream to reduce emissions.[27] In Appendix B.9 we extend the above analysis and conclusions to more general supply chain networks as in Section 5.3, while in the next subsection we consider a simple example.

6.3 Two Sector Example

The relative effectiveness of downstream greenification is well-illustrated in the special case of a two-layer supply chain, with only an upstream and a downstream sector. In that

26. This is trivial when $j > i$. For $j < i$,we differentiate (28) and obtain

$$\begin{aligned} -\frac{\partial^2 \ell_d}{\partial \chi_i \partial \chi_j} &= \frac{r_i(1-\alpha_i)}{1+\tau} \left(\prod_{k=j+1}^{i-1} \chi_k (1-\alpha_k) - \sum_{l=1}^{j-1} \prod_{k=l+1}^{i-1} \frac{\chi_k (1-\alpha_k)(1-\chi_l)}{\chi_j} \right) \\ &= \frac{r_i(1-\alpha_i)}{(1+\tau)} \left(\prod_{k=j+1}^{i-1} \chi_k (1-\alpha_k) \right) \left(1 - (1-\alpha_j) \sum_{l=1}^{j-1} \prod_{k=l+1}^{j-1} \chi_k (1-\alpha_k)(1-\chi_l) \right), \end{aligned}$$

which is weakly positive for the same reason as $-\frac{\partial \ell_d}{\partial \chi_i} \geq 0$.

27. Input shares also play a role in the sectoral heterogeneity of emission reductions. Low input shares downstream proportionally dampen emission reductions independent of the level of greenification elsewhere, but low input shares upstream—as well as in one's own sector—enhance emission reductions by reducing the offsetting effect. Therefore, an economy characterized by low input shares throughout the supply chain will favor downstream greenification as these low input shares penalize upstream sectors by reducing their market size but reward downstream sectors by reducing their total demand for dirty inputs.

case, Equation (28) implies

$$-\frac{\partial \ell_d}{\partial \chi_2} = \frac{1-(1-\alpha_2)(1-\chi_1)}{1+\tau} \text{ and } -\frac{\partial \ell_d}{\partial \chi_1} = \frac{\chi_2(1-\alpha_2)}{1+\tau}.$$

Downstream greenification always reduces emissions, but upstream greenification only reduces emissions as long as downstream is already greenified ($\chi_2 > 0$). While greenification in each sector is more effective when the other one is more greenified, the upstream sector is more "vulnerable" to low greenification of its counterpart. If nobody buys electric vehicles (EVs), greenifying the production of EV batteries has little effect on emissions.

We can better understand this asymmetry by sketching the line where the two sectors are equally effective at reducing emissions:

$$\chi_2 = \frac{\alpha_2}{1-\alpha_2} + \chi_1.$$

When χ_2 is greater than this line, upstream greenification is more effective at reducing emissions, while downstream is more effective otherwise. The slope is always one, but the intercept is decreasing in the downstream input share, as a larger input share strengthens the offsetting effect for downstream, while raising the market size for upstream. In the extreme case of an input share equal to one, the situation is symmetric and greenifying the lagging sector is more effective. But as the input share shrinks below one, the space where upstream greenification is better at reducing emissions shrinks. Once the input share reaches one half, downstream greenification is always more effective (for any χ).

7 Horizontal Misallocation

In Section 4, we showed that industrial policy can bring large welfare gains. Yet, misdirected green industrial policies can also bring welfare losses if it "picks the wrong winner". In this section, we entertain the possibility of "horizontal" misallocations of public greenification investment. More specifically, we consider an extended version of our basic model with two layers, where on top of using labor, the downstream dirty industrial process also uses inputs from an upstream sector that can be greenified.[28] In that context, greenification in the two upstream sectors are strategic substitutes, and over-investing in the greenification of the upstream sector associated with the dirty downstream process may derail the overall transition towards a clean economy. This reinforces the argument that (if limited) industrial policy should prioritize downstream sectors.

28. For instance, fossil fuel engines can be produced in a cleaner way or become more fuel efficient.

7.1 A Simple Two-Legs Model

We consider a simple two-layer network, with a single downstream sector and two upstream sectors denoted by $1a$ and $1b$. The downstream sector uses input $1a$ when producing using the dirty technology and input $1b$ when producing using the clean technology. More formally, a non-greenified downstream variety is produced according to

$$y_2(v) = \left(\frac{l_{2t}^d(v)}{\alpha}\right)^{\alpha}\left(\frac{m_{1at}(\nu)}{1-\alpha}\right)^{1-\alpha}$$

whereas a greenified downstream variety is produced according to

$$y_2(v) = \left(\frac{l_{2t}^d(v)}{\alpha}\right)^{\alpha}\left(\frac{m_{1at}(\nu)}{1-\alpha}\right)^{1-\alpha} + \left(\frac{e^z l_{2t}^c(v)}{\alpha}\right)^{\alpha}\left(\frac{m_{1bt}(\nu)}{1-\alpha}\right)^{1-\alpha}$$

Varieties in the upstream sectors are produced according to

$$y_{1j}(v) = \begin{cases} l_{1j}^d(v) & \text{if not greenified} \\ l_{1j}^d(v) + e^z l_{1j}^c(v) & \text{if greenified} \end{cases} \quad \text{for } j \in \{a, b\}.$$

Finally, greenification involves the fixed cost distributions F_2, F_{1a}, and F_{1b}.[29]

7.2 Equilibrium

We solve for the equilibrium following similar steps to Section 3.

Prices. As before, in the upstream sectors, the price of a variety greenified by the previous period is $p_{1jt}(\nu) = e^{-z}$ for $j \in \{a, b\}$, whereas the price of a newly greenified or non-greenified variety is $p_{1jt}(\nu) = 1 + \tau_t$. We then get that in the downstream sector, the price of a variety is given by

$$p_{2t}(\nu) = \begin{cases} \min\left[(1+\tau)^{\alpha} p_{1a,t}^{1-\alpha}, e^{-\alpha z} p_{1b,t}^{1-\alpha}\right] & \text{if the variety has been greenified by time } t-1, \\ (1+\tau)^{\alpha} p_{1a,t}^{1-\alpha} & \text{otherwise.} \end{cases} \tag{29}$$

We still assume that $Z = z + \ln(1+\tau) > 0$, so the upstream sectors use the clean production process when greenified. Therefore, the price of the upstream input $1j$ at date

29. A generalization would involve the clean and dirty industrial processes using both upstream goods with Cobb-Douglas shares: σ_a^d and σ_b^d denoting respectively the use of sector $1a$ and $1b$ in the dirty downstream process as well as σ_a^c and σ_b^c denoting the corresponding shares for the clean process. Then the case with $\sigma_a^d = \sigma_b^d = 0$ corresponds to the original model and its results based on strategic complementarity extend directly for $\sigma_a^d < \sigma_a^c$ and $\sigma_b^d < \sigma_b^c$. If $\sigma_a^d = \sigma_a^c$ and $\sigma_b^d = \sigma_b^c$, then there is no strategic complementarity. If $\sigma_a^d > \sigma_a^c$ and $\sigma_b^d < \sigma_b^c$ (as here), there is strategic substitutability.

t follows the same formula as in the baseline model, namely: $p_{1jt} = (1+\tau)\, e^{-Z\chi_{1j,t-1}}$.

The assumption $Z > 0$ is no longer sufficient to ensure that the downstream sector always uses the clean production process since the dirty input may be cheaper than the clean one if it is itself sufficiently greenified. Instead, the downstream sector uses the clean production process if and only if

$$e^{-\alpha z} p_{1b,t}^{1-\alpha} < (1+\tau)^{\alpha}\, p_{1a,t}^{1-\alpha},$$

or equivalently (when $Z > 0$) if and only if $\mu_{2,t-1} > 0$, where we adjust the definition of $\mu_{2,t}$ relative to the baseline model:

$$\mu_{2,t-1} \equiv \alpha + (\chi_{1b,t-1} - \chi_{1a,t-1})\,(1-\alpha)\,. \tag{30}$$

For expositional purposes, we let $\alpha > 1/2$ so the above condition always holds.

Equilibrium Revenues and Profits. As in our baseline model, the profit margin is equal to $1 - e^{-Z}$ in the upstream sectors $1a$ and $1b$. In the downstream sector 2, the profit margin is given by

$$1 - \frac{e^{-\alpha z} p_{1b,t}^{1-\alpha}}{(1+\tau_t)^{\alpha}\, p_{1a,t}^{1-\alpha}} = 1 - e^{-Z\mu_{2,t-1}},$$

which is positive since by assumption, we always have $\mu_{2,t} > 0$ for all t. This is again the same expression as in the baseline model.

We now derive the revenue accruing to each variety. In the downstream sector 2, revenues obey, as before, $r_{2,t} = p_t c_t = 1$. Revenues $r_{1b,t}$ for the upstream sector $1b$ follow exactly the same logic as the upstream sector in the baseline model:

$$r_{1b,t} = \underbrace{\chi_{2,t-1} r_{2,t}\,(1-\alpha)}_{\text{sales to previously greenified varieties}} + \underbrace{(\chi_{2,t} - \chi_{2,t-1})\, r_{2,t} e^{-Z\mu_{2,t-1}}\,(1-\alpha)}_{\text{sales to newly greenified varieties}} = \widetilde{\chi}_{2,t}\,(1-\alpha)\,,$$

again with $\widetilde{\chi}_{2,t}$ defined by ((8)). Consider now the upstream sector $1a$: all the revenues accruing to greenified varieties in that sector come from still non-greenified varieties in sector 2, so that

$$r_{1at} = (1-\chi_{2,t})\, r_{2,t}\,(1-\alpha) = (1-\chi_{2,t})\,(1-\alpha)\,.$$

The greenification rents in the three sectors $2, 1a, 1b$ are then given by the profit margins times the revenues; namely, they follow (10) for $i \in \{2, 1a, 1b\}$,with $\mu_{1a,t} = \mu_{1b,t} = 1$ as in the baseline model.

Greenification Levels. As in the baseline model, the producer of a variety in sector i greenifies at time t if and only if the rents from greenification π_{it} are bigger than the fixed cost of greenification. We obtain the equilibrium conditions for greenification:[30]

$$\chi_{1a,t} = F_{1a}\left(\left(1-e^{-Z}\right)\left(1-\chi_{2,t}\right)(1-\alpha)\right) \text{ and } \chi_{1b,t} = F_{1b}\left(\left(1-e^{-Z}\right)\widetilde{\chi}_{2,t}\left(1-\alpha\right)\right) \tag{31}$$

$$\chi_{2,t} = F_2\left(1-e^{-Z\mu_{2,t-1}}\right). \tag{32}$$

This system is recursive: given that $\mu_{2,t-1}$ is predetermined, ((8)) determines χ_{2t}, from which we uniquely determine χ_{1at} and χ_{1bt} using ((31)). Thus,the equilibrium is still unique given initial greenification shares in the three sectors.

Steady-States. The steady-state greenification levels are now defined by[31]

$$\chi_{1a} = F_{1a}\left(\left(1-e^{-Z}\right)(1-\chi_2)(1-\alpha)\right), \quad \chi_{1b} = F_{1b}\left(\left(1-e^{-Z}\right)\chi_2(1-\alpha)\right), \quad \chi_2 = F_2\left(1-e^{-Z\mu_2}\right),$$

with $\mu_2 = \alpha - (1-\alpha)\chi_{1a} + \chi_{1b}(1-\alpha)$. The first equation establishes that χ_{1a} is a decreasing function of χ_2. The second equation establishes that χ_{1b} is increasing in χ_2, and the third equation establishes that χ_2 is increasing in χ_{1b}, but decreasing in χ_{1a}. Therefore, greenification in $1b$ and 2 are strategic complements, but greenification in $1a$ versus $1b$ and 2 are strategic substitutes. In other words, greenifying sector $1a$ diverts effort away from greenifying sectors 2 and $1b$ since it reduces the cost of the dirty industrial process in the downstream sector compared to the clean industrial process.

The strategic substitutability between $1a$ and 2 plus $1b$ has three important implications: First, an industrial policy that favors $1a$ may backfire and halt the greenification that would have otherwise happened in 2 and $1b$ without government intervention—thereby reducing long-term welfare compared to no intervention (see Appendix B.11). Second, the laissez-faire equilibrium may involve too much greenification of $1a$ compared to the ex-ante optimum. But, given the history dependence in the dynamics of greenification shares over time, delaying greenification in 2 and $1b$ in turn can lead to irreversible long-term consequences (see Appendix B.12). Third, we note that strategic substitutability provides another argument for prioritizing greenification in the downstream sector: An increase

30. Technically, greenification levels can never decrease so they are given by the minimum of the right-hand side and the levels in the previous period exactly as in the baseline model.

31. Again, we focus on steady-states where the three conditions below hold as equalities. The full set of steady-states is larger since it includes all $\{\chi\}$ such that the left-hand side is greater than the right-hand side as greenification cannot be undone.

in greenification in the downstream sector, incentivizes greenification in the "right" upstream sector ($1b$). In contrast, and as just argued, a policy that targets the wrong upstream sector ($1a$) discourages greenification downstream.

8 Numerical Example

We have showed theoretically that coordination issues along the supply chain can generate low greenification traps impeding the green transition. But, is this a real concern in practice? To answer this question, we present a quantitative illustration of our model to long-range and heavy duty (LRHD) transportation, namely trucking, aviation, shipping, and rail, which account for around half of global transport-related CO_2 emissions. We consider "greenification" via a switch to hydrogen (H_2, either in fuel cells or direct combustion), a key technology to help decarbonize LRHD transportation (IPCC 2024).[32] Indeed, hydrogen-based models are emerging or under development for trucks (e.g. Hyundai XCIENT, Nikola), trains (e.g. Alstom Coradia iLint, Siemens Mireo Plus H), and passenger aircraft (e.g. Airbus ZEROe, Embraer Energia), though their market share remains close to zero. Importantly, hydrogen can itself be produced using either fossil fuels or (green) electricity. Currently, less than 1% of the world's hydrogen production is green even if fossil-based production with carbon capture is included (IEA 2024). Consequently, the greenification of the full value chain is precisely what is required in this context.

We calibrate the model to the US economy with a 2-sector structure: gaseous hydrogen (GH_2) production and distribution as upstream sector 1, and LRHD transportation as downstream sector 2. We assume that a model period corresponds to 25 years. The quantitative model generalizes slightly the benchmark theoretical framework to allow for heterogeneity in the relative input efficiency parameter z_i and the emission rate ξ_i across sectors and in adding total factor productivity (TFP) parameters A_i to each sector i's pro-

32. In contrast to light duty vehicles such as passenger cars, battery electrification is often less viable for LRHD applications due to, e.g., the weight of the batteries and the high charging loads that would be required (e.g. Gross 2020; Tiwari et al. 2024). Even for rail, where electrification is advanced in some parts of the world (though not the United States), the high capital cost of catenary systems can favor hydrogen as a way of decarbonizing diesel locomotive areas already under current conditions (Ahluwalia et al. 2021). Of course, in reality, there is also technology crossover in light- and heavy-duty applications, as there are both some hydrogen-powered passenger vehicles and electric trucks on the market.

duction function, so that (2) becomes:

$$y_{it}(\nu) = A_i \left[\ell_{dit}(\nu) + \gamma_{it}(v) \left(\frac{e^{z_i} \ell_{cit}(\nu)}{\alpha_i} \right)^{\alpha_i} \left(\frac{m_{it}(\nu)}{1-\alpha_i} \right)^{1-\alpha_i} \right].$$

We also introduce a labor disutility parameter into (1) to match observed wage and employment data. Appendix C presents the quantitative model structure in further detail and describes our calibration of initial wages and labor supplies to US data. All dollar values are in constant $2022 unless noted otherwise.

8.1 Calibration: LRHD Transportation

Our downstream sector differentiates eight varieties: trucks–class 7-8, trucks–class 4-6, air–narrow-body/short-medium range, air–wide-body/long-range, rail–passenger, rail–freight, water–passenger, and water–freight. Among these varieties, the majority of both value-added and carbon emissions are due to trucking (71% and 62%, respectively) and aviation (21% and 24%, respectively).[33] Our approach relies on engineering and industry estimates to quantify production and cost parameters. We briefly summarize the calibration here and provide further details in Appendix C.

First, we quantify the cost share of delivered gaseous hydrogen as $1 - \alpha_2 = 0.18$ based on the value-added-share weighted average across 26 estimates of future levelized costs (LCOs) of H_2 LRHD transport.[34] Second, we use pairs of LCO estimates using fossil fuels or hydrogen to calculate the value of z_2 implied by the relevant marginal cost ratio in the model, yielding a weighted average of $z_2 = -2.58$ (across 21 estimates). Third, we set A_2 to match calibration year (2022) data on prices and effective taxes, and calculate the baseline emissions intensity based on aggregate emissions, output, and prices. Finally, we include three types of costs in the fixed costs $\phi_2(.)$: technology development costs (e.g. of hydrogen aircraft), certain infrastructure adjustment costs (e.g. of refueling stations to accommodate hydrogen storage and dispensation), and the *excess costs* of current vs. future hydrogen-fueled technologies due to anticipated learning-by-doing and other technological advancements.

Figure 3 showcases our estimates of fixed costs across sectors, normalized as the per-

33. Based on 2022 data from the Bureau of Transportation Statistics (BTS 2024) and Environmental Protection Agency (EPA 2024).

34. Given that we define Sector 1 output as *delivered* GH_2—rather than *dispensed* H_2—we adjust any LCO estimates that use a "dispensed" H_2 cost measure accordingly.

cent increase in per-unit costs relative to current fossil fuel-based costs (over a 25-year model period).[35] Encouragingly, the estimates appear consistent with the empirically observed reality that H_2 development so far has focused more on lower-cost varieties such as passenger rail, trucks, and narrow-body aircraft compared to the higher-cost varieties of freight rail, wide-body aircraft, or maritime transport.

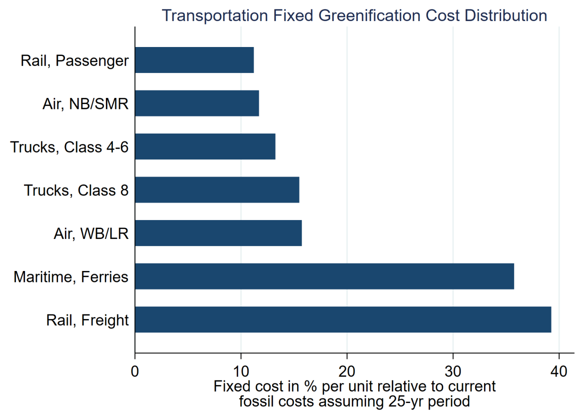


Figure 3. Distribution of fixed greenification costs in the downstream sector

8.2 Calibration: Hydrogen Production and Distribution

Our upstream sector encompasses the production of (gaseous) hydrogen and its distribution across six US regions.[36] For H_2 production, we consider electrolysis from dedicated renewables[37] as "green" and steam-methane reforming with natural gas as the polluting benchmark technology. While green H_2 may become cost-competitive in the future due to learning-by-doing and technological advancements (e.g. BloombergNEF 2023), at present it is associated with substantially higher production costs. We again consider these *excess initial costs* as a non-recurring cost that must be incurred for greenification. For distribution, we consider tube-trailer trucks as the polluting benchmark and hydrogen pipelines

35. Our calibration focuses on the US economy. In reality, the market for new technologies like hydrogen aircraft may include other countries. We thus adjust the fixed LRHD technology development costs downward (by 30%) to reflect that US revenues need not cover the full fixed costs of technology development. Our results are robust to relaxing this assumption as described in the Appendix.

36. The motivation for modeling hydrogen production and distribution as having some substitutability is that hydrogen can be produced locally (such as near refueling stations) with lower efficiency, or produced at scale and in preferable locations (with, e.g., higher solar potential) but then requiring more distribution.

37. While technically possible, grid-based electrolysis is generally estimated to be more expensive than with dedicated (or "islanded") renewable electricity (European Hydrogen Observatory 2023).

as the "green" option that can be accessed after incurring a fixed cost (of legal and technological development and construction).[38]

For the production function parameters, $\alpha_1 = 1$ is given and we set $z_1 = 1.608$ based on a weighted average across paired estimates of clean and dirty upstream costs from BloombergNEF and the Argonne National Laboratory's Hydrogen Delivery Scenario Analysis Model (HDSAM v4.5). Productivity A_1 matches 2022 data on prices, wages, and effective taxes. The emissions rate is based on IEA (2023) and the HDSAM. Finally, fixed costs $\phi_1(.)$ for hydrogen production capture *excess initial costs* (from BloombergNEF) and we estimate fixed costs for hydrogen pipelines in each region with the HDSAM (Brown et al. 2022). Appendix C provides further details, including how we re-scale fixed costs to account for demand outside our model, and presents the resulting distribution of fixed costs across upstream varieties.

8.3 Results

This section highlights three main results. First, steady-states multiplicity is empirically relevant. Figure 4 panel (a) presents an empirical counterpart similar to Figure 1b for a uniform carbon tax corresponding to the current effective average rate in the US ($13/t$CO_2$, OECD 2023). Already at this low tax, there are multiple steady-states, including the current stable steady-state of ($\chi_{1,0} = 0, \chi_{2,0} = 0$), an unstable intermediate steady-state, and a high steady-state with greenification rates around 90% in both sectors.

Second, it is difficult to escape the low initial greenification steady-state using a carbon price alone. We consider the introduction of a carbon price of $300/t$CO_2$, which is between recent estimates of the social cost of carbon from the US EPA ($213/t$CO_2$ in $2022,[39] EPA 2022) and those of some European countries (e.g. €300 ~ $340/t$CO_2$ in Germany, UBA 2024; SFR 430 ~ $526/t$CO_2$ in Switzerland). Figure 4 panel (b) shows that even a US carbon price of $300/t$CO_2$ is not sufficient to eliminate (0,0) as a steady-state. The welfare stakes of this multiplicity are large: Assuming an annual utility discount factor of $\beta_{yr} = 0.97$, we estimate a welfare difference between the low (0,0) and the stable high steady-state (0.89,

38. Pipelines are estimated to reduce H_2 distribution emissions by 85-95% (Demir et al. 2018; diLullo et al. 2022; Frank et al. 2021). We disregard the possibility of liquefied H_2 distribution via truck due to its limited scalability (e.g. Steer 2023). For aviation, we include liquefaction in the downstream costs of switching to and operating H_2 aircraft.

39. We convert the benchmark $190/t$CO_2$ in $2020 figure from EPA (2022) into $2022 based on the relevant US GDP deflator values from the St. Louis Fed's FRED database.

0.99) of around $8.4 trillion (initial period consumption equivalent variation).[40] Escaping this multiplicity with carbon prices alone would require a tax of around \$440/tCO_2 to ensure a high greenification steady-state.

Third, we confirm that a temporary downstream subsidy may be sufficient to induce greenification and is preferable to an upstream subsidy. We specifically model the economy's transition path from the current (0,0) steady-state assuming that a temporary greenification subsidy is provided either downstream or upstream in addition to a Pigouvian carbon price of \$300/tCO_2. Figure 4 panel (c) shows that a two-period 50% downstream greenification subsidy succeeds in triggering greenification in both sectors, but that this is not the case for a comparable upstream subsidy.[41] In terms of welfare, the stakes are large: Over the next 50 years alone, adding the downstream greenification subsidy decreases US emissions by almost 16 billion metric tons CO_2 (undiscounted total)—equivalent to almost half of the world's annual energy-related CO_2 emissions. The welfare gain associated with the downstream (vs. upstream) subsidy is estimated to be $3.2 trillion. Finally, we also find that the downstream subsidy + carbon price policy package achieves welfare gains of around $324 billion compared to a "high carbon price only" policy that seeks to induce full greenification without a clean technology subsidy (with a \$440/tCO_2 tax).

9 Conclusion

In this paper we analyzed a model of green technological transition along a supply chain. We then provided a quantitative application of our model to decarbonization of long-range and heavy-duty transportation via hydrogen, and showed that even with a uniform carbon price set equal to the social cost of carbon, the US economy could remain stuck in the "wrong" steady-state with CO_2 emissions far above the social optimum, whereas adding a temporary downstream greenification subsidy can induce high levels of greenification and yield large welfare gains.

Our analysis in this paper could be extended in several directions. A first and primary extension would be to develop a quantitative macroeconomic model of green transition of

40. We calculate the counterfactual steady-state welfare assuming that the government provides sufficient subsidies to reach the high steady-state greenification shares in the first period, so that the economy is in the steady-state from the second period as in the proof of Proposition 4. Both steady-states are modeled with a \$300/tCO_2 carbon price.

41. Note that i) the 50% downstream subsidy is sub-optimal, and ii) a uniform subsidy that solely corrects for innovators' myopia is insufficient to induce even moderate levels of greenification.

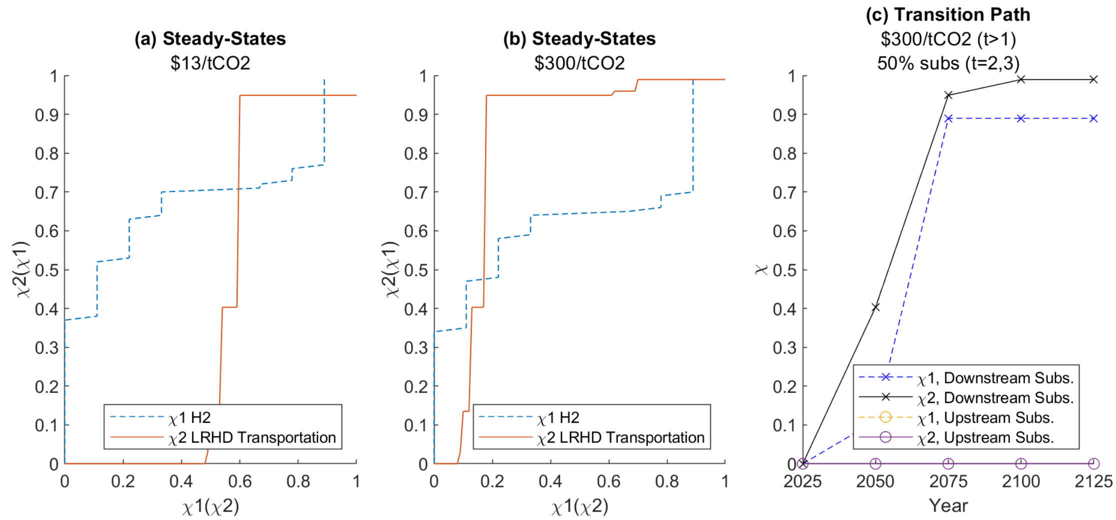


Figure 4. Panels (a) and (b) show steady-state greenification shares consistent with the current (\$13/tCO_2, (a)) and Pigouvian (\$300/tCO_2, (b)) carbon price, respectively. Panel (c) shows transition dynamics with a Pigouvian carbon price and temporary 50% greenification subsidy applied downstream (x-marker) or upstream (circle marker).

the overall economy seen as a grand network comprising multiple parallel supply chains. Another extension would be to look at coordination and multiple steady states not only across sectors and layers within a country but also along international value chains. A third extension would be to explore the extent to which allowing for vertical integration between different layers in the production chain affects our main conclusions. A fourth extension would be to use our framework to compute the overall elasticities of substitution between clean and dirty inputs to produce final goods once the supply chains involved in these technologies are fully taken into account. We know from previous work (e.g. Acemoglu et al. 2012, 2023; Donald 2023) that these elasticities play a major role in the design of optimal policies, yet rigorous methodologies to compute them remain to be found. These and other extensions are left for future research.

References

Acemoglu, Daron, Philippe Aghion, Lint Barrage, and David Hémous. 2023. "Climate Change, Directed Innovation and Energy Transition: The Long-run Consequences of the Shale Gas Revolution." NBER working paper 31657.

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hémous. 2012. "The Environment and Directed Technical Change." *The American Economic Review.*

Acemoglu, Daron, Ufuk Akcigit, Douglas Hanley, and William Kerr. 2016. "The Transition to Clean Technology." *Journal of Political Economy* 124 (1): 52–104.

Agency, US Environmental Protection. 2024. *Fast Facts on Transportation Greenhouse Gas Emissions.* Technical report.

Ahlualia, R.K, J.K. Peng, F. Cetinbas, D.D. Papadias, X. Wang, J. Kopasz, and T. Krause. 2021. "Rail, Aviation, and Maritime Metrics," no. TA034.

Bhashyam, Adithya. 2023. "2023 Hydrogen Levelized Cost Update." *BloombergNEF.*

Brown, Daryl, Krishna Reddi, and Amgad Elgowainy. 2022. "The Development of Natural Gas and Hydrogen Pipeline Capital Cost Estimating Equations." *International Journal of Hydrogen Energy* 47 (79): 33813–33826.

Buera, Francisco, and Nicholas Trachter. 2024. "Sectoral Development Multipliers." *Working Paper.*

Colmer, Jonathan, Ralf Martin, Mirabelle Muûls, and Ulrich J. Wagner. 2024. "Does Pricing Carbon Mitigate Climate Change? Firm-Level Evidence From the European Union Emissions Trading Scheme." *The Review of Economic Studies* 92 (3): 1625–1660.

Crouzet, Nicolas, Apoorv Gupta, and Filippo Mezzanotti. 2023. "Shocks and Technology Adoption: Evidence from Electronic Payment Systems." *Journal of Political Economy.*

Davis, Stacy C., and Robery Gary Boundy. 2020. "Transportation Energy Data Book: Edition 38."

Demir, Murat Emre, and Ibrahim Dincer. 2018. "Cost Assessment and Evaluation of Various Hydrogen Delivery Scenarios." *International Journal of Hydrogen Energy.*

Devulder, Antoine, and Noemie Lisack. 2020. *Carbon Tax in a Production Network: Propagation and Sectoral Incidence.* Working papers 760. Banque de France.

Di Lullo, Giovanni, Temitayo Giwa, Ayodeji Okunlola, Matthew Davis, T Mehedi, AO Oni, and A Kumar. 2022. "Large-Scale Long-Distance Land-Based Hydrogen Transportation Systems: A Comparative Techno-Economic and Greenhouse Gas Emission Assessment." *International Journal of Hydrogen Energy* 47 (83): 35293–35319.

Donald, Eric. 2023. "Spillovers and the Direction of Innovation: An Application to the Clean Energy Transition." *Working Paper.*

Dugoua, Eugenie, and Marion Dumas. 2021. "Green Product Innovation in Industrial Networks: A Theoretical Model." *Journal of Environmental Economics and Management* 107:102420.

———. 2024. "Coordination Dynamics Between Fuel Cell and Battery Technologies in the Transition to Clean Cars." *Proceedings of the National Academy of Sciences.*

Environmental Protection Agency. 2022. "EPA External Review Draft of Report on the Social Cost of Greenhouse Gases: Estimates Incorporating Recent Scientific Advances."

Frank, Edward, Amgad Elgowainy, Krishna Reddi, and Adarsh Bafana. 2021. "Life-Cycle Analysis of Greenhouse Gas Emissions from Hydrogen Delivery: A Cost-Guided Analysis." *International Journal of Hydrogen Energy* 46 (43): 22670–22683.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski. 2014. "Optimal Taxes on Fossil Fuel in General Equilibrium." *Econometrica* 82:41–88.

Greaker, Mads, and Kristoffer Midttomme. 2016. "Network effects and Environmental Externalities: Do Clean Technologies Suffer from Excess Inertia?" *Journal of Public Economics.*

Greenwald, Bruce, and Joseph Stiglitz. 2006. "Helping Infant Economies Grow: Foundations of Trade Policies for Developing Countries." *American Economic Review.*

Gross, Samantha. 2020. "The Challenge of Decarbonizing Heavy Transport." *Foreign Policy.*

International Energy Agency. 2023. *Comparison of the emissions intensity of different hydrogen production routes.* Technical report.

———. 2024. *Global Hydrogen Review 2024.* Technical report.

Jaramillo, P., S. Kahn Ribeiro, P. Newman, S. Dhar, O.E. Diemuodeke, T. Kajino, D.S. Lee, et al. 2024. "Climate Change 2022: Mitigation of Climate Change."

Juhàsz, Rèka, Nathan Lane, and Dani Rodrik. 2023. *The New Economics of Industrial Policy.* Working Paper, Working Paper Series 31538. National Bureau of Economic Research, August.

King, Maia, Bassel Tarbush, and Alexander Teytelboym. 2019. "Targeted Carbon Tax Reforms." *European Economic Review* 119:526–547.

Liu, Ernest. 2019. "Industrial Policies in Production Networks." *The Quarterly Journal of Economics* 134 (4): 1883–1948.

Liu, Ernest, and Song Ma. 2021. *Innovation Networks and R&D Allocation.* Working Paper, Working Paper Series 29607. National Bureau of Economic Research.

Liu, Ernest, and Aleh Tsyvinski. 2024. "A Dynamic Model of Input-Output Networks." *Review of Economic Studies.*

Mahen, Shane. 2025. "The Clean Transition in a Network Economy." *Working Paper.*

Martin, Ralf, Mirabelle Muûls, and Thomas Stoerk. 2023. "The Effects of Carbon Pricing along the Production Network." *Working Paper.*

Murphy, Kevin, Andrei Shleifer, and Robert Vishny. 1989. "Industrialization and the Big Push." *Journal of Political Economy* 97 (5): 1003–1026.

Nordhaus, William. 1994. *Managing the Global Commons: The Economics of Climate Change.* MIT Press. Cambridge, Massachusetts.

OECD. 2023. *Effective Carbon Rates 2023.* Technical report.

Rapson, David, and Erich Muehlegger. 2023. "Global Transportation Decarbonization." *Journal of Economic Perspectives* 37 (3): 163–188.

Rosenstein-Rodan, P. N. 1943. "Problems of Industrialisation of Eastern and South-Eastern Europe." *The Economic Journal* 53 (210/211): 202–211.

Smulders, Sjak, and Sophie Zhou. 2024. "Self-Fulfilling Prophecies in the Transition to Clean Technology." *Working Paper.*

Steer. 2023. *Analysing the costs of hydrogen aircraft.* Technical report 24135101. Transport & Environment.

Sturm, John. 2023. "How to Fix a Coordination Failure: A "Super-Pigouvian" Approach." *Working Paper.*

Tiwari, Saurav, Michael Pekris, and John Doherty. 2024. "A Review of Liquid Hydrogen Aircraft and Propulsion Technologies." *International Journal of Hydrogen Energy.*

Transportation Bureau of Transportation Statistics, US Department of. 2024. *Gross domestic product (GDP) attributed to transportation by mode.* Technical report.

Zeira, Joseph. 1998. "Workers, Machines, and Economic Growth." *Quarterly Journal of Economics* 113 (4): 1091–1117.

A Main Appendix: Proof of Proposition 3

We proceed in five steps: 1) We write output as a function of the labor allocation and technology, 2) we derive the optimal labor allocation, 3) we show that disutility from labor and pollution is a constant, 4) we derive the simplified Social Planner problem of Equation (18), and 5) we derive the first-order conditions.

Step 1: With $(1+\xi)e^z > 1$, the Social Planner uses the clean production process whenever it is available. As argued in the text, all greenification in the optimum happens immediately, so that the share of greenified varieties $\{\chi_{it}\}$ is constant over time. Therefore, the allocation of labor to production is also constant over time, and we can drop time subscript. Then, for any $k < N$, we get

$$y_{k+1} = \left(\frac{\ell_{d(k+1)}}{1-\chi_{k+1}}\right)^{1-\chi_{k+1}} \frac{1}{\chi_{k+1}^{\chi_{k+1}}} \left(\frac{e^z \ell_{c(k+1)}}{\alpha_{k+1}}\right)^{\chi_{k+1}\alpha_{k+1}} \left(\frac{y_k}{1-\alpha_{k+1}}\right)^{\chi_{k+1}(1-\alpha_{k+1})}. \quad \text{(A.1)}$$

Assume that for sector k, we have

$$\ln y_k = \sum_{i=1}^{k} \prod_{j=i+1}^{k} \chi_j (1-\alpha_j) \left[\begin{array}{c} -(\chi_i \ln \chi_i + (1-\chi_i)\ln(1-\chi_i)) + \chi_i \alpha_i \ln\left(\frac{e^z \ell_{ci}}{\alpha_i}\right) \\ -\chi_i (1-\alpha_i)\ln(1-\alpha_i) + (1-\chi_i)\ln \ell_{di} \end{array} \right].$$

Then taking the log of (A.1), it is immediate that sector $k+1$ follows the same formula. Therefore, defining $\omega_i \equiv \prod_{j=i+1}^{N} \chi_j (1-\alpha_j)$, we can rewrite output as[A.1]

$$\ln y_N = \sum_{i=1}^{N} \omega_i \left[\begin{array}{c} -(\chi_i \ln \chi_i + (1-\chi_i)\ln(1-\chi_i)) + \chi_i \alpha_i \ln\left(\frac{e^z \ell_{ci}}{\alpha_i}\right) \\ -\chi_i (1-\alpha_i)\ln(1-\alpha_i) + (1-\chi_i)\ln \ell_{di} \end{array} \right]. \quad \text{(A.2)}$$

Step 2: Taking first-order conditions with respect to clean and dirty labor inputs in the social planner problem (16), we immediately get that

$$\ell_{ci} = \chi_i \alpha_i \omega_i, \qquad \ell_{di} = (1-\chi_i)\,\omega_i / (1+\xi). \quad \text{(A.3)}$$

Step 3: Assume that $\sum_{j=1}^{i} [(1+\xi)\ell_{dj} + \ell_{cj}] = \omega_i$ holds for i, then the same relationships holds for $i+1$ as:

$$\begin{aligned} \sum_{j=1}^{i+1} [(1+\xi)\ell_{dj} + \ell_{cj}] &= \chi_{i+1}\alpha_{i+1}\omega_{i+1} + (1-\chi_i)\,\omega_{i+1} + \omega_i \\ &= [\chi_{i+1}\alpha_{i+1} + (1-\chi_i) + \chi_{i+1}(1-\alpha_{i+1})]\,\omega_{i+1} = \omega_{i+1}. \end{aligned}$$

Since this is true for $i = 1$ from using (A.3), this relationship holds by induction for all i.

A.1. To economize notations, we keep $(1-\alpha_1)\ln(1-\alpha_1)$ in (A.2) but treat it as a zero (as $\alpha_1 = 1$).

Given that $\omega_N = 1$, we then get that $\sum_{j=1}^{N} [(1+\xi)\,\ell_{dj} + \ell_{cj}] = 1$.

Step 4: Plugging the labor allocation Equations (A.3) into Equation (A.2) we get

$$\begin{aligned}
\ln y_N &= \sum_{i=1}^{N} \omega_i \left[-(1-\alpha_i)\,\chi_i \ln \chi_i + \chi_i \alpha_i \ln (e^z \omega_i) - \chi_i (1-\alpha_i) \ln (1-\alpha_i) + (1-\chi_i) \ln \frac{\omega_i}{1+\xi} \right] \\
&= \left(\sum_{i=1}^{N} \omega_i \chi_i \alpha_i \right) z - \left(\sum_{i=1}^{N} \omega_i (1-\chi_i) \right) \ln (1+\xi) + \sum_{i=1}^{N} \omega_i \left[\begin{array}{c} -(1-\alpha_i)\,\chi_i \ln (\chi_i (1-\alpha_i)) \\ +(1-\chi_i+\chi_i\alpha_i) \ln \omega_i \end{array} \right]
\end{aligned} \tag{A.4}$$

We note that the following two relationships:

$$\sum_{i=1}^{N} (1-\chi_i)\,\omega_i + \sum_{i=1}^{N} \omega_i \chi_i \alpha_i = \sum_{i=1}^{N} \omega_i - \sum_{i=1}^{N} \omega_i \chi_i (1-\alpha_i) = \sum_{i=1}^{N} \omega_i - \sum_{i=2}^{N} \omega_{i-1} = \omega_N = 1;$$

and
$$\begin{aligned}
&\sum_{i=1}^{N} \omega_i \left[-(1-\alpha_i)\,\chi_i \ln (\chi_i (1-\alpha_i)) + (1-\chi_i+\chi_i\alpha_i) \ln \omega_i \right] \\
&= \sum_{i=1}^{N} \omega_i \ln \omega_i - \sum_{i=1}^{N} \omega_i (1-\alpha_i)\,\chi_i \ln (\chi_i (1-\alpha_i)\,\omega_i) = \sum_{i=1}^{N} \omega_i \ln \omega_i - \sum_{i=1}^{N-1} \omega_i \ln \omega_i = 0.
\end{aligned}$$

Using both relationships in (A.4), we can rewrite

$$\ln y_N = \left(\sum_{i=1}^{N} \omega_i \chi_i \alpha_i \right) (z + \ln (1+\xi)) - \ln (1+\xi). \tag{A.5}$$

Dropping constants and multiplying the original problem by $1-\beta$, we then get that the Social Planner solves the simplified Social Planner problem given Equation (18).

Step 5: An interior solution $\{\chi_i\}$ must satisfy the first-order conditions given by

$$\frac{\ln ((1+\xi)\,e^z)}{1-\beta} \sum_{j=1}^{i} \frac{\omega_j \alpha_j \chi_j}{\chi_i} = \mathcal{F}_i' (\chi_i).$$

We note that $\sum_{j=1}^{i} \frac{\omega_j \alpha_j \chi_j}{\chi_i} = \mu_i \omega_i$, which delivers (19). To see this, note that for $i > 1$,

$$\chi_i \omega_i \mu_i = \chi_i \omega_i (\alpha_i + (1-\alpha_i)\,\chi_{i-1} \mu_{i-1}) = \chi_i \omega_i \alpha_i + \chi_{i-1} \mu_{i-1} \omega_{i-1} = \sum_{j=1}^{i} \omega_j \alpha_j \chi_j.$$

This establishes Step 5 and thus completes the proof of Proposition 3. □

Supplemental Appendix

Transition to Green Technology along the Supply Chain

Philippe Aghion, Lint Barrage, Eric Donald, David Hémous, and Ernest Liu

B Theory Derivations and Extensions

B.1 Proof of Proposition 1

We proceed by induction. Suppose we know the sequence of greenification shares $\{\chi_{it-1}\}$ at time $t-1$. Then, we can compute the sequence $\{\mu_{it}\}$ recursively from upstream moving downstream using the equations $\mu_{1t} = 1$ and (6). Next, given $\chi_{N,t-1}$ and the μ's at time $t-1$, one can compute χ_{Nt} using that $r_{Nt} = 1$ as:

$$\chi_{Nt} = \max\left\{\chi_{N,t-1}, F_N\left(\left(1 - e^{-\mu_{N,t-1}Z}\right)\right)\right\}.$$

For given downstream greenification shares χ_{jt} for $j \in \{i+1, N\}$, we can compute the upstream equilibrium greenification share χ_{it}using (8) and

$$\chi_{it} = \max\left\{\chi_{i,t-1}, F_i\left(\left(1 - \left[e^{z}\left(1+\tau_t\right)\right]^{-\mu_{it-1}}\right)\prod_{j=i+1}^{N}\widetilde{\chi}_{jt}\left(1-\alpha_j\right)\right)\right\}.$$

By induction, the equilibrium sequence of shares at date t, $\{\chi_{it}\}_{i=1}^{N}$, is uniquely determined. □

B.2 Proof of Proposition 2

When $N = 1$, the steady-state equilibrium greenification share χ_1 satisfies the equation $\chi_1 = F_1\left(1 - e^{-Z}\right)$, which has a unique solution.

Assume now that $N \geq 2$. To establish the result, we simply need to build an example with multiple steady-state equilibria. We do so by assuming that the distribution of greenification costs is a mass point at some value ϕ. We further assume that all the α_i 's are equal to the same α (except for $i = 1$). We derive conditions under which there is a steady-state where all sectors fully greenify and another one where no sector greenifies.

Consider first the case where no firm greenifies in any sector. In this case, the profit from greenification in all sectors $i < N$ is zero due to zero demand. The rent from greenifying in the most downstream sector N is $\pi_N = 1 - e^{-\alpha Z}$. Thus there will be no greenification in sector N whenever $1 - e^{-\alpha Z} < \phi$.

Next, suppose that all but a measure zero of firms greenify in all sectors. In this case, the profit from greenification in sector i is $\pi_i = (1-\alpha)^{N-i}\left[1-e^{-Z}\right]$. For all firms to have an incentive to greenify, we need that $(1-\alpha)^{N-1}\left[1-e^{-Z}\right] > \phi$.

Hence, both the full and no greenification steady-states exist if ϕ satisfies

$$1-e^{-\alpha Z} < \phi < (1-\alpha)^{N-1}\left[1-e^{-Z}\right].$$

This is possible as soon as there exist values of z, τ, and α such that

$$1-e^{-\alpha Z} < (1-\alpha)^{N-1}\left[1-e^{-Z}\right].$$

For instance, for small Z ($e^Z \approx 1+z$) this inequality boils down to $\alpha < (1-\alpha)^{N-1}$, which is satisfied for α sufficiently small. This completes the proof. □

B.3 Example with Multiple Steady-States in the Cap-and-Trade Case

In this section, we show that in the presence of a cap-and-trade system with a cap $\bar{\ell}_d$, there exist multiple steady-states over a non-empty open set of parameters whenever $N \geq 2$, while there is a unique steady-state when $N = 1$.

Revenues of the dirty production process get allocated to the payment of labor in these processes and emission permits. Given a price on emissions τ_t, and using that revenues of each sector are given by (9), we get that

$$(1+\tau_t)\,\ell_{dt} = \sum_{i=1}^{N}(1-\chi_{it})\prod_{j=i+1}^{N}\widetilde{\chi}_{jt}\,(1-\alpha_j)\,, \tag{B.1}$$

under the maintained assumption that $(1+\tau_t)\,e^z > 1$. If the cap does not bind, then $\tau_t = 0$, and if it binds, $\ell_{dt} = \bar{\ell}_d$ and the previous equation uniquely determines the price of emissions for given technology levels (noting that $\widetilde{\chi}_{jt}$ decreases in τ_t).

With $N = 1$, a steady-state is then uniquely characterized by

$$\chi_1 = F_1\left(1-\frac{e^{-z}}{1+\tau}\right) \text{ and } (1+\tau_t)\,\ell_{dt} = 1-\chi_1.$$

To show that there can be multiple steady-states for $N \geq 2$, we build an example with two sectors. We consider parameter values for which the cap always binds. A steady-state is a pair $\{\chi_1, \chi_2\}$ and a price on emissions τ, which satisfy (13), (15), and (B.1) such that

$$(1+\tau)\,\bar{\ell}_d = (1-\chi_1)\,\chi_2\,(1-\alpha_2) + (1-\chi_2) \tag{B.2}$$

$$\chi_1 = F_1\left[\left(1-\frac{e^{-z}}{1+\tau}\right)\chi_2\,(1-\alpha_2)\right] \tag{B.3}$$

$$\chi_2 = F_2\left(1 - \left[\frac{e^{-z}}{1+\tau}\right]^{\mu_2}\right) \text{ with } \mu_2 = \alpha_2 + \chi_1(1-\alpha_2). \tag{B.4}$$

We construct non-knife edge examples where one steady-state features $\chi_1 = 0$, $\chi_2 > 0$ and the other one features $\chi_1^\dagger = 1$, $\chi_2^\dagger > \chi_2$.

In the first steady-state, $\mu_2 = \alpha_2$ and given (B.2), $1+\tau = (1-\alpha_2\chi_2)/\bar{\ell}_d$, hence (B.3) and (B.4) give

$$0 = F_1\left[\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right)\chi_2(1-\alpha_2)\right] \tag{B.5}$$

$$\chi_2 = F_2\left(1 - \left[\frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right]^{\alpha_2}\right). \tag{B.6}$$

In the second steady-state, $\mu_2^\dagger = 1$ and given (B.2), $1+\tau = \left(1-\chi_2^\dagger\right)/\bar{\ell}_d$, hence (B.3) and (B.4) give

$$1 = F_1\left[\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right)\chi_2^\dagger(1-\alpha_2)\right] \tag{B.7}$$

$$\chi_2^\dagger = F_2\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right). \tag{B.8}$$

The right-hand sides of (B.6) and (B.8) are decreasing in χ_2 and $\chi_2^\dagger$ respectively. For a sufficiently low cap $\bar{\ell}_d$, we have $1 - \left[\frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right]^{\alpha_2} < 1 - \frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}$. Then, (B.6) and (B.8) imply that $\chi_2^\dagger > \chi_2$, and one can build F_2 such that there is a large gap between $\chi_2^\dagger$ and χ_2. Again, for a sufficiently low cap, we get $\left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\alpha_2\chi_2}\right)\chi_2 < \left(1 - \frac{e^{-z}\bar{\ell}_d}{1-\chi_2^\dagger}\right)\chi_2^\dagger$. Building F_1 such that all the mass of the distribution is between these two values, we can satisfy both (B.5) and (B.7). This shows that multiple steady-states are possible.

B.4 Proof of Proposition 4

Consider any initial allocation $\{\chi_{i,0}\}$ and assume that the Social Planner imposes a Pigouvian tax $\tau_t = \xi$ and a set of sector specific subsidies $\{q_{i,t}\}$. Then, the equilibrium level of greenification at time t is given by

$$\chi_{i,t} = F_i\left(\frac{\left(1 - e^{-\mu_{i,t-1}Z}\right)}{1-q_{i,t}}\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{j,t}(1-\alpha_j)\right)\right).$$

For sector N at time 1, we can always set $q_{N,1}$ such that

$$\chi_N^{SP} = F_N\left(\frac{\left(1-e^{-\mu_{N,0}Z}\right)}{1-q_{N,1}}\right),$$

where χ_N^{SP} is the optimal level of greenification in sector N and $\mu_{N,0}$ is predetermined. Assume now that the Social Planner uses a set of sector specific subsidies $\{q_{j,1}\}$ for $j > i$, in order to implement the optimal level of greenification χ_j^{SP} for $j > i$ at $t = 1$. Then for sector i, the Social Planner can choose $q_{i,1}$ such that

$$\chi_i^{SP} = F_i\left(\frac{\left(1 - e^{-\mu_{i,0}Z}\right)}{1 - q_{i,1}} \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{j,1}(1 - \alpha_j)\right)\right),$$

since $\mu_{i,0}$ is again pre-determined and $\widetilde{\chi}_{j,1} = \chi_{j,0} + \left(\chi_j^{SP} - \chi_{j,0}\right) e^{-\mu_{j,0}Z}$ is also given. Then, by induction, the Social Planner can implement the socially optimal levels of greenification χ_i^{SP} in all sectors from the most downstream to the most upstream at time $t = 1$.

At time $t = 2$, there is no more incentives to greenify when $q_{i,2} = 0$, because if $\chi_{j,2} = \chi_j^{SP}$, we get that

$$\chi_i^{SP} = F_i\left(\frac{\mu_i^{SP}Z}{1 - \beta} \prod_{j=i+1}^{N} \left(\chi_j^{SP}(1 - \alpha_j)\right)\right) > F_i\left(\left(1 - e^{-\mu_{i,1}Z}\right) \prod_{j=i+1}^{N} \left(\widetilde{\chi}_{j,2}(1 - \alpha_j)\right)\right),$$

where the equality stems from the fact that χ_i^{SP} is the optimum, but the inequality uses that $\mu_{i,1} = \mu_i^{SP}$, $\widetilde{\chi}_{j,2} = \chi_j^{SP}$ if there is no further greenification and that $\frac{\mu_i^{SP}Z}{1-\beta} > \left(1 - e^{-\mu_{i,1}Z}\right)$. The overall inequality implies that there is no further incentive to greenify in the decentralized economy. Since there is no greenification, there are no profits either, and the social optimum is implemented from $t = 2$ onward. This completes the proof. □

B.5 Example: Small Interventions can have Large Effects

We assume that the distributions of fixed costs F_1 and F_2 have mass points at ϕ_1 and ϕ_2, respectively. We first derive conditions under which there are three steady-states characterized by no greenification, full greenification, and an interior level of greenification. We then derive conditions under which moving from the no greenification to the interior level of greenification only requires a small intervention.

First, assume that the economy features no greenification, so there is no incentive to greenify downstream (sector 2) provided that $1 - e^{-\alpha_2 Z} < \phi_2$. In that case, there is no market for sector 1 and no greenification upstream either as long as $\phi_1 > 0$. Because these are strict inequalities, there are still no incentives to greenify for χ's slightly different from 0, so this steady-state is stable.

Second, full greenification is also a steady-state provided that $\left(1 - e^{Z}\right)(1 - \alpha_2) > \phi_1$, which ensures full greenification upstream, and $\left(1 - e^{-Z}\right) > \phi_2$, which ensures full greenification downstream. For the same reason as before, this steady-state is also stable.

Then the no-greenification and full-greenification steady-states coexist provided that

$$1-e^{-\alpha_2 Z}<\phi_2<1-e^{-Z} \text{ and } 0<\phi_1<\left(1-e^{-Z}\right)(1-\alpha_2).$$

Third, an interior steady-state equilibrium (χ_1^*, χ_2^*) must satisfy

$$\left(1-e^{-Z}\right)\chi_2^*(1-\alpha_2) = \phi_1 \tag{B.9}$$

$$1-e^{-\left(\alpha_2+\chi_1^*(1-\alpha_2)\right)Z} = \phi_2. \tag{B.10}$$

Given that $1-e^{-\alpha_2 Z}<\phi_2<1-e^{-Z}$, there always exists a $\chi_1^* \in (0,1)$ which satisfies the second equation. Similarly, given that $\left(1-e^{-Z}\right)(1-\alpha_2)>\phi_1>0$, there also always exists a $\chi_2^* \in (0,1)$ that satisfies the first equation. Since the left hand side of (B.9) is increasing in χ_2 and the left-hand side of (B.10) is increasing in χ_1, while the right-hand sides are fixed at ϕ_1 and ϕ_2, this interior steady-state is necessarily unstable. Therefore, a small increase in χ_1 and/or χ_2 starting from (χ_1^*, χ_2^*) will lead to further greenification.

Next, we derive a set of subsidies sufficient to ensure that the economy moves from the no greenification to the interior steady-states at time 1. For greenification in sector 1 to be interior, we need a subsidy $q_{1,1}$ which satisfies

$$\frac{1-e^{-Z}}{1-q_{1,1}}\widetilde{\chi}_{2,1}(1-\alpha_2)=\phi_1 \text{ with } \widetilde{\chi}_{2,1}=\chi_2^* e^{-\alpha_2 Z},$$

which, using (B.9), yields $q_{1,1}=1-e^{-\alpha_2 Z}$. Similarly, for greenification in sector 2 to be interior, we need a subsidy $q_{2,1}$ such that

$$\frac{1-e^{-\alpha_2 Z}}{1-q_{2,1}}=\phi_2 \Longrightarrow q_{2,1}=1-\frac{1-e^{-\alpha_2 Z}}{\phi_2}.$$

Overall, the total amount of subsidies to move the economy to the interior unstable steady-state, is given by

$$Q_1=\chi_1^* q_{1,1}+\chi_2^* q_{2,1}=\chi_1^*\left(1-e^{-\alpha_2 Z}\right)+\chi_2^*\left(1-\frac{1-e^{-\alpha_2 Z}}{\phi_2}\right)$$

If ϕ_2 is above but close to $1-e^{-\alpha_2 Z}$, then $q_{2,1}$ is small. In addition, in that case, (B.10) implies that χ_1^* is small too, which ensures that the total amount spent Q_1 is small. This establishes the result described above: if greenification costs are just a little too high downstream, then a small intervention is enough to push the economy toward the interior steady-state and eventually the full greenification one as well.[B.1]

B.6 Escaping the No Greenification Trap

We provide a formal treatment of the example of Section 5.1 in the following proposition:

Proposition 7. *Consider a supply chain with $N \geq 3$ sectors, and suppose that the basic parameters and the F_i's are such that no greenification in all sectors is a steady-state. Suppose also that initially the economy is stuck in this no greenification steady-state and that the government can directly greenify a positive mass of varieties in one sector only. Then, provided that greenification costs are not too high, (i) greenification starts propagating immediately only if the government intervenes in the most downstream sector N; (ii) greenification starts propagating with a one-period delay if the government intervenes in sector $N-1$; and (iii) greenification never propagates if the government intervenes in a sector which is more upstream than sector $N-1$.*

Proof. **Part (i):** Suppose the government greenifies sector N at the beginning of time t, so that $\chi_{N,t} > 0$. Then, greenification in sector $N-1$ at t is given by

$$\chi_{N-1.t} = F_{N-1}\left(\left(1 - e^{-\alpha_{N-1}Z}\right)\chi_{N,t}e^{-\alpha_{N-1}Z}\left(1-\alpha_{N-1}\right)\right).$$

With $\chi_{N,t} > 0$, we get $\chi_{N-1,t} > 0$ (as long as greenification costs at $N-1$ are not too high: $F_{N-1}\left(\left(1 - e^{-\alpha_{N-1}Z}\right)\chi_{N,t}e^{-\alpha_{N-1}Z}\left(1-\alpha_{N-1}\right)\right) > 0$). In contrast at $t-1$, pre-intervention, we had $\chi_{N,t-1} = \chi_{N-1,t-1} = 0$.

Now suppose greenification did propagate to sectors $j > i$ (i.e. $\chi_{j,t} > 0$). Then it also propagates to sector i since:

$$\chi_{it} = F_i\left(\left(1 - e^{-\alpha_i Z}\right)\prod_{j=i+1}^{N}\chi_{j,t}e^{-\alpha_j Z}\left(1-\alpha_j\right)\right) \text{ for } i < N,$$

which is positive if greenification costs are not too high. Hence greenification propagates all the way from the most downstream sector N to the most upstream sector 1 at time t.

At time $t+1$, we have

$$\chi_{i,t+1} = F_i\left(\left(1 - e^{-\mu_{i,t}Z}\right)\prod_{j=i+1}^{N}\left(\widetilde{\chi}_{j,t+1}\left(1-\alpha_j\right)\right)\right),$$

B.1. This logic does not extend to the case where greenification costs are just a little too high upstream: If ϕ_1 is positive but close to 0, then χ_2^* is close to 0 and subsidies spent for sector 2 are small. However, there is no guarantee that subsidies spent for greenification in sector 1 are small without additional assumptions.

with $\widetilde{\chi}_{j,t+1}$ given by (8) and μ_{it} by (6). Then, $\mu_{it} \geq \alpha_i$ and $\widetilde{\chi}_{j,t+1} \geq \chi_{j,t} > \chi_{j,t} e^{-\alpha_j Z}$, which implies that further greenification occurs in all sectors at time $t+1$ (as long as F_i has positive mass around the relevant range), and this continues in subsequent periods until we reach a steady-state with positive greenification in all sectors.

Part (ii): Suppose now that the government starts greenifying in sector $N-1$. Then we get that $\mu_{N,t-1} = \alpha_N$ (i.e. the pre-intervention value) so that χ_{Nt} must satisfy

$$\chi_{Nt} = F_N \left(1 - e^{-\alpha_N Z}\right) = \chi_{N,t-1} = 0.$$

In other words, greenifying first in sector $N-1$ at time t does not immediately propagate to the most downstream sector N. Consider now sector $i < N-1$. Since, $\chi_{N,t} = 0$, $r_{it} = 0$ and there is no greenification in any sector besides $N-1$.

Consider now time $t+1$. In sector N, we get $\mu_{N,t} = \alpha_N + \chi_{N-1,t}\alpha_{N-1}\left(1-\alpha_N\right) > \alpha_N$ and greenification incentives in sector N obey

$$\chi_{N,t+1} = F_N \left(1 - e^{-\mu_{N,t} Z}\right).$$

Since $\mu_{N,t} > \alpha_N$, then provided that greenification costs are not too high, we can have $F_N\left(1 - e^{-\mu_{N,t}Z}\right) > F_N\left(1 - e^{-\alpha_N Z}\right) = 0$, such that $\chi_{N,t+1} > 0$, in which case greenification propagates to sector N.

Moving back to sector $N-1$, we have

$$\chi_{N-1,t+1} = \max\left\{\chi_{N-1,t}, F_{N-1}\left(\left(1 - e^{-\alpha_{N-1}Z}\right)\widetilde{\chi}_{N,t+1}\left(1-\alpha_N\right)\right)\right\},$$

with $\widetilde{\chi}_{N,t+1} = \chi_{N,t+1}\left(e^z\left(1+\tau_t\right)\right)^{-\mu_{N,t}}$. Now moving to sector $N-2$, we get that

$$\chi_{N-2,t+1} = F_i\left(\left(1 - e^{-\alpha_{N-2Z}Z}\right)\widetilde{\chi}_{N-1,t+1}\left(1-\alpha_{N-1}\right)\widetilde{\chi}_{N,t+1}\left(1-\alpha_N\right)\right),$$

with $\widetilde{\chi}_{N-1,t+1} = \chi_{N-1,t} + \left(\chi_{N-1,t+1} - \chi_{N-1,t}\right)e^{-\mu_{N-1,t}Z} \geq \chi_{N-1.t}$ and $\widetilde{\chi}_{N,t+1} > 0$ if $\chi_{N,t+1} > 0$. This in turn implies that greenification also propagates to sector $N-2$, provided that the distribution of fixed costs F_i has positive mass in the relevant range. The logic extends to all sectors $j > N-1$ so that greenification propagates to all sectors at time $t+1$. And greenification intensifies in subsequent periods until we reach a steady-state with positive greenification in all sectors.

Part (iii): Now suppose that the government starts greenifying in a sector j more upstream than $N-1$, i.e. $j < N-1$, at time t. Consider first sector N at time t. Given that greenification incentives in that sector only depend upon $\mu_{N,t-1}$, they are the same as pre-intervention so that $\chi_{N,t} = 0$. As a result $r_{it} = 0$ for any $i < N$ and greenification does not propagates: $\chi_{k,t} = 0$ for all $k \neq j$. At time $t+1$, since $\chi_{N-1,t} = 0$, $\mu_{N,t} = \mu_{N,t-1}$, and greenification incentives in sector N are the same as in period t, i.e. $\chi_{N,t+1} = 0$. It follows that we also have $\chi_{k,t+1} = 0$ for all $k \neq j$. Since the same reasoning carries over to all future periods, greenification never propagates. This establishes the proposition. $\square$

B.7 Proof of Proposition 5

Part i) is trivial. To establish Part ii), we first derive $\frac{\partial \mu_i}{\partial \chi_k}$. We immediately note that $\frac{\partial \mu_i}{\partial \chi_k} = 0$ for $k > i$. Further, we have that $\frac{\partial \mu_i}{\partial \chi_{i-1}} = (1-\alpha_i)\,\mu_{i-1}$, while for $k < i-1$, we get $\frac{\partial \mu_i}{\partial \chi_k} = \chi_{i-1}\,(1-\alpha_i)\,\frac{\partial \mu_{i-1}}{\partial \chi_k}$. Iterating on j such that $i-j$ goes down to $k+1$, we get

$$\frac{\partial \mu_i}{\partial \chi_k} = \begin{cases} \frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\chi_k} & k < i \\ 0 & \text{otherwise} \end{cases}. \tag{B.11}$$

For $k > i$, it is then immediate that $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = 1$. While for $k < i$, we get

$$\frac{\partial \ln \pi_i}{\partial \ln \chi_k} = \frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} \frac{\partial \ln \mu_i}{\partial \ln \chi_k},$$

which combined with (B.11) gives (21).

We note that $\frac{\mu_i Z e^{-\mu_i Z}}{1-e^{-\mu_i Z}} < 1 \iff 1 + \mu_i Z < e^{\mu_i Z}$, which is true for $Z > 0$. Next, for $k = i-1$, we get using (6) that

$$\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} = \frac{(1-\alpha_i)\chi_{i-1}\mu_{i-1}}{\mu_i} = \frac{\mu_i - \alpha_i}{\mu_i} < 1.$$

In addition, for any $k < i-1$, then, using again (6), we get

$$\begin{aligned} \frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} &= \frac{\left(\prod_{j=0}^{i-(k+1)-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)(\mu_{k+1}-\alpha_{k+1})}{\mu_i} \\ &< \frac{\left(\prod_{j=0}^{i-(k+1)-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_{k+1}}{\mu_i}. \end{aligned}$$

Then, we get that $\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i}$ is increasing in k for $k \le i-1$, and by induction, $\frac{\left(\prod_{j=0}^{i-k-1}(1-\alpha_{i-j})\chi_{i-j-1}\right)\mu_k}{\mu_i} < 1$ for all $k \le i-1$. As both fractions on the right-hand side of (21) are smaller than 1, $\frac{\partial \ln \pi_i}{\partial \ln \chi_k} < 1$, establishing Part ii). Part iii) is then trivial.

B.8 Proof of Proposition 6

Applying Step 1 of Appendix A gives us a mapping from technology and the labor allocation to output via Equation (A.2). This mapping implies that the profit maximization conditions governing the allocation of labor follow

$$\ell_{ci} = \chi_i \alpha_i \omega_i, \qquad \ell_{di} = \frac{(1-\chi_i)\,\omega_i}{1+\tau},$$

where we have used the fact that household optimization implies that $p_N \frac{\partial y_N}{\partial x} = \frac{\partial \ln(y_N)}{\partial x}$. Step 3 then implies that total labor costs, inclusive of the carbon price, equal one, while Step 4 implies that the Planner's problem can be rewritten as

$$\max_{\{\chi_j\}} \ln\left((1+\tau)\, e^z\right) \sum_{i=1}^{N} \omega_i \chi_i \alpha_i - (\xi - \tau) \sum_{i=1}^{N} \ell_{di} - (1-\beta) \sum_{i=1}^{N} \mathcal{F}_i(\chi_i).$$

This gives us the first-order conditions

$$\frac{\ln\left((1+\xi)\, e^z\right)}{1-\beta} \sum_{j=1}^{i} \frac{\omega_j \alpha_j \chi_j}{\chi_i} + \frac{\xi - \tau}{1-\beta}\left(-\frac{\partial \ell_d}{\partial \chi_i}\right) = \mathcal{F}_i'\left(\chi_i\right).$$

Step 5 implies that $\sum_{j=1}^{i} \frac{\omega_j \alpha_j \chi_j}{\chi_i} = \mu_i \omega_i$, leading to Equation ((27)). $\square$

B.9 Incomplete Carbon Prices in a General Supply Chain

We have argued that downstream greenification is more effective at reducing emissions, so we will again examine the generality of this argument by considering a general supply chain network like the one described in Section 5.3. Note that the optimal policy results of Proposition 6 carry over to this more general environment. The substantive difference will be in the equilibrium emission reductions: $-\frac{\partial \ell_d}{\partial \chi_i}$.

Total emissions in this more general setting are given by

$$\ell_d = \frac{1}{1+\tau} \boldsymbol{r}'(\mathbf{1} - \boldsymbol{\chi}).$$

That is, emissions $\times\,(1+\tau)$ equal equilibrium revenue for each sector multiplied by its dirty share. If we define the revenue flow matrix $\Theta_{ij} \equiv \chi_i \sigma_{ij}$, we can again ask how emissions will be affected by greenification in a given sector, which is given by

$$-\frac{\partial \ell_d}{\partial \chi_i} = \frac{1}{1+\tau} \boldsymbol{r}' \left[\boldsymbol{e_i} - \frac{\partial \boldsymbol{\Theta}}{\partial \chi_i} \left(\mathbf{I} - \boldsymbol{\Theta}\right)^{-1} (\mathbf{1} - \boldsymbol{\chi})\right], \tag{B.12}$$

where $\boldsymbol{e_i}$ is a standard unit vector for the ith dimension.[B.2] Note that $\frac{\partial \boldsymbol{\Theta}}{\partial \chi_i}$ will be all zeros but for the ith row composed of the input shares σ_{ij}. Therefore, Equation (B.12) has a very similar interpretation to Equation (28), with a term reflecting a sector's total demand out front and square brackets that include both a direct reduction in emissions as well as an offsetting increase in emissions due to demand for the inputs of other sectors.

As before, the effectiveness of upstream greenification for reducing emissions is more "vulnerable" to low levels of greenification. In particular, let $n(j)$ denote the minimum number of steps through the network needed before the goods from sector j are used in the final good. That is, $n(N) = 0$, $n(j) = 1$ for any sector that supplies to final good ($\sigma_{Nj} > 0$), $n(j) = 2$ for any sector that supplies the one-step sectors (but not the final good), and so on. In this more general setting, n provides us with a measure of a sector's upstreamness. If we again consider $\chi_j = \epsilon x_j$ and take ϵ to zero, then the term in square brackets will converge to $[1 - (1 - \alpha_i)]\boldsymbol{e_i}$. Moreover, the revenue of a sector will be $\mathcal{O}(\epsilon^{n(j)})$ because revenue will have to pass through $n(j)$-many steps before it reaches sector j. Thus, as in the vertical network case, we have that more upstream sectors are less effective at reducing emissions for low levels of greenification.

B.10 Utility Flow in the Extended Model

We show that in the extended model of Section 7. The steady-state utility flow is given by

$$\begin{aligned} & \ln y_2 - \sum_{i \in \{1a,1b,2\}} \left((1+\xi)\, \ell_{di} + \ell_{ci} \right) \\ = \; & \left[\chi_2 \alpha + \chi_2 (1-\alpha)\, \chi_{1b} + (1-\chi_2)(1-\alpha)\, \chi_{1a} \right] Z - \ln(1+\xi) - 1. \end{aligned} \tag{B.13}$$

Proof. The utility flow is given by $\ln y_{2t} - \sum_{i \in \{1a,1b,2\}} \left((1+\xi)\, \ell_{dit} + \ell_{cit} \right)$. In steady-state, all inputs are priced at marginal costs. Since downstream production is Cobb-Douglas between the clean and dirty production processes, steady-state output in sector 2 obeys

$$y_2 = \frac{\left(e^{\alpha z} \ell_{c2}^{\alpha} y_{1b}^{1-\alpha} \right)^{\chi_2} \left(\ell_{d2}^{\alpha} y_{1a}^{1-\alpha} \right)^{1-\chi_2}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} \chi_2^{\chi_2} (1-\chi_2)^{1-\chi_2}}, \tag{B.14}$$

where we kept the assumption that downstream greenified producers prefer to use the clean production process. In sector $1a$ and $1b$, we similarly have

$$y_{1k} = \frac{\left(e^{z} \ell_{c1k} \right)^{\chi_{1k}} \ell_{d1k}^{1-\chi_{1k}}}{\chi_{1k}^{\chi_{1k}} (1-\chi_{1k})^{1-\chi_{1k}}} \text{ for } k \in \{a, b\}\,. \tag{B.15}$$

B.2. To show this, combine $\boldsymbol{r}' = \boldsymbol{e}'_{\boldsymbol{N}} (\mathbf{I} - \boldsymbol{\Theta})^{-1}$ and $\frac{\partial (\mathbf{I} - \boldsymbol{\Theta})^{-1}}{\partial \chi_i} = (\mathbf{I} - \boldsymbol{\Theta})^{-1} \frac{\partial \boldsymbol{\Theta}}{\partial \chi_i} (\mathbf{I} - \boldsymbol{\Theta})^{-1}$.

Since wages and revenues are both equal to one, clean labor commands income shares equal to its overall factor share in production. Thus, we have

$$\ell_{c2} = \chi_2\alpha,\ \ell_{c1b}=\chi_2\left(1-\alpha\right)\chi_{1b} \text{ and } \ell_{c1a}=\left(1-\chi_2\right)\left(1-\alpha\right)\chi_{1a}. \tag{B.16}$$

With Pigouvian taxation, the cost of using dirty labor is $1+\xi$, such that we obtain

$$\left(1+\xi\right)\ell_{d2} = \left(1-\chi_2\right)\alpha,\ \left(1+\xi\right)\ell_{d1b}=\chi_2\left(1-\alpha\right)\left(1-\chi_{1b}\right) \text{ and } \left(1+\xi\right)\ell_{d1a}=\left(1-\chi_2\right)\left(1-\alpha\right)\left(1-\chi_{1a}\right). \tag{B.17}$$

Using these expressions, we get that the total disutility of labor in production plus pollution is always equal to one:

$$\left(1+\xi\right)\sum_i \ell_{di} + \sum_i \ell_{ci} = 1. \tag{B.18}$$

Plugging (B.15) into (B.14) and taking logs, we can express log output as

$$\begin{aligned}
\ln y_2 \quad = \quad & -\left(\chi_2\ln\chi_2 + \left(1-\chi_2\right)\ln\left(1-\chi_2\right)\right) - \alpha\ln\alpha - \left(1-\alpha\right)\ln\left(1-\alpha\right) \\
& +\chi_2\alpha z + \chi_2\alpha\ln\ell_{c2} + \left(1-\chi_2\right)\alpha\ln\ell_{d2} \\
& +\chi_2\left(1-\alpha\right)\left[\chi_{1b}z + \chi_{1b}\ln\ell_{c1b} - \chi_{1b}\ln\chi_{1b} + \left(1-\chi_{1b}\right)\ln\ell_{d1bt} - \left(1-\chi_{1b}\right)\ln\left(1-\chi_{1b}\right)\right] \\
& +\left(1-\chi_2\right)\left(1-\alpha\right)\left[\chi_{1a}z + \chi_{1a}\ln\ell_{c1a} - \chi_{1a}\ln\chi_{1a} + \left(1-\chi_{1a}\right)\ln\ell_{d1a} - \left(1-\chi_{1a}\right)\ln\left(1-\chi_{1a}\right)\right].
\end{aligned}$$

Substituting the labor allocations, (B.16) and (B.17), into the previous expression gives

$$\begin{aligned}
\ln y_2 \quad = \quad & -\left(\chi_2\ln\chi_2 + \left(1-\chi_2\right)\ln\left(1-\chi_2\right)\right) - \alpha\ln\alpha - \left(1-\alpha\right)\ln\left(1-\alpha\right) \\
& +\chi_2\alpha z + \chi_2\alpha\ln\left(\chi_2\alpha\right) + \left(1-\chi_2\right)\alpha\ln\frac{\left(1-\chi_2\right)\alpha}{\left(1+\xi\right)} \\
& +\chi_2\left(1-\alpha\right)\left[\chi_{1b}z + \ln\chi_2 + \ln\left(1-\alpha\right) - \left(1-\chi_{1b}\right)\ln\left(1+\xi\right)\right] \\
& +\left(1-\chi_2\right)\left(1-\alpha\right)\left[\chi_{1a}z + \ln\left(1-\chi_2\right) + \ln\left(1-\alpha\right) - \left(1-\chi_{1a}\right)\ln\left(1+\xi\right)\right].
\end{aligned}$$

Rearranging terms, we then obtain

$$\begin{aligned}
\ln y_2 \quad = \quad & \chi_2\alpha z - \left(1-\chi_2\right)\alpha\ln\left(1+\xi\right) + \chi_2\left(1-\alpha\right)\left[\chi_{1b}z - \left(1-\chi_{1b}\right)\ln\left(1+\xi\right)\right] \\
& +\left(1-\chi_2\right)\left(1-\alpha\right)\left[\chi_{1a}z - \left(1-\chi_{1a}\right)\ln\left(1+\xi\right)\right].
\end{aligned}$$

Using this expression together with (B.18) delivers (B.13). □

B.11 Backfiring Industrial Policy

Here we build an example where an industrial policy initially focused on the upstream sector $1a$ backfires: given initial conditions and the greenification cost functions, the laissez-faire is associated with full greenification in sectors 2 and $1b$, whereas a (misguided) industrial policy focusing on sector $1a$ in the initial period reduces long-run welfare by preventing full greenification in sectors 2 and $1b$. Key to our example is the fact that greenification in sector $1a$ reduces the cost-advantage of greenified varieties in sector 2, which reduces the incentives to greenify both sector 2 and its upstream sector, $1b$.

Example 2. We assume that the economy initially features no greenification in all three sectors ($\chi_{i,0} = 0$ for $i \in \{2, 1a, 1b\}$). We consider two potential policies: In the first one, the government does not subsidize greenification. In the second one, the government subsidizes greenification in sector $1a$ at time $t = 1$, such that $\chi_{1a,1} = 1$. In both cases, the government implements a Pigouvian carbon tax: $\tau = \xi$.

Consider first the case with no government intervention. Then greenification at time 1 in sector 2 follows from (32): with $\mu_{2,0} = \alpha$, where $\chi_{2,1}$ is given by $\chi_{2,1} = F_2\left(1 - e^{-Z\alpha}\right)$. In addition, following (8), $\widetilde{\chi}_{2,1} = \chi_{2,1}e^{-Z\alpha}$ so that, using (31), $\chi_{1b,1}$ satisfies

$$\chi_{1b,1} = F_{1b}\left(\left(1 - e^{-Z}\right)\chi_{2,1}e^{-Z\alpha}\left(1 - \alpha\right)\right).$$

We assume that this is positive, i.e. the smallest fixed cost of greenification in sector $1b$ lies below $(1 - e^{-Z})\chi_{2,1}e^{-Z\alpha}\left(1 - \alpha\right)$. For sector $1a$, we then get

$$\chi_{1a,1} = F_{1a}\left(\left(1 - e^{-Z}\right)\left(1 - \chi_{2,1}\right)\left(1 - \alpha\right)\right),$$

which we also assume to be equal to zero. This in turn will be the case if the smallest fixed cost of greenification in sector $1a$ is greater than $\left(1 - e^{-Z}\right)\left(1 - \alpha\right)$.

We now consider period $t = 2$. Since $\chi_{2,t}$ is non-decreasing, the incentives to greenify sector $1a$ are weakly smaller, so that we still have $\chi_{1a,2} = 0$. Using (30), we get $\mu_{2,1} = \alpha + \chi_{1b,1}\left(1 - \alpha\right)$. Therefore, following (32), we get that greenification in sector 2 is given by $\chi_{2,2} = F_{2b}\left(1 - e^{-Z\mu_{2,1}}\right)$, which we take to be equal to one. That is, all greenification costs in sector 2 are below $1 - e^{-Z\mu_{2,1}}$. We now get $\widetilde{\chi}_{2,2} = \chi_{2,1} + \left(1 - \chi_{2,1}\right)e^{-Z\mu_{2,1}}$ so that, using (31), $\chi_{1b,2}$ satisfies

$$\chi_{1b,2} = F_{1b}\left(\left(1 - e^{-Z}\right)\left(\chi_{2,1} + \left(1 - \chi_{2,1}\right)e^{-Z\mu_{2,1}}\right)\left(1 - \alpha\right)\right).$$

Again, we take this to be equal to one: namely, all greenification costs in sector $1b$ are below the term at which F_{1b} is evaluated in the expression above. With full greenification in sectors $1b$ and 2, the economy has reached a steady-state with $\chi^*_{1a} = 0$ and $\chi^*_{1b} = \chi^*_2 = 1$. Not greenifying sector $1a$ in this context comes at no cost, since the input from that sector is used only for dirty production in sector 2 which disappears from $t = 3$ onwards. Using (B.13), the corresponding utility flow is

$$\ln y_2 - 1 = z - 1.$$

Now, suppose instead that starting from no greenification in all sectors, the government decides to fully greenify the upstream sector $1a$ at time $t = 1$, i.e. sets $\chi^{\dagger}_{1a,1} = 1$ (we add † to denote variables under this alternative scenario). Since greenification incentives only move downstream with a lag, this does not change $\chi_{2,1}$ and $\chi_{1b,1}$, which remain the same as without the policy: $\chi^{\dagger}_{2,1} = \chi_{2,1}$ and $\chi^{\dagger}_{1b,1} = \chi_{1b,1} = 0$.

Consider now time $t = 2$. Using (30), we get that: $\mu^{\dagger}_{2,1} = \alpha - (1 - \chi_{1b,t-1})(1 - \alpha) < \mu_{2,0}$. As a result, there is no further greenification in sector 2, namely: $\chi^{\dagger}_{2,2} = \widetilde{\chi}^{\dagger}_{2,2} = \chi^{\dagger}_{2,1} = \chi_{2,1}$. In sector $1b$, using (31), we get

$$\chi^{\dagger}_{1b,2} = F_{1b}\left(\left(1 - e^{-Z}\right)\chi_{2,1}(1 - \alpha)\right).$$

We assume that this is still equal to $\chi_{1b,1}$: That is, the distribution of greenification costs in sector $1b$ is such that a positive mass of varieties have fixed costs below $\left(1 - e^{-Z}\right)\chi_{2,1}e^{-Z\alpha}(1 - \alpha)$, no varieties have fixed costs in the interval $\left(1 - e^{-Z}\right)(1 - \alpha) \times \left(\chi_{2,1}e^{-Z\alpha}, \chi_{2,1}\right)$, and all remaining varieties have their fixed costs in the interval $\left(1 - e^{-Z}\right)(1 - \alpha) \times \left(\chi_{2,1}, \chi_{2,1} + (1 - \chi_{2,1})\, e^{-Z\mu_{2,1}}\right)$. At this point, the economy has reached a steady-state and no further greenification occurs. This gives a flow utility

$$\ln y_2 - 1 = \left[\chi_{2,1}\alpha + \chi_{2,1}(1 - \alpha)\chi_{1b,1} + (1 - \chi_{2,1})(1 - \alpha)\right] Z - \ln(1 + \xi) - 1.$$

For $\chi_{2,1}$ sufficiently small, which is clearly feasible, this is strictly less than the flow utility $z - 1$ under laissez-faire. That is, industrial policy backfires.

Note, however, that industrial policy cannot backfire when the decentralized economy is initially already stuck in a steady-state. Since the steady-state utility flow is weakly increasing in all the $\chi's$, the only cost associated with industrial policy in that case are the costs of the subsidies themselves (i.e. the costs associated with the corresponding increase in the $\mathcal{F}_i(\chi_i)$'s).

B.12 Excessive Greenification and Path Dependence

We now build an example where, given initial conditions and the greenification cost functions, the laissez-faire allocation involves positive greenification in sector $1a$, but no greenification in sectors 2 and $1b$, whereas the optimal allocation involves less greenification in sector $1a$, but positive greenification in sectors 2 and $1b$. In other words, the laissez-faire economy exhibits excessive greenification in sector $1a$.

Example 3. We still assume that a Pigouvian carbon tax is in place. We choose the cost functions so that there exists a steady-state $(\chi_{1a}^*, \chi_{1b}^*, \chi_2^*)$ with $\chi_{1a}^* > 0$ and $\chi_{1b}^* = \chi_2^* = 0$:

$$\chi_{1a}^* = F_{1a}\left(\left(1 - e^{-Z}\right)(1-\alpha)\right) \text{ and } \chi_2^* = 0 = F_2\left(1 - e^{-Z\left(\alpha - (1-\alpha)\chi_{1a}^*\right)}\right).$$

The latter condition requires that the lowest fixed cost in sector 2 be above $1 - e^{-Z\left(\alpha-(1-\alpha)\chi_{1a}^*\right)}$.

Suppose now that the economy starts with greenification shares $\chi_{1a,0} = \chi_{1a}^* - \varepsilon$ (with $\varepsilon > 0$ but small) and $\chi_{1b,0} = \chi_{2,0} = 0$. Then, we get that $\chi_{1a,1}$ and $\chi_{2,1}$ solve

$$\chi_{1a,1} = F_{1a}\left(\left(1 - e^{-Z}\right)(1-\alpha)\right) = \chi_{1a}^* \text{ and } \chi_{2,1} = F_2\left(1 - e^{-Z(\alpha-(1-\alpha)\chi_{1a,0})}\right) = 0$$

whenever ε is sufficiently small and the smallest fixed greenification cost in sector 2 lies significantly above $1 - e^{-Z\left(\alpha-(1-\alpha)\chi_{1a}^*\right)}$ relative to ε. Then, the laissez-faire economy will reach the steady-state $(\chi_{1a}^*, \chi_{1b}^* = 0, \chi_2^* = 0)$.

Let us compare this laissez-faire equilibrium with the social optimum. We note first that the Social Planner always wants to implement the socially optimal steady-state immediately. Then, provided that greenification costs are bounded above, a sufficiently patient Social Planner will seek to maximize steady-state utility flow given by (B.10), which is maximized with $\chi_{1b}, \chi_2 = 1$ regardless of χ_{1a}. Therefore, the Social Planner immediately sets greenification to $(\chi_{1a,0}, 1, 1)$. The corresponding χ_{1a} is lower than under laissez-faire: there is excessive greenification of $1a$ in laissez-faire compared to the social optimum.

C Details on the Quantification

C.1 Calibrated Model

We briefly present the model that we calibrate in Section 8. We modify the model such that each sector may feature heterogeneity in the relative productivity of clean and dirty technologies z_i, the emission rate associated with the use of the dirty production process ξ_i and a TFP parameter A_i. We present the model for $N \geq 2$, though we will have $N = 2$ in the calibration. Therefore a variety ν in sector i is now produced according to

$$y_{it}(\nu) = A_i\left[\ell_{dit}(\nu) + \gamma_{it}(\nu)\left(\frac{e^{z_i}\ell_{cit}(\nu)}{\alpha_i}\right)^{\alpha_i}\left(\frac{m_{it}(\nu)}{1-\alpha_i}\right)^{1-\alpha_i}\right].$$

We also now allow for a more general utility function

$$\sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \kappa \ell_t - a_t\right) \text{ with } p_t c_t = w_t \ell_t + \pi_t.$$

This allows us to set the labor disutility parameter κ to match the data at the consumer's optimality condition, $w_t = \kappa p_t c_t$.

Following the same logic as in the baseline model, the price of production process ν in sector i is given by:

$$p_{it}(\nu) = \begin{cases} \min\left(w_t^{\alpha_i} e^{-\alpha_i z_i} p_{i-1,t}^{1-\alpha_i}, w_t\left(1+\tilde{\tau}_{it}\right)\right)/A_i & \text{if electrified by time } t-1 \\ w_t\left(1+\tilde{\tau}_{it}\right)/A_i & \text{otherwise} \end{cases}, \tag{C.1}$$

where $\alpha_1 = 1$, and the term $p_{i-1,t}^{1-\alpha_i}$ drops from the previous expression. Letting τ^C denote the carbon price per ton of CO_2, we define $\tilde{\tau}_{it} = \xi_i \tau^C A_i / w_t$ as the relevant tax policy parameter in the model (as the corresponding proportional tax on each good). We can then solve for the price index in each sector as

$$\frac{p_{1t}}{w_t} = \frac{1+\tilde{\tau}_{1t}}{A_1} \min\left\{\left(\frac{e^{-z_1}}{1+\tilde{\tau}_{1t}}\right)^{\chi_{1,t-1}}, 1\right\} \tag{C.2}$$

$$\frac{p_{it}}{w_t} = \frac{1+\tilde{\tau}_{it}}{A_i} \min\left\{\left(\frac{e^{-\alpha_i z_i}}{1+\tilde{\tau}_{it}}\left(\frac{p_{i-1,t}}{w_t}\right)^{1-\alpha_i}\right)^{\chi_{i,t-1}}, 1\right\} \text{ for } i > 1. \tag{C.3}$$

With heterogeneous sectors in z and τ, we cannot introduce the variable μ_i that allowed for closed form solutions as before, but this does not affect the logic of the model.

An innovator only greenifies if the marginal cost of using the clean production process is lower than that of the dirty production process. Assuming that this is the case and following the same steps as in the baseline model, we get that if a variety from sector i is greenified at time t, the innovator obtains a profit margin given by $1 - \frac{e^{-\alpha_i z_i}}{1+\tilde{\tau}_{it}}\left(\frac{p_{i-1,t}}{w_t}\right)^{1-\alpha_i}$ (or $1 - \frac{e^{-z_1}}{1+\tau_{1t}}$ for sector 1). That is, an innovator obtains profits given by

$$\pi_{it}(\nu) = r_{it}\left[1 - \frac{e^{-\alpha_i z_i}}{1+\tilde{\tau}_{it}}\left(\frac{p_{i-1,t}}{w_t}\right)^{1-\alpha_i}\right], \tag{C.4}$$

where, r_{it} still denotes the revenue of a variety in sector i at time t. We now get that in sector N, $r_{Nt} = p_t c_t \equiv E_t$, and we can obtain revenues for sector $i < N$ recursively from

$$r_{it} = E_t \prod_{j=i+1}^{N} \widetilde{\chi}_{jt} \left(1 - \alpha_j\right),$$

where $\widetilde{\chi}_{j,t}$ is now defined as

$$\widetilde{\chi}_{j,t} \equiv \chi_{j,t-1} + \left(\chi_{j,t} - \chi_{j,t-1}\right) \frac{e^{-\alpha_j z_j}}{1 + \tilde{\tau}_{jt}} \left(\frac{p_{j-1,t}}{w_t}\right)^{1-\alpha_j}. \tag{C.5}$$

Importantly, we introduce the possibility that the government subsidizes greenification in sector i at rate q_{it}. An equilibrium is then defined as follows:[C.1]

Definition. Given initial greenification shares $\{\chi_{i0}\}$, an equilibrium with a sequence of carbon taxes (relative to wages)$\{\tilde{\tau}_{it}\}$ and greenification subsidies $\{q_{it}\}$ is a sequence $\{\chi_{it}, \widetilde{\chi}_{it}, p_{it}\}_{t>0}$ such that p_{it} obey (C.2) and (C.3), $\widetilde{\chi}_{it}$ is given by (C.5), and χ_{it} obeys

$$\chi_{it} = \max\left\{ F_i\left(\left[1 - \frac{e^{-\alpha_i z_i}}{1 + \tilde{\tau}_{it}} \left(\frac{p_{i-1,t}}{w_t}\right)^{1-\alpha_i}\right] \frac{E_t \prod_{j=i+1}^{N} \widetilde{\chi}_{jt} \left(1 - \alpha_j\right)}{1 - s_{it}} \right), \chi_{i,t-1} \right\}. \tag{C.6}$$

As before, the equilibrium is unique. In turn, a steady-state is characterized by:

Definition. For given carbon taxes $\{\tau_i\}$ and greenification subsidies $\{q_i\}$, a steady-state is a vector of greenification shares and prices $\{\chi_i, p_i\}$ such that

$$\frac{p_1}{w} = \frac{1 + \tilde{\tau}_1}{A_1} \min\left\{ \left(\frac{e^{-z_1}}{1 + \tilde{\tau}_1}\right)^{\chi_1}, 1 \right\}, \quad \frac{p_i}{w} = \frac{1 + \tilde{\tau}_i}{A_i} \min\left\{ \left(\frac{e^{-\alpha_i z_i}}{1 + \tilde{\tau}_i} \left(\frac{p_{i-1}}{w_t}\right)^{1-\alpha_i}\right)^{\chi_i}, 1 \right\} \tag{C.7}$$

$$\chi_i \geq F_i\left(\left(1 - \frac{e^{-\alpha_i z_i}}{1 + \tilde{\tau}_i} \left(\frac{p_{i-1}}{w}\right)^{1-\alpha_i}\right) \frac{E \prod_{j=i+1}^{N} \chi_j \left(1 - \alpha_j\right)}{1 - s_i} \right). \tag{C.8}$$

As before, the inequality in (C.8) results from the fact that greenification can never decrease and we focus on interesting steady-states where (C.8) holds with equality.

To derive output in each sector, we start from the output for each variety, which using the Cobb-Douglas aggregator must obey $y_{it}(\nu) = r_{it}/p_{it}(\nu)$. We assume here that marginal costs are such that greenified varieties use the clean production process (as this is the relevant case for our computations). Output of a non-greenified or newly greenified variety is $y_{it}(\nu) = \frac{A_i r_{it}}{w_t(1+\tilde{\tau}_{it})}$, while the employment of dirty labor for non-greenified varieties is $\frac{r_{it}}{w_t(1+\tilde{\tau}_{it})}$. The output of a previously greenified variety is $y_{it}(\nu) = \frac{A_i r_{it}}{w_t^{\alpha_i} e^{-\alpha_i z_i} p_{i-1,t}^{1-\alpha_i}}$.

C.1. If the clean production process is more expensive than the dirty one, there is no greenification. As $\pi_{it}(\nu)$ in (C.4) is negative in that case, then (C.6) still holds.

We then get that the output of sectoral good i is

$$y_{it} = A_i \left(\frac{w_t}{p_{i-1,t}}\right)^{\chi_{i,t-1}(1-\alpha_i)} \frac{e^{\alpha_i z_i \chi_{i,t-1}} r_{it}}{w_t \left(1+\tilde{\tau}_{it}\right)^{1-\chi_{i,t-1}}}.$$

We can calculate dirty and clean labor in each sector, respectively, via

$$\ell_{dit} = \frac{1-\chi_{it}}{w_t\left(1+\tilde{\tau}_{it}\right)} r_{it}, \quad \ell_{cit}, = \frac{\widetilde{\chi}_{i,t}}{w_t} r_{i,t}\alpha_i.$$

Finally, disutility from emissions in sector i is given by

$$a_{it} = \iota \cdot \xi_i A_i \frac{1-\chi_{it}}{w_t\left(1+\tilde{\tau}_{it}\right)} r_{it},$$

where the emissions disutility parameter ι ensures that the household's willingness to pay to avoid another ton of CO_2 emissions equals the social cost of carbon (SCC) via $\iota = \frac{\kappa}{w} SCC$. Total emissions disutility is then given by $a = a_1 + a_2$.

C.2 Calibration Details

Initial Wages, Prices, Productivities, and Emissions: The initial wage w_0 equals the value-added per worker-year in LRHD transportation services in 2022 (= \$362,220 based on initial output of \$833.1 billion (BTS 2024) and employment of 2.3 million worker-years (BLS 2024 counting air, rail, water, and truck transportation (NAICS 481-484)).[C.2] The initial price of transportation services $p_{20} = \$0.23$ is calculated as a Cobb-Douglas price index over the revenue-share weighted average freight price per ton-mile of \$0.26 and the revenue-share weighted average passenger price per passenger-mile of \$0.13 in 2022 (both calculated for air, truck, railroad, and water transport from BTS Tables 1-50, 3-21, 1-40, and 3-20), where the expenditure share of freight (vs. passenger) miles is set to 0.81 in line with its value for 2022 in the data. Transportation TFP A_2=1.5914$\times 10^6$ is calculated from (C.3), where we set τ_{20} based on the average effective carbon price in the United States in 2021 (\$13/tCO_2, OECD 2023) and the sector's emissions intensity (0.0008 tCO_2e/\$ calculated from total emissions data from EPA 2024 and output data from BTS 2024) and assume that $\chi_{20} = 0$. The initial price of delivered GH_2 is set to $p_{10} = \$4.65$/kg based on the current (2023) estimated US fossil hydrogen production cost of \$1.45/kg from BloombergNEF (2023) plus current fossil (truck) distribution costs of \$3.20/kg estimated using the Argonne National Laboratory's HDSAM (v4.5, Elgowainy et al. 2024, as de-

C.2. We include in "heavy duty" some trucks that would technically be categorized as "medium duty" under the US Department of Transportation classification based on gross vehicle weight ratings.

scribed further below) and converted to \$2022 (using US GDP deflator data from FRED). Hydrogen TFP $A_1 = 8.8779 \times 10^4$ is calculated from (C.2) analogous to the calculation of A_2 but with $\tau_{1,0}$ based on an estimated emissions intensity of 10.75 $kgCO_2/kgH_2$with 9 $kgCO_2/kgH_2$ from natural gas-based hydrogen production (median value from IEA 2023) plus 1.75 $kgCO_2/kgH_2$from distribution (from the HDSAM). Here we also assume that $\chi_{1,0} \approx 0$ based on estimates from the IEA (2024).

Transportation Parameters α_2 and z_2: First, the studies used to calibrate α_2 (described in Section 8.1) include Ahluwalia et al. (2021, 2020), Hoelzen et al. (2023), Ledna et al. (2024), Gilleon et al. (2022), Burnham et al. (2021), Hunter et al. (2021), and Steer (2023). Second, we infer the implied value of $z_{2,i}$ from each literature paired estimate i of the levelized costs of producing a transportation service with fossil vs. hydrogen fuels via the associated ratio of clean vs. dirty marginal costs:

$$\frac{mc_{c,i}}{mc_{d,i}} = \frac{(w_0)^{\alpha_2} e^{-\alpha_2 z_{2,i}} (p_{1,i})^{1-\alpha_2} / A_2}{w_0(1 + \widetilde{\tau}_{2,i})/A_2},$$

with $p_{1,i}$ based on the hydrogen input price assumed in each study i (with adjustments where necessary to reflect the cost of *distributed* and not dispensed hydrogen as explained below), $\widetilde{\tau}_{2,i}$ based on the tax rates assumed, and the other parameters are set as described above. We then average across $z_{2,i}$'s for each variety and compute the value added-share weighted average across varieties.

Aviation Fixed Costs: For aircraft development costs, we first take an industry estimate of \$16.4 billion (Steer 2023) for a H_2 narrow-body short- to medium-range (NB/SMR) aircraft similar to the Airbus A320 and scale it up based on typical NB/SMR aircraft family expenditure shares to yield an overall cost of \$55 billion (\$2022).[C.3] For the wide-body long-range (WB/LR) market, development costs are generally expected to be higher as the lower volumetric energy density of hydrogen necessitates additional changes in aircraft designs to accommodate increased fuel storage requirements (ICAO 2022a). We thus adjust our per-aircraft family development cost estimate upwards based on estimated hydrogen-gas turbines and blended wing-body aircraft development costs (from Balles-

C.3. Using US BTS Air Carrier Financial (Schedule P-5.2) operating expenditure data—which include both passenger and cargo operations—for 2023 we calculate an NB/SMR market expenditure share of 25.2% for the A320 family (including both A318 and A319 aircraft but not the A321/LR due to its distinctly larger size and range). Given Airbus' smaller market share in the United States, we also consider the Boeing 737 (again excluding longer range variants such as the 737-900ER) for which the corresponding expenditure share is 45.6%. We thus assume that the \$16.4 billion development costs cover 30% of the NB/SMR market.

teros et al. 2022) to yield a per-aircraft family cost of \$21.8 billion and scale this figure up based on typical aircraft family market shares for WB/LR aircraft, which we take to be 15%,[C.4] yielding \$145 billion in overall WB/LR aircraft development costs. For airport infrastructure adjustments, we assume a cost increase of 9.1% per revenue passenger kilometer during the adjustment period, in line with other estimates.[C.5]

Other Transportation Fixed Costs: For trucking, rail, and water–passenger, we quantify the excess costs of initial clean technology adoption based on pairs of LCO estimates for current vs. future hydrogen-fueled transportation within each variety. For refueling infrastructure, we infer (capital) cost shares from Bracci et al. (2024) and Reddi et al. (2017) and apply these to the relevant estimates of *dispensed* H_2 costs underlying each LCO estimate from the literature. For water-freight, we lack sufficient data to quantify fixed costs analogously to the other varieties. Given the challenges associated with the use of H_2 as a fuel in long-distance and cargo shipping (EMSA 2023; Ahluwalia et al. 2020), we thus set the fixed costs for this variety as double the fixed costs of passenger water travel (ferries).

Hydrogen Productivity Parameter z_1: We again construct pairs of levelized cost estimates i from fossil vs. clean production to infer $z_{1,i}$ from the corresponding clean vs. dirty marginal cost ratios (via $\frac{mc_{c,i}}{mc_{d,i}} = e^{-z_{1,i}}(1 + \widetilde{\tau}_{1,i})$). For H_2 production, we consult BloombergNEF (2023) estimates for US hydrogen production using SMR today vs. hydrogen in the future (2050). For H_2 distribution, we run the Argonne HDSAM model (v 4.5) for the Medium/Heavy-Duty market assuming either Tube-Trailer or Pipeline distribution for each region.[C.6] We then average the resulting $z_{1,i}$ estimates for distribution across

C.4. BTS data suggest WB/LR expenditure shares range from, e.g., 9.2% for older models such as the B757 to 18.1% for the B777 family.

C.5. Hoelzen et al. (2023) estimate that airport refueling system adjustment and liquefaction plant costs will add around \$0.48 to the levelized cost per kg of dispensed liquid hydrogen (LH_2) at a typical airport. We combine this estimate with projected future hydrogen aircraft fuel economy (ICCT 2022) and reference aircraft operating costs (Hoelzenen et al. 2022) to calculate the corresponding cost increase per passenger kilometer. For comparison, Steer (2023) project an 11% future increase in airport charges in a typical flight due to the investment costs required to accommodate hydrogen aircraft.

C.6. We assume a typical transmission distance of 640 km, in line with, e.g., Tayarani and Ramji (2022) and in the central range of the broader H_2 transmission literature. For example, Frank et al. (2021) consider 50, 550, and 1500 km; diLullo et al. (2022) consider 100-3000 km, and Demir et al. (2018) consider 100km. We also assume a typical throughput of 150 tonnes/day and leave the remaining parameters at their HDSAM benchmarks. To isolate the levelized cost of H_2 transmission, we subtract from the HDSAM's resulting cost estimates (in \$/kgH_2) the costs associated with refueling stations (accounted for in downstream fixed costs) and the capital cost component of the pipelines (included in the upstream fixed costs).

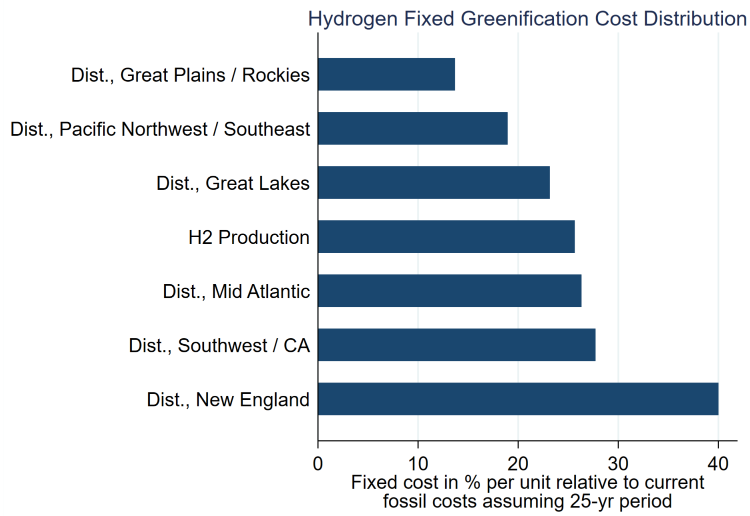


Figure C.1. Distribution of fixed greenification costs in the upstream sector

regions and compute a weighted average value across production and distribution based on the current cost share of each in the levelized cost of (fossil) H_2, yielding $z_1 = 1.608$.

Hydrogen Sector Fixed Costs: First, we calculate the fixed costs $\phi_1(.)$ for *production* based on *excess initial* current vs. future (2050) costs. We adjust the raw projected cost difference in three ways: First, by removing projected cost declines due to improvements in renewable electricity generation (which we take to occur outside the model), and second by accounting for the exponential nature of the cost declines projected by BloombergNEF over the next 25 years. Second, for distribution, we take the capital costs (per kg H_2) for pipeline construction in each region from the HDSAM model.[C.7] Third, we adjust the magnitude of the fixed costs downward based on the projected share of transportation in future hydrogen demand (which we take to be around 40% based on IEA 2023) to account in reduced form for other downstream sectors which demand H_2. Figure C.1 showcases the estimated distribution of fixed greenification costs (in percent relative to current per unit fossil costs) in the hydrogen production and distribution sector. Fixed pipeline distribution costs are lowest in the Great Plains and highest in New England due to, e.g., the relatively flat vs. rocky nature of the terrain .

Robustness: Our preferred quantification re-scales fixed costs to account for addi-

C.7. We assume greenfield hydrogen pipeline development. Our quantification could, however, also accommodate the possibility of retrofitting existing natural gas pipelines.

tional potential revenue sources outside our model—such as non-US demand for H_2 aircraft—that could, in reality, help offset some of the fixed technology development costs in our calibration. We now discuss the robustness of our results to this assumption. First, for transportation services, removing the scaling factor—that is, assuming that 100% of the fixed technology development costs must be compensated by US domestic demand for H_2 transportation—does not change the results meaningfully. Figure C.2 presents a version of Figure 4.b) with the re-scaling factor removed, which reveals a very similar pattern of multiplicity in steady-states. It also remains the case that a temporary downstream technology development subsidy can lead to sustained and high levels of greenification, whereas the same level of upstream subsidy cannot. While the *level* of the subsidy is higher with the re-scaling factor removed (increasing form 50% to 65% to achieve greenification levels of 89% and 95% in Sectors 1 and 2, respectively), the relative effectiveness of downstream vs. upstream targeting is unchanged.

Second, we consider alternative scaling for fixed costs in hydrogen production and distribution. As there already exists other demand for H_2 in the US economy, such as from ammonia fertilizer production, we first consider a change from 40% to 60% as the implied share of clean H_2 technology development costs that must be borne by the LRHD transportation sector. Here we again find (i) multiplicity of steady-states as shown in Figure C.2 and (ii) that a temporary downstream subsidy is more effective at inducing greenification (with, e.g., a 65% two-period subsidy inducing greenification shares of 78% upstream and 99% downstream) than an equivalent upstream subsidy (which achieves no greenification). Finally, for a hypothetical 100% cost share from transportation (no rescaling), we find that, at a carbon price of \$300/tCO_2, (0,0) is now the only steady-state. However, in the dynamic analysis, we again find that a downstream fixed cost subsidy is more effective (with a 75% two-period subsidy leading to greenification shares of 11% upstream and 99% downstream) than an equivalently sized upstream subsidy (which achieves no greenification), showcasing the robustness of this result.

C.3 Stylized Quantification: Two Downstream Sectors

This appendix presents two stylized quantifications of the profit elasticity conditions in the model version with two downstream sectors (22). The upstream sector is electricity

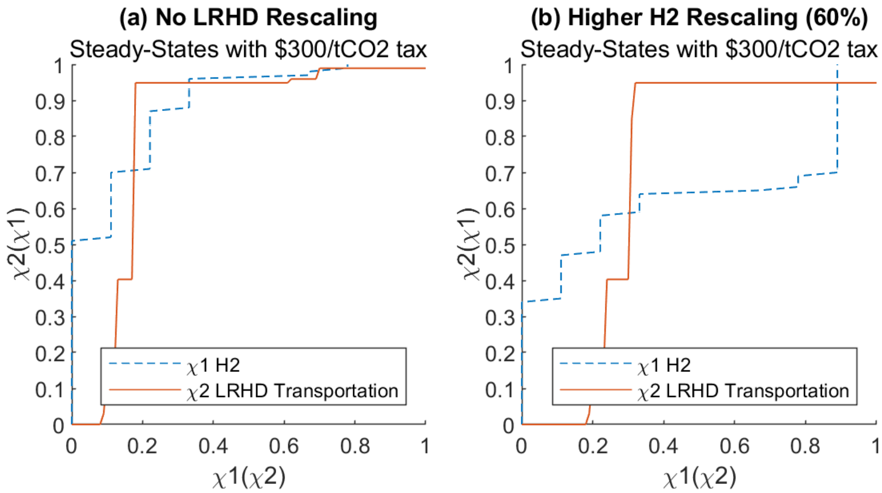


Figure C.2. Steady-state greenification shares (a) without re-scaling of LRHD Transportation fixed costs to account for global demand and (b) with less re-scaling of H_2 fixed costs (60% instead of 40%) to account for other H_2 demand

generation in both examples, for which we take a baseline green share of $\chi_1 = 0.4$.[C.8]

First, we consider energy use in US manufacturing, where significant greenification potential exists through, e.g., the electrification of certain industrial boilers especially in non-durable goods producing industries such as food, paper, and chemicals (Schoenenberger et al. 2022). We define the two downstream sectors in (22) as durable ("D") and non-durable ("ND") manufacturing. We quantify their current greenification shares from the EIA's Manufacturing Energy Consumption Survey[C.9] as $\chi_{2,D} = 0.294$ and $\chi_{2,ND} = 0.133$, respectively, and their expenditure shares based on the US BEA's Real Value Added by Industry tables (for 2022) as $\lambda_D = 0.56$ and $\lambda_{ND} = 0.44$, respectively, where we adopt the BEA's delineation of durable vs. non-durable goods. Finally, while energy cost shares are generally estimated to be in the single digits both in aggregate and in US manufacturing (Ganapati et al. 2020), we assume a lower labor share value of $\alpha = 0.9$ both to account for potentially higher energy cost shares after electrification and to err on the

C.8. We use US EIA data, counting nuclear and renewables as "green" in "US electricity generation by major energy source 1950-2023", obtained from (accessed February 2025): [https://www.eia.gov/energyexplained/electricity/electricity-in-the-us-generation-capacity-and-sales.php].

C.9. We specifically compute the share of net electricity in total energy consumption for each industry from the 2018 MECS Table 3.2 obtained from (accessed February 2025): [https://www.eia.gov/consumption/manufacturing/data/2018/].

side of finding larger effects of upstream greenification. For the same reason, we consider a range of values for Z (from 0.0001 to 10) and report the highest resulting estimate of $\frac{\partial \ln \pi_{2,k}}{\partial \ln \chi_1}$, which is 0.0426. The corresponding upstream profit elasticities with respect to downstream greenification are $\frac{\partial \ln \pi_1}{\partial \ln \chi_{2,D}} = 0.7378$ and $\frac{\partial \ln \pi_1}{\partial \ln \chi_{2,ND}} = 0.2622$. Consequently, in this example, it is still the case that upstream profits are more elastic to downstream greenification than vice versa.

Our second example considers the broader US economy with four downstream sectors: industry, commercial, residential, and transportation, with respective current electrification shares of $\chi_{2,I} = 0.13$, $\chi_{2,C} = 0.5$, $\chi_{2,R} = 0.44$, and $\chi_{2,T} = 0.01$.[C.10] The respective expenditure shares are calculated from BEA data on Value Added by Industry (in 2022) as $\lambda_I = 0.178$, $\lambda_C = 0.650$, $\lambda_R = 0.136$, and $\lambda_R = 0.036$.[C.11] For this example, we adopt a standard aggregate value of non-energy cost share of $\alpha = 0.96$. The results indicate a maximum value of $\frac{\partial \ln \pi_{2,k}}{\partial \ln \chi_1} = 0.0164$, whereas the corresponding elasticities with respect to downstream greenification range from 0.00088 (transportation) to 0.7959 (commercial). These results indicate that, in this case, the profit elasticity *can* be higher for upstream greenification, but also that downstream greenification may still induce a substantially higher upstream profit effect even in this setting.

References

Agency, International Energy. 2023. *Comparison of the emissions intensity of different hydrogen production routes.* Technical report.

Agency, US Environmental Protection. 2024. *Fast Facts on Transportation Greenhouse Gas Emissions.* Technical report.

Ahlualia, R.K, D. Papadias, and X. Wang. 2020. *Rail and Maritime Metrics.* Technical report TA034. Argonne National Laboratory.

Ahlualia, R.K, J.K. Peng, F. Cetinbas, D.D. Papadias, X. Wang, J. Kopasz, and T. Krause. 2021. "Rail, Aviation, and Maritime Metrics," no. TA034.

C.10. Data for 2023 is from the US EIA's Monthly Energy Review (accessed February 2025): [https://www.eia.gov/energyexplained/us-energy-facts/images/consumption-by-source-and-sector.pdf].

C.11. We map the four sectors into corresponding NAICS codes based on the EIA documentation for industry (31-33, 11, 21, 23), and assume NAICS (48-49) for transportation, (53) for residential, and attribute the remaining output to commercial. We re-scale all output shares to be net of utilities (NAICS 22).

Ballesteros, Marta, and Ricardo et al. Neiva. 2022. *Research for TRAN Committee - Investment scenario and roadmap for achieving aviation Green Deal objectives by 2050.* Technical report.

Bhashyam, Adithya. 2023. "2023 Hydrogen Levelized Cost Update." *BloombergNEF.*

Bracci, Justin, Mariya Koleva, and Mark Chung. 2024. *Levelized Cost of Dispensed Hydrogen for Heavy-Duty Vehicles.* Technical report.

Brown, Daryl, Krishna Reddi, and Amgad Elgowainy. 2022. "The Development of Natural Gas and Hydrogen Pipeline Capital Cost Estimating Equations." *International Journal of Hydrogen Energy* 47 (79): 33813–33826.

Burnham, Andrew, David Gohlke, Luke Rush, Thomas Stephens, Yan Zhou, Mark A. Delucchi, and Alicia et al. Birky. 2021. *Comprehensive Total Cost of Ownership Quantification for Vehicles with Different Size Classes and Powertrains.* Technical report ANL/ESD-21/4. Argonne National Laboratory.

Demir, Murat Emre, and Ibrahim Dincer. 2018. "Cost Assessment and Evaluation of Various Hydrogen Delivery Scenarios." *International Journal of Hydrogen Energy.*

Di Lullo, Giovanni, Temitayo Giwa, Ayodeji Okunlola, Matthew Davis, T Mehedi, AO Oni, and A Kumar. 2022. "Large-Scale Long-Distance Land-Based Hydrogen Transportation Systems: A Comparative Techno-Economic and Greenhouse Gas Emission Assessment." *International Journal of Hydrogen Energy* 47 (83): 35293–35319.

European Maritime Safety Agency. 2023. *Potential of Hydrogen as Fuel for Shipping.* Technical report.

Frank, Edward, Amgad Elgowainy, Krishna Reddi, and Adarsh Bafana. 2021. "Life-Cycle Analysis of Greenhouse Gas Emissions from Hydrogen Delivery: A Cost-Guided Analysis." *International Journal of Hydrogen Energy* 46 (43): 22670–22683.

Ganapati, Sharat, Joseph Shapiro, and Reed Walker. 2020. "Energy Cost Pass-Through in US Manufacturing: Estimates and Implications for Carbon Taxes." *American Economic Journal: Applied Economics* 12 (2): 303–342.

Gilleon, Spencer, Michael Penev, and Chad Hunter. 2022. "Powertrain Performance and Total Cost of Ownership Analysis for Class 8 Yard Tractors and Refuse Trucks."

Hoelzen, Julian, M Flohr, Daniel Silberhorn, Jonas Mangold, A Bensmann, and Richard Hanke-Rauschenbach. 2022. "H2-powered aviation at airports–Design and economics of LH2 refueling systems." *Energy Conversion and Management: X.*

Hoelzen, Julian, L Koenemann, Lukas Kistner, F Schenke, A Bensmann, and Richard Hanke-Rauschenbach. 2023. “H2-Powered Aviation–Design and Economics of Green LH2 Supply for Airports.” *Energy Conversion and Management: X* 20:100442.

Hunter, Chad, Michael Penev, Evan Reznicek, Jason Lustbade, Alicia Birky, and Chen Zhang. 2021. *Spatial and Temporal Analysis of the Total Cost of Ownership for Class 8 Tractors and Class 4 Parcel Delivery Trucks.* Technical report.

IEA. 2023. *World Energy Outlook 2023.* Technical report.

International Energy Agency. 2024. *Global Hydrogen Review 2024.* Technical report.

Labor Statistics, US Bureau of. 2024. *Industries at a Glance: Transportation and Warehousing NAICS 48-49.* Technical report.

Ledna, Catherine, Matteo Muratori, Arthur Yip, Paige Jadun, Christopher Hoehne, and Kara Podkaminer. 2024. “Assessing Total Cost of Driving Competitiveness of Zero-Emission Trucks.” *iScience* 27 (4).

Mukhopadhaya, Jayant, and Dan Rutherford. 2022. *Performance Analysis of Evolutionary Hydrogen-Powered Aircraft.* Technical report.

OECD. 2023. *Effective Carbon Rates 2023.* Technical report.

Reddi, Krishna, Amgad Elgowainy, Neha Rustagi, and Erika Gupta. 2017. “Impact of Hydrogen Refueling Configurations and Market Parameters on the Refueling Cost of Hydrogen.” *International Journal of Hydrogen Energy* 42 (34): 21855–21865.

Schoeneberger, Carrie, Jingyi Zhang, Colin McMillan, Jennifer Dunn, and Eric Masanet. 2022. “Electrification Potential of US Industrial Boilers and Assessment of the GHG Emissions Impact.” *Advances in Applied Energy* 5:100089.

Steer. 2023. *Analysing the costs of hydrogen aircraft.* Technical report 24135101. Transport & Environment.

Tayarani, Hanif, and Aditya Ramji. 2022. “Life Cycle Assessment of Hydrogen Transportation Pathways via Pipelines and Truck Trailers: Implications as a Low Carbon Fuel.” *Sustainability* 14 (19): 12510.

Transportation Bureau of Transportation Statistics, US Department of. 2024. *Gross domestic product (GDP) attributed to transportation by mode.* Technical report.