Lemon Cycles[*]

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Abstract

We build a model of endogenous credit cycles arising from the dynamics of adverse selection. Heterogeneous entrepreneurs trade productive assets in an anonymous market subject to financial frictions. Cream-skimming rent-seekers create lemon assets that can be traded. Lemon assets are indistinguishable ex-ante from the productive assets but have no productive value ex-post. The average quality of assets is the key state variable of the economy. High asset prices today attract the creation of more lemons, thereby exacerbating adverse selection and depressing the future reallocation of productive assets and asset prices. Productive and lemon assets therefore exhibit predator-prey dynamics, and the quality of assets evolves endogenously over time. The equilibrium may feature endogenous cycles and chaos, with the credit market freezing and thawing recurrently and with deterministic ups and downs in asset prices and the volume of trades. We show that a social planner has incentives to tax credit market activities in order to reduce lemon assets and to eliminate endogenous cycles.

Keywords: dynamic adverse selection, endogenous cycles, credit expansion, systemic risk, predator-prey dynamics.

JEL Classification: D82, E32, E44, G01

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1 Introduction

A large and recent empirical literature shows that financial and economic crises are often predictable by the preceding booms in credit and asset prices (Mendoza and Terrones (2008), Schularick and Taylor (2012), Boissay, Collard and Smets (2016), Gorton and Ordonez (2016), Krishnamurthy and Muir (2017), Greenwood, Hanson, Shleifer and Sørensen (2022), Mian, Sufi and Verner (2017), Mian, Sufi and Verner (2020), Boissay, Collard, Galí and Manea (2021)). The usual story is that the availability of cheap credit during booms gives rise to low asset quality, thereby sowing the seeds of the subsequent crises. Existing models that capture this story rely on exogenous shocks to initiate the credit boom-and-bust cycles.

This paper builds a simple model of credit and asset price cycles arising from the dynamics of adverse selection, where deterministic, endogenous equilibrium boom-and-bust cycles occur in the absence of exogenous (either payoff-relevant or sunspot) shocks. In the model, heterogeneous entrepreneurs trade productive assets in an anonymous market subject to financial frictions. Cream-skimming rent-seekers create lemon assets that can be traded. Lemon assets are indistinguishable ex-ante from the productive assets but have no productive value ex-post. Both assets are durable, and the average quality of assets is the key state variable of the economy. When assets are of high quality—when there are few lemons—so are asset prices. High asset prices today attract the creation of more lemons, thereby exacerbating adverse selection in the future credit market and depressing the reallocation of productive assets and asset prices. Productive and lemon assets therefore exhibit predator-prey dynamics, and the quality of assets evolves endogenously over time. The equilibrium may feature endogenous cycles, with the credit market freezing and thawing recurrently and with predictable ups and downs in asset prices and the volume of trades, as booms sow the seeds of future busts.

Standard reasoning suggests that relaxing financial frictions can improve resource allocation by channeling assets to entrepreneurs with higher productivity. This is true in our model: holding fixed the quantity of lemon assets, relaxing financial frictions—allowing productive firms to increase leverage and borrow more—improves static outcomes. On the other hand, we show that higher leverage can attract the creation of more lemon assets, thereby exacerbating the predator-prey dynamics and amplify the endogenous cyclical movements in asset prices and adverse selection. We characterize the equilibrium cyclicality and demonstrate that relaxing financial frictions may give rise to higher periodicity and even chaos (Li and Yorke (2004)), whereby the equilibrium dynamics follow deterministic but seemingly random, aperiodic trajectories, and small differences in the initial condition can result in large differences in the dynamic equilibrium path.

A key difficulty in modeling the predator-prey dynamics between productive and lemon assets due to adverse selection is that when both asset types are durable, forward-looking agents make consumption-saving decisions in anticipation of future credit market conditions. The two stocks of assets are therefore both slow-moving state variables, and the interplay between future states and current choices may become intractable even in the absence of exogenous shocks. Given this challenge, existing models with endogenous asset quality following predator-prey dynamics (Caramp (2017), Fukui (2018), and Neuhann (2019)) do not model asset quality as a persistent variable. We overcome this key challenge by adopting a microfoundation where forward-looking entrepreneurs and rent-seekers both have logarithm intertemporal utility, and the production technology is linear in wealth (Moll (2014)). In this formulation, the consumption-saving decision has a closed-form solution, as all agents save a constant fraction of current wealth. It is precisely the durability of assets that give rise to the predator-prey dynamics and the endogenous lemon cycles.

The endogenous lemon cycles originate from the pecuniary externality in the adversely selected credit market. As productive entrepreneurs use leverage to borrow assets, they ignore the fact that high asset prices attract lemons, thereby worsening the adverse selection in the future and negatively affecting the future credit market participants. We characterize the constrained optimal allocation and show that a social planner subject to the same information asymmetry has incentives to tax credit market activities in order to reduce lemon assets. The planner's solution never features cycles, reminiscent of the classic Turnpike theorem (McKenzie (1976)).

Our paper contributes to the large literature on adverse selection in credit markets (Guerrieri and Shimer (2014), Zryumov (2015), Guerrieri and Shimer (2018), Chang (2018), Chiu and Koeppl (2016), Ikeda and Phan (2016) House and Masatlioglu (2015), Maurin (2022)), especially in dynamic general equilibrium models with capital accumulation (Eisfeldt (2004), House (2006), Kurlat (2013), Bigio (2015), and Ikeda (2019)). A key feature of our model is the predator-prey dynamics on the endogenous creation of assets with differing quality: asset price booms attract the creation of lemons, leading to the deterioration of future asset quality. This key feature is also present in the important contributions of Caramp (2017), Fukui (2018), and Neuhann (2019), but none of these papers feature persistent asset quality. Both Caramp (2017) and Fukui (2018) feature three-period models where the lemon assets are traded once. Neuhann (2019) features a dynamic model where assets are traded repeatedly, but asset quality responds to the unobserved contemporaneous effort and is thus not a state variable. By contrast, the average quality of assets (i.e., the fraction of lemon assets) is a state variable of our dynamic economy and is the key for generating the predator-prey dynamics and endogenous credit cycles.

There is also a literature on the mechanism design approach to policy interventions (see, e.g., Tirole (2012), Philippon and Skreta (2012) and Fuchs and Skrzypacz (2019)). An important recent contribution is Williams (2021), which provides a unified framework to analyze retention and liquidity jointly in adverse selection markets with multidimensional information. Our analysis of the constrained-optimal allocation differs from the mechanism design approach in that the market features anonymous trading, and the planner can only adopt simple linear taxes on asset trades.

Our paper also contributes to the literature on endogenous cycles (see, e.g., Suarez and Sussman (1997), Aghion, Banerjee and Piketty (1999), Matsuyama (1999) and Martin (2005); and see Boldrin and Woodford (1990) for a survey), where deterministic cycles arise from non-monotonicities in the law of motion for the key state variables. Especially related is the recent literature focusing on credit cycles featuring endogenous fluctuations in the quantity of credit (Azariadis and Smith (1998); Matsuyama (2007); Gorton and He (2008); Myerson (2012); Gu, Mattesini, Monnet and Wright (2013), Azariadis, Kaas and Wen (2016), Cui and Kaas (2020), and Farboodi and Kondor (2021)). Relative to these papers, our model is motivated by the evidence on the cyclical fluctuations in asset quality (Schularick and Taylor (2012), Mendoza and Terrones (2012), and Reinhart and Rogoff (2009)), and cycles arise in our setting through the novel mechanism that high asset prices attract the endogenous creation of lemon assets. We further characterize how relaxing financial frictions can amplify the equilibrium cyclicality and give rise to chaotic dynamics.

We also share the key question of interest, i.e., how a change in the availability of credit may induce booms and busts, with the literature on collateral based credit cycles (Kiyotaki and Moore (1997); Lorenzoni (2008); Mendoza (2010); Gorton and Ordonez (2014)). Different from these papers, where exogenous shocks are amplified and become persistent through the net worth channel and collateral constraints, exogenous shocks play no role in generating long-run, recurrent cyclical fluctuations in credit quality and leverage. Note that cycles in our models arise deterministically and are also not driven by sunspot shocks; the model features no scope for coordination or multiple equilibria. Our model is thus different from those where fluctuations occur due to sunspot shocks that switch equilibria (Asriyan, Fuchs and Green (2019); Lee and Neuhann (2023)). Finally, our paper also relates to the literature on screening, including important recent works that feature dynamic models with asset quality as the persistent state variable (Hu (2022); Fishman, Parker and Straub (2020)). Relative to this literature, our contribution is to have a tractable model to analyze deterministic cycles arising from predator-prey dynamics between productive and lemon assets.

The rest of the paper is organized as follows. Section 2 sets up the model and characterizes the equilibrium. Section 3 characterizes the existence of the endogenous credit cycles and analyzes their properties. Section 4 analyzes the constrained optimal allocation and studies policy interventions. Section 5 concludes.

2 Model and Equilibrium

Time is discrete and infinite. There are two types of firms: productive ("entrepreneurs") and non-productive ("rent-seekers"), each with a continuum of measure one.

Each entrepreneur i enters the period t with k_{it} units of productive assets. The assets can be either used internally or lent out to other firms. The firm draws an independent, idiosyncratic productivity $z_{it} \sim F$ and then decides whether to participate in the credit market to borrow or lend capital at the given market rate, generating output z_{it} times the amount of productive capital in possession during the period.

Each rent-seeker enters the period with x_{it} units of non-productive assets ("lemon capital"), which cannot be used productively but can nevertheless be lent out in the credit market in exchange for rental income. There is adverse selection: to the borrower, the lemon capital is indistinguishable from productive assets until after production takes place.

The credit market is competitive and anonymous. The rental rate and trading volume of capital are determined in equilibrium and depend on the demand and supply. The demand of capital is from the entrepreneurs with high productivity draws. The supply is from both rent-seekers and those entrepreneurs with low productivity draws who choose to lend. Because of adverse selection, when the supply of lemon capital is sufficiently high, the credit market may endogenously break down with no trades taking place.

Both types of agents are forward-looking with preferences $\sum_{t=0}^{\infty} \beta^t \ln c_{it}$. At the end of each period, both types of agents make consumption-saving decisions. Entrepreneurs can invest and accumulate productive capital; rent-seekers can invest and accumulate lemons. Both types of capital depreciate at rate δ, and time moves on.

2.1 The Credit Market Equilibrium Within Each Period

We first describe within-period part of the model formally and analyze the credit market and production outcomes, taking the distribution of productive and lemon capital levels k_{it} and x_{it} as given. For this part of the analysis we drop the time subscript t. In the next section we describe aggregation and the consumption-saving decisions that give rise to the endogenous capital dynamics.

Entrepreneurs. Let b_i and ℓ_i respectively denote the amount of capital that entrepreneur i chooses to borrow from or lend to the credit market at rate r. We impose the constraint that the total amount borrowed cannot exceed $\frac{\phi}{1-\phi} > 0$ times the firm's net worth:

$$b_i \in \left[0, \frac{\phi}{1-\phi} k_i\right]. \tag{1}$$

The leverage constraint is parametrized by $\phi \in [0,1)$ such that when the entrepreneur borrows to the maximum ($b_i = \frac{\phi}{1-\phi} k_i$), $\phi = \frac{b_i}{b_i + k_i}$ is the ratio between the amount borrowed and the total amount of capital used by the firm.

The firms can lend up to its entire capital stock:

$$\ell_i \in [0, k_i]. \tag{2}$$

Because of adverse selection, only an endogenous $\rho \in [0,1]$ fraction of the borrowed capital is productive. We refer to ρ as the quality of assets in the credit market. Within each period, an entrepreneur takes the rental rate p and asset quality ρ as given and chooses ℓ_i, b_i to maximize the income:

$$y_i \equiv \max_{\ell_i, b_i} \quad z_i \underbrace{(k_i - \ell_i + \rho b_i)}_{\substack{\text{total productive capital} \\ \text{in possession at time } t}} - p \underbrace{(b_i - \ell_i)}_{\text{net borrowing}} \qquad \text{subject to (1) and (2).}$$

The first term is the total productive income; the second term is the net rental expense.

Lemma 1. *A productive firm's decision to participate in the credit market depends on two cutoffs* $(\underline{z}, \bar{z}) \equiv (p, p/\rho)$*, such that*

$$(b_i, \ell_i) = \begin{cases} (0, k_i), & \textit{if } z_i < \underline{z} \\ (0, 0), & \textit{if } z_i \in (\underline{z}, \overline{z}) \\ \left(\frac{\phi}{1-\phi} k_i, 0\right) & \textit{if } z_i \geq \overline{z} \end{cases}.$$

The income of the productive firm is

$$y_i = \left[p + \max\{z_i - p, 0\} + \frac{\phi}{1-\phi} \max\{z_i \rho - p, 0\}\right] k_i. \tag{3}$$

Because the producer's problem features linear objective and constraints, the optimal participation in the credit market, as characterized by Lemma 1, features cut-off strategies. Those with productivity levels below the rental rate of capital ($z_i < \underline{z} \equiv p$) would prefer to lend out their capital stock (thus choosing $\ell_i = k_i$ and no borrowing, $b_i = 0$) for the rental income instead of in-house production. On the other hand, those with sufficiently high productivity levels would borrow as much as possible. Because of adverse selection, the effective rental rate per productive unit of capital is p/ρ; hence, those with $z_i \geq p/\rho \equiv \bar{z}$ would choose to borrow $b_i = \frac{\phi}{1-\phi} k_i$ (and the constraint 1 binds). Finally, those with intermediate productivity levels would abstain from participating in the credit market.

Rent-seekers. Within each period, a rent-seeker's decision is simple: because lemon assets cannot be used productively, they prefer to lend out all lemon assets to the credit market for any rental rate $p \geq 0$.

Equilibrium in the Credit Market. Denote $K \equiv \int_0^1 k_i \, \mathrm{d}i$ as the aggregate stock of productive capital and $X \equiv \int_0^1 x_i \, \mathrm{d}i$ the stock of lemon capital. Let $\chi \equiv X/K$ denote the relative stock of lemon capital.

A credit market equilibrium is the pair (p, ρ)—and the associated participation cutoffs $(\underline{z}, \bar{z}) \equiv (p, p/\rho)$ as characterized by Lemma 1—such that:

1. The demand of capital never exceeds supply:

$$\underbrace{\frac{\phi}{1-\phi}\left[1-F\left(\bar{z}\right)\right]K}_{\text{total amount borrowed}} \leq \underbrace{F\left(\underline{z}\right)K + X}_{\text{total amount lent}}, \text{ with equality if } p > 0. \tag{4}$$

2. The asset quality ρ satisfies:

$$\rho = \frac{F\left(\underline{z}\right)K}{F\left(\underline{z}\right)K + X}. \tag{5}$$

From equation (4) get

$$\frac{\phi}{1-\phi}\left[1-F\left(\bar{z}\right)\right] = F\left(\underline{z}\right) + \chi \tag{6}$$

where $\bar{z} = \underline{z}/\rho$, so we can write $\bar{z}$ as a function of $\underline{z}$:

$$\frac{\phi}{1-\phi}\frac{1-F\left(\bar{z}\right)}{\bar{z}} = \frac{F\left(\underline{z}\right)}{\underline{z}} \tag{7}$$

Equation (4) states that the total assets borrowed by the entrepreneurs with productivity above $\bar{z}$ cannot exceed the total supply of assets, which consists both the stock of lemon assets and the productive assets lent by entrepreneurs with productivities below $\underline{z}$. Equation (4) shows that the assets quality in the credit market is endogenous, even taking as given the total stock of productive and lemon assets K and X. When the cutoff $\underline{z}$ is higher, more productive assets are lent to the credit market, and the average quality ρ is higher. The cutoffs $(\underline{z}, \bar{z})$ for entrepreneurs' participation in the credit market in turn depend on equilibrium asset quality and the supply and demand.

Assumption 1. *The productivity distribution is uniform:* $U[0,1]$.

We impose Assumption 1 throughout the rest of the paper for expositional simplicity. In Appendix B we extend our analysis to a general distribution F with bounded support, and all of our key results go through.

Lemma 2. *The credit market equilibrium is characterized by the rental rate p and asset quality ρ that satisfy*

$$p = \max\{\phi - \chi, 0\}, \qquad \rho = \max\left\{\frac{\phi - \chi}{\phi}, 0\right\}.$$

Whether there is trading in the credit market depends on the relative stock of lemon capital, $\chi \equiv X/K$:

1. *When $\chi < \phi$, the productivity cutoffs are $(\underline{z}, \bar{z}) = (\phi - \chi, \phi)$: entrepreneurs lend iff $z_i < \phi - \chi$ and borrow iff $z_i \geq \phi$.*

2. *When $\chi \geq \phi$, the credit market breaks down, with rental rate and asset quality both equal to zero, and no entrepreneurs borrow.*

The lemma shows that the relative stock of lemon capital, $\chi \equiv X/K$, is the key aggregate state variable that characterizes the credit market outcome. When the stock of lemon capital is high, the credit market is more adversely selected (lower quality ρ), and the rental rate p is low. For sufficiently high levels of lemon capital stock (when $\chi \geq \phi$), the credit market breaks down completely: it becomes so adversely selected that the rental rate falls to zero, and no productive capital is traded.

Aggregate Output. We next characterize the aggregate output. By independence of productivity draws, the aggregate output is

$$Y = Z \cdot K$$

where $K \equiv \int_0^1 k_i \, \mathrm{d}i$ is the aggregate stock of productive capital, and

$$Z = (1 - F(\underline{z}))\,\mathbb{E}[z|z \geq \underline{z}] + F(\underline{z})\,\mathbb{E}[z|z \geq \bar{z}] = \frac{1 + (1 - \underline{z})\,\underline{z} + \underline{z}\bar{z}}{2}. \tag{8}$$

The effective aggregate productivity Z thus depends on the degree of capital reallocation from the less productive (those with $z_i < \underline{z}$) to the more productive entrepreneurs (those with $z_i \geq \bar{z}$). Lemma 2 implies the following.

Lemma 3. *The total output is $Y = Z \cdot K$, where K is aggregate stock of productive capital, and Z is the effective aggregate productivity satisfying $Z = \frac{1+(1+\chi)\max\{\phi-\chi,0\}}{2}$. The total fraction of productive capital reallocated is $\max\{\phi - \chi, 0\}$.*

The effective aggregate productivity in equilibrium monotonically declines in the relative stock of lemon capital, $\chi \equiv X/K$. The total fraction of productive capital reallocation also declines in χ.

2.2 Dynamics

We are now ready to describe the dynamic component of the model. We can write the problem of a productive entrepreneur with productivity z and capital stock k given the state variable χ (the relative stock of lemon capital in the aggregate) as follows:

$$V(k, z, \chi) = \max_{c,k'} \{\ln c + \beta \mathbb{E} V(k', z', \chi')\}$$

subject to the budget constraint

$$c + k' = \left[p(\chi) + \max\{z_i - p(\chi), 0\} + \frac{\phi}{1-\phi} \max\{z_i \rho(\chi) - p(\chi), 0\}\right] k + (1-\delta) k$$

where the first term on the right-hand side of the budget constraint is the flow income characterized by Lemma 1.

The problem of a rent-seeker with a stock of lemon asset x is

$$V^L(x, \chi) = \max_{c^L, x'} \left\{\ln c^L + \beta \mathbb{E} V^L(x', \chi')\right\}$$

subject to

$$\frac{x' - (1-\delta) x}{\mu} = p(\chi) x - c^L,$$

where the first term on the right-hand side of the budget constraint is the rental income earned by the lemon asset, and the entire right-hand side captures savings in the period. The unit cost of creating lemon assets is μ^{-1} measured in terms of the consumption good. The parameter $\mu > 1$ captures the notion that lemon capital is less costly to produce than productive capital.

Because the flow income is linear in the capital stock for both types of entrepreneurs, the consumption-saving decision has a closed-form solution, with a constant saving rate. The corresponding law of motions for capital are characterized by the following lemma.

Lemma 4. *At the firm-level, productive capital evolves according to*

$$k'(z, \chi) = \beta \left[1 - \delta + p(\chi) + \max\{z_i - p(\chi), 0\} + \frac{\phi}{1-\phi} \max\{z_i \rho(\chi) - p(\chi), 0\}\right] k$$

and lemon capital evolves according to

$$x'(\chi) = \beta [1 - \delta + \mu p(\chi)] x.$$

In aggregate, capital evolves according to

$$K'(\chi) = \beta [Z(\chi) - p(\chi) \chi + 1 - \delta] K,$$
$$X'(\chi) = \beta [\mu p(\chi) + 1 - \delta] X, \tag{9}$$

where $Z(\chi) \equiv \frac{1 + (1+\chi) \max\{\phi - \chi, 0\}}{2}$ *following Lemma 3.*

Because of log-utility and that the flow income is linear in the current wealth, both entrepreneurs and rent-seekers consume $(1-\beta)$ fraction of the current wealth and save β fraction for the next period. That each entrepreneur's capital stock evolves linearly facilitates aggregation—the wealth distribution among entrepreneurs and among rent-seekers are irrelevant for characterizing the aggregate outcomes of the economy. Our next result characterizes the evolution of the key state variable χ, the relative stock of lemon capital in the aggregate.

Proposition 1. *The law of motion for χ is*

$$\chi_{t+1} \equiv \Gamma(\chi_t) = \begin{cases} \frac{\mu(\phi-\chi_t)+1-\delta}{(1+(\phi-\chi_t)(1-\chi_t))/2+1-\delta}\chi_t, & \textit{if } \chi_t < \phi \\ \frac{1-\delta}{1/2+(1-\delta)}\chi_t, & \textit{otherwise.} \end{cases}$$

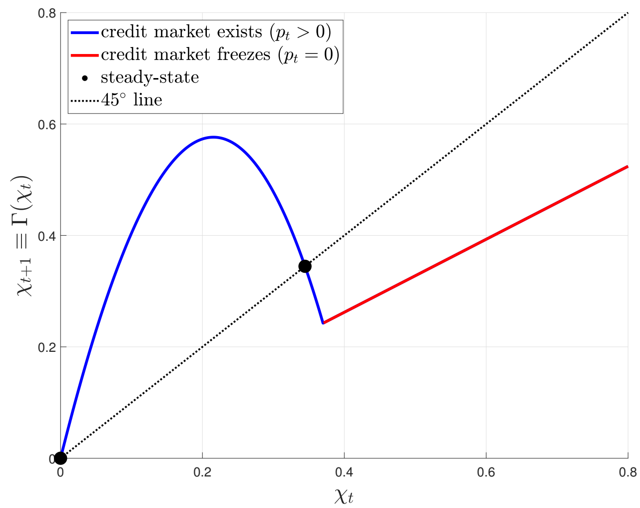
Proposition 1 shows that the evolution of χ_t depends on whether the credit market functions or breaks down in period t. When $\chi_t < \phi$, the credit market opens, with capital reallocation and positive rental rate. When $\chi_t \geq \phi$, the rental rate falls to zero, and no capital reallocation occurs.

Figure 1 illustrates the law of motion. The blue portion of the curve correspond to the region where $\chi_t < \phi$, whereas the red portion of the curve is where $\chi_t \geq \phi$. We highlight three features of the law of motion. First, the blue curve is concave and non-monotone: χ_{t+1} is increasing (in χ_t) when χ_t is low and decreasing when χ_t is high. To understand this, note that because of saving rates are constant and identical between the entrepreneurs and the rent-seekers, the evolution of χ_t depends on the relative income between the two groups. When χ_t is high, the credit market is more adversely selected, the fraction ρ of productive assets is low, and the rental rate is also low (c.f. Lemma 2). Because the income of rent-seekers depends solely on renting lemons through the credit market, the income per unit of lemon assets is decreasing in χ_t. The blue curve is therefore concave in χ_t, and, for sufficiently high χ_t, the rental rate is so low that χ_{t+1} becomes decreasing in χ_t.

Second, for sufficiently high χ_t, the credit market breaks down, and the relative stock of lemon capital declines over time ($\chi_{t+1} < \chi_t$; i.e., the red line is below the 45-degree line).

Third, the blue curve intersects the 45-degree line twice. These intersections corresponds to steady-states of the economy. There is a trivial steady-state $\chi = 0$, and another interior steady-state denoted as χ^*. However, neither steady-states has to be stable or attractive. Indeed, both steady-states depicted in Figure 1 are unstable, meaning an economy that does not start in a steady-state never converges to one. We analyze the economy's dynamic behavior in the next section.

Figure 1. Law of motion for χ_t



Notes: Parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$ and $\phi = 0.37$.

3 Endogenous Cycles

In this section we analyze the cyclical properties embodied in the law of motion $\chi_{t+1} = \Gamma(\chi_t)$ (c.f. Proposition 1). The next result first establishes that there is at most one interior steady-state $\chi^* > 0$. The lemma also characterizes conditions under which the interior steady-state is locally unstable, thereby implying the emergence of endogenous cycles.

Lemma 5. *(i) When $\phi(2\mu - 1) > 1$, the economy has a single interior steady-state $\chi^* > 0$ satisfying $\chi^* = \Gamma(\chi^*)$. Otherwise, the only steady-state is $\chi = 0$.*

(ii) If $\Gamma'(\chi^*) < -1$, the interior steady-state is unstable, and the economy features endogenous cycles.

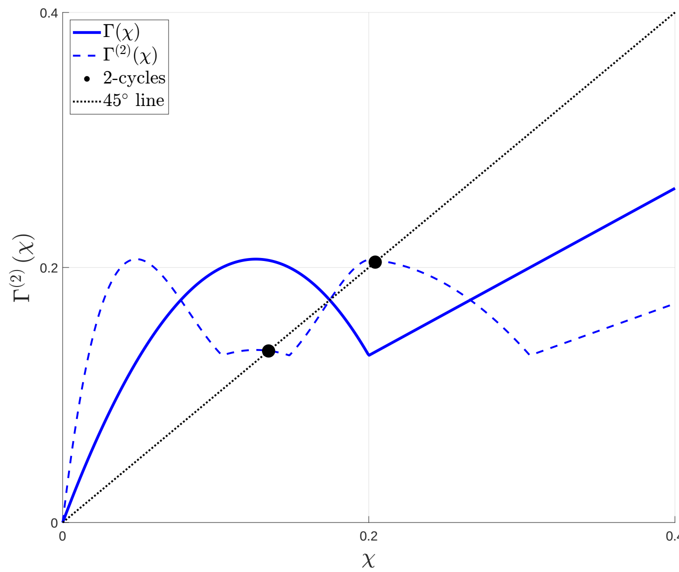
(iii) $\partial\Gamma'(\chi^*)/\partial\phi < 0$: a higher ϕ (more leverage) results in a more negative slope of $\Gamma(\cdot)$ at the interior steady-state.

To understand endogenous cycles in the model, Figure 2 shows $\Gamma(\chi)$ in the dashed line and $\Gamma^{(2)}(\chi) \equiv \Gamma(\Gamma(\chi))$ in the solid line. The two black dots indicate locations where $\Gamma^{(2)}$ intersects the 45-degree line, representing χ^a and χ^b such that

$$\chi^a = \Gamma^{(2)}(\chi^a), \quad \chi^b = \Gamma^{(2)}\left(\chi^b\right).$$

Suppose at time t, $\chi_t = \chi^a$; then $\chi_{t+2} = \chi^a$. The dashed line $\Gamma(\chi)$ also indicates that neither χ^a nor χ^b are steady-states; hence $\chi_{t+1} = \chi_{t+3} \neq \chi^a$. It therefore must be the case that $\chi_{t+1} = \chi_{t+3} = \chi^b$; that is, the equilibrium path must alternate between χ^a and χ^b ad infinitum, which indicates an endogenous, deterministic cycle of period 2. The equilibrium path of the rental rate and asset quality also exhibit period-2 cycles, as shown in Figure A.3.

Figure 2. The existence of period-2 cycles



Notes: Parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$ and $\phi = 0.2$ so there exists one attracting 2-cycle.

What gives rise to the endogenous cycle? Intuitively, lemon and productive assets exhibit predator-prey dynamics. When the stock of lemon assets is relatively low (low χ^a), credit market functions well, and a large fraction $\phi - \chi$ of productive assets are reallocated. Because of high rental income earned from the credit market, more lemon assets accumulate (χ^b), exacerbating the adverse selection over the next period and reducing the degree of capital reallocation in the credit market. The rental income falls, fewer lemon assets are created, and the cycle continues.

The stability of cycles can also be examined from Figure 2. The period-2 cycle is globally attractive—meaning, starting from any initial condition $\chi_0 > 0$, the economy eventually converges to the period-2 cycle—iff the absolute values of the slope of $\Gamma^{(2)}(\cdot)$, evaluated at χ^a and χ^b, are less than one.

Note that, while our model is cast in discrete time—so that endogenous cycles arise with a single state variable χ—predator-prey dynamics can also be modeled in continuous time with two state variables (i.e., we have to keep track of the stock of productive and lemon assets separately, with the two variables following what is known as the "Lotka-Volterra" equations).

Finally, note that at χ^b, the relative stock of lemon assets is sufficiently high so that the credit market breaks down (χ^b is to the right of the kink of $\Gamma(\chi)$). The period-2 endogenous cycles therefore features alternative periods of the credit market opening and closing down, with potentially large fluctuations in asset prices and trading volumes.

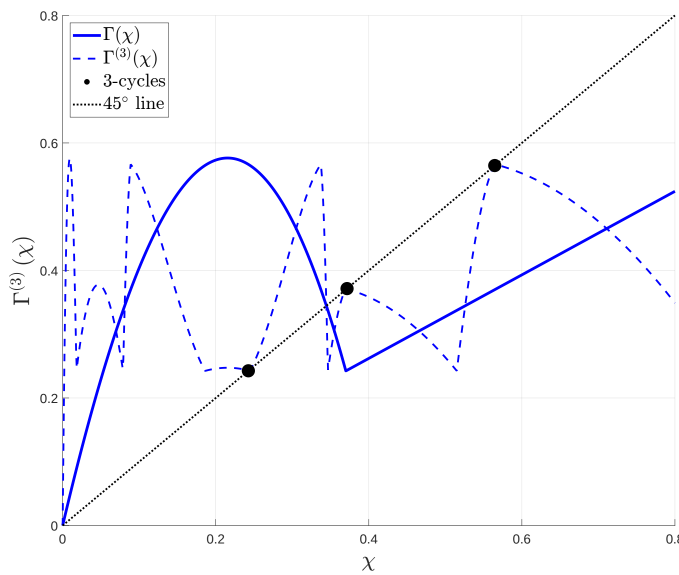
3.1 Periodicity and Chaos

In Lemma 5, we find that as leverage ϕ increases, the slope of the law of motion $\Gamma(\cdot)$ at the steady state decreases and could become unstable as 2-cycle emerges when $\Gamma'(\chi^*) < -1$. By the same analysis, the 2-cycle is globally stable if the absolute value of the slope of $\Gamma^{(2)}(\cdot)$ at the cycle is less than one. However, as ϕ increases further, the 2-cycles could also become unstable, and the model may feature a 3-cycle. Figure 3 shows $\Gamma^{(3)}(\chi) \equiv \Gamma(\Gamma(\Gamma(\chi)))$ in solid line (and $\Gamma(\chi)$ in dashed line). The three black dots indicate locations where $\Gamma^{(3)}$ intersects the 45-degree line, representing χ^a, χ^b and χ^c such that

$$\chi^a = \Gamma^{(3)}(\chi^a), \quad \chi^b = \Gamma^{(3)}(\chi^b), \quad \chi^c = \Gamma^{(3)}(\chi^c).$$

Then χ^a, χ^b and χ^c denotes the 3-cycle where $\chi_{t+3} = \chi_t$ while $\chi_{t+1} \neq \chi_t$ and $\chi_{t+2} \neq \chi_t$. A transition path as the economy converges to a 3-cycle can be seen in Figure A.4.

Figure 3. The existence of 3-cycles



Notes: Parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$ and $\phi = 0.37$ so there exists one unique 3-cycle.

For the set of parameters under which the model exhibits 3-periods cycles, the Li-Yorke theorem and Sarkovskii's theorem imply that under the same parameters, there are initial conditions χ_0 for which the equilibrium converges to regular cycles of every periodicity as well as complete chaos. Formally, for any natural number k, there exists a set of initial conditions such that as $t \to \infty$, the equilibrium converges to a period-k cycle. As $k \to \infty$, the equilibrium may appear aperiodic and chaotic.

3.2 Credit Loosening and Chaotic Dynamics

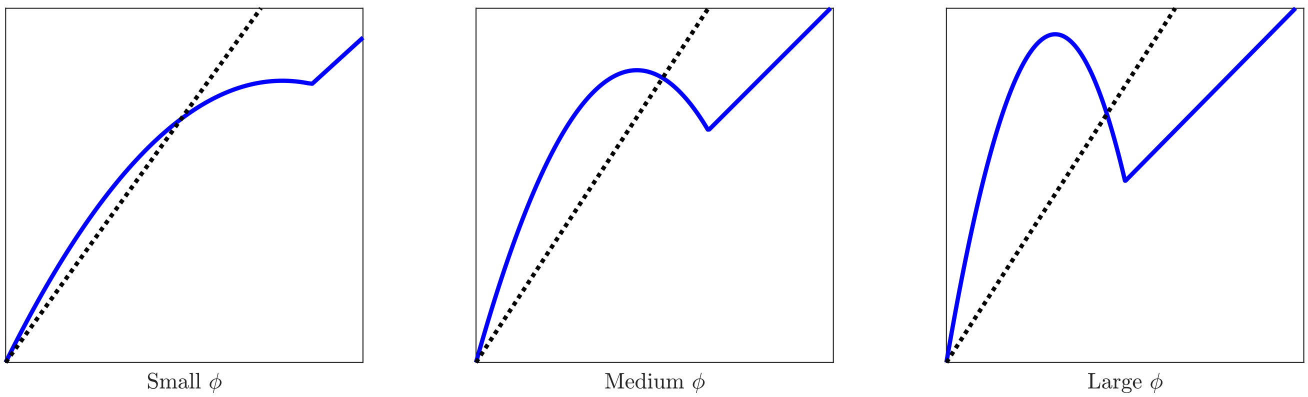
As we have demonstrated, the leverage constraint parametrized by ϕ—higher ϕ implies higher leverage—is crucial for the model dynamics. Figure 4 shows that the function $\Gamma(\chi;\phi)$ under three different levels of ϕ. Lemma 5 implies, the slope $\Gamma'(\chi^*)$ decreases in ϕ. When ϕ is low as in panel (a) of Figure 4, $\Gamma'(\chi^*) > 0$, meaning that the steady state (or period 1-cycle) is stable and globally attractive, and with any initial value $\chi_0 > 0$ the system will converge monotonically toward the steady state χ^* (i.e., χ_t is monotone in t and $\lim_{t\to\infty}\chi_t = \chi^*$). As ϕ increases as in panel (b), $\Gamma'(\chi^*) \in (-1, 0)$. The steady-state is still stable and globally attractive, but there exists a neighborhood $\mathcal{X}$ around χ^* such that if $\chi_t \in \mathcal{X}$, then χ_{t+s} still converges to χ^* ($\lim_{s\to\infty}\chi_{t+s} = \chi^*$) but oscillates around χ^* along the path of convergence (i.e., $\chi_{t+s} < \chi^*$, $\chi_{t+s+1} > \chi^*$, $\chi_{t+s+2} < \chi^*$, and so on). As ϕ increases further as in panel (c), the slope $\Gamma'(\chi^*) < -1$, and the interior steady-state becomes unstable; that is when a period-2 cycle emerges. Finally, as ϕ further increases, the slope of $\Gamma^{(2)}$ also becomes smaller than -1 at χ^a and χ^b; this implies that the period-2 cycle is no longer attractive, and the model may feature higher order cycles. As shown in Figure 3, with $\phi = 0.37$ the model may exhibit a period-3 cycle, and the Li-Yorke theorem implies that the system have regular cycles of every periodicity as well as complete chaos, and an example of a chaotic equilibrium path is shown in Figure A.5.

To illustrate that credit loosening can generate chaos, we show a sample bifurcation diagram of the function $\Gamma(\cdot)$. The terminology "bifurcation" refers to the phenomenon where as ϕ increases, a stable cycle disappears and a new cycle of different periodicity emerges. For instance, a "period-doubling" bifurcation is said to occur at the level of ϕ under which $\Gamma'(\chi^*) = -1$; a small further increase in ϕ implies that the interior steady-state becomes unstable, and a period-2 cycle starts to emerge.

Figure 5 shows the bifurcation diagram. We fix an initial condition $\chi_0 = 0.95$ (the stock of lemon assets is 0.95 times the stock of productive capital at time zero), run the economy forward, and plot the long-run distribution of the state variable χ_t. Specifically, we run the economy for 1000 periods and collect the sample path χ_t between $t = 1000$ and 2000, and we plot the distribution of $\{\chi_t\}_{t=1000}^{2000}$ for varying levels of the leverage constraint ϕ. That is, the X-axis in Figure 5 shows the corresponding leverage constraint ϕ; the Y-axis shows the scatter plot of the sample path χ_t between $t = 1000$ and 2000. Red points are when the credit market freezes; blue points are when the credit market opens and reallocates assets. As can be seen, for small values of $\phi < 0.15$, the economy converges to the interior steady-state, as $\{\chi_t\}_{t=1000}^{2000}$ concentrates on a single point along the Y-axis for each value of ϕ. A period-doubling bifurcation occurs at around $\phi = 0.18$, and a 2-cycle emerges. As ϕ further increases, the equilibrium periodicity changes further (e.g., period-4 cycle starts to emerge at around $\phi = 0.4$). For three intervals of ϕ (around

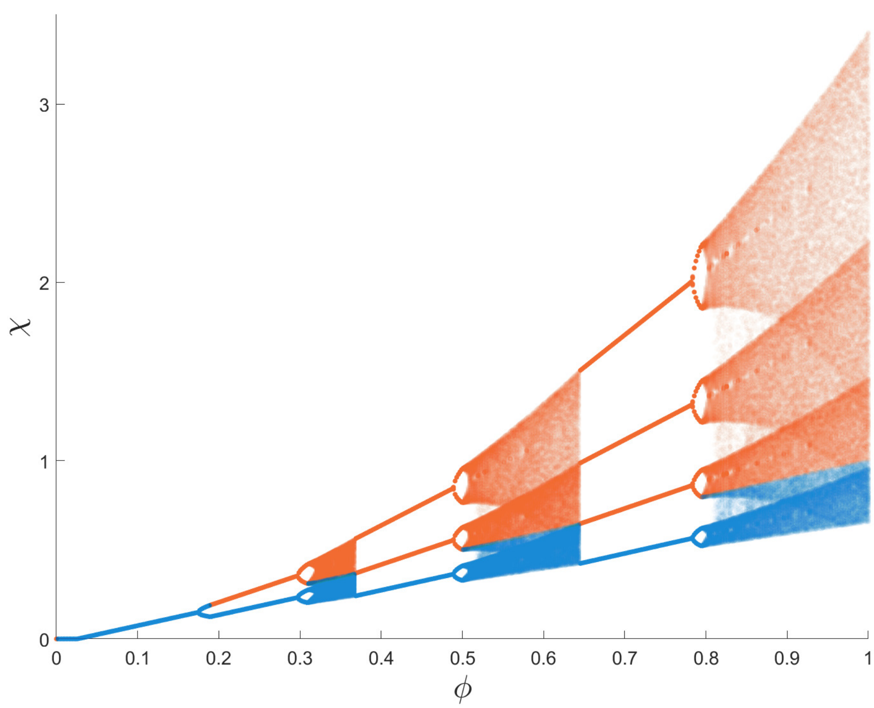
$[0.32, 0.36]$, $[0.51, 0.64]$, and $[0.81, 1)$), the dynamic system does not seem to converge to a fix-period cycle after 1000 periods; instead, the state variable features seemingly chaotic movements, as $\{\chi_t\}_{t=1000}^{2000}$ scatters across many distinct values along the Y-axis. Correspondingly, the credit market experiences seemingly chaotic booms and busts, with asset prices and the quality of traded assets characterized by Lemma 2.

Figure 4. Credit loosening and law of motion



Notes: law of motion of χ_t with different levels of ϕ.

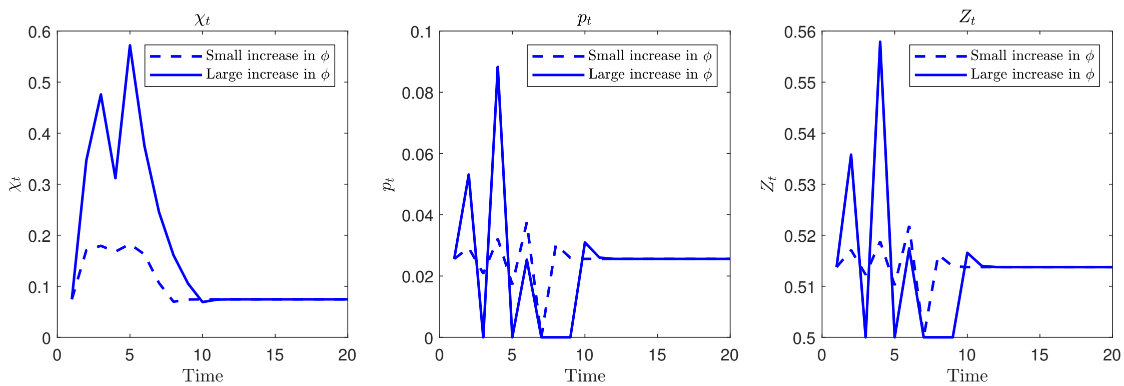
Figure 5. Credit loosening and limit cycles



Notes: Bifurcation diagram of χ_t with respect to ϕ. Other parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$.

Figure 5 shows that a credit loosening (increase in ϕ) may lead to long-run limit cycles and chaotic dynamics. It is worthwhile to study the impulse response of a temporary shock in ϕ and a permanent shock in ϕ. The impulse responses of a temporary increase in ϕ (where we assume that the level of ϕ increases for 5 periods and then goes back to the initial level) are shown in Figure 6, where we plot the transition dynamics for the state variable χ_t (relative stock of lemons; left panel), the prices of traded assets (middle panel), and the effective aggregate productivity (right panel). We find that a temporary credit loosening will lead to temporary higher lemon asset fraction χ_t, and the economy gradually recovers after the shock diminishes. With a larger shock, the asset quality decreases faster, and recovers slower. The impulse responses under permanent shocks of ϕ are shown in Figure 7, and we see that a permanent credit loosening may lead to higher χ_t, and even may drive the economy into a cycle or even a chaotic region.

Figure 6. Impulse Response under Temporary Shock of ϕ



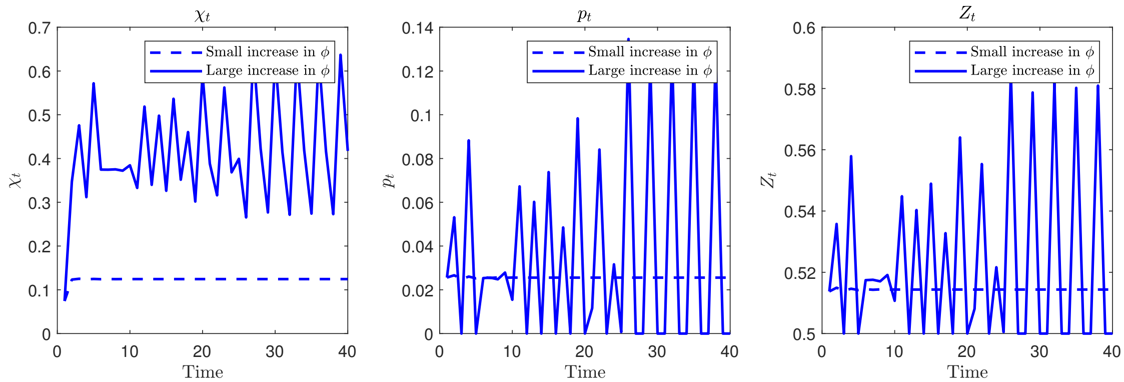
Notes: For a small shock, we assume that ϕ increases from 0.1 to 0.2, and for a large shock of ϕ, we assume that ϕ increases to 0.4. Other parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$. A temporary shock of ϕ means that the increase takes 5 periods and then goes back to the initial level.

4 Welfare Analysis

The endogenous lemon cycles originate from the pecuniary externality in the adversely selected credit market. As productive entrepreneurs use leverage to borrow assets, they ignore the fact that high asset prices attract lemons, thereby worsening the adverse selection in the future and negatively affecting the future credit market participants.

In this section, we characterize the constrained optimal allocation and show that a social planner subject to the same information asymmetry should lean against asset price movements, subsidizing trades when adverse selection is severe and taxing trades when the leverage restrictions during good times with high asset prices. The planner's solution never features cycles,

Figure 7. Impulse Response under Permanent Shock of ϕ



Notes: For a small shock, we assume that ϕ increases from 0.1 to 0.2, and for a large shock of ϕ, we assume that ϕ increases to 0.4 so that there exists a limit cycle. Other parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$.

reminiscent of the classic Turnpike theory (McKenzie (1976)).

Specifically, we consider a planner with Pareto weight $(1 - \alpha)$ on entrepreneurs and α on rent-seekers. We give the planner access to a proportional rental tax τ, such that entrepreneurs who rent capital from the credit market have to pay $p\tau$. To balance the budget, we assume the planner rebates the collected taxes to each entrepreneur as a lumpsum transfer. Setting $\tau = 1$ restores the decentralized equilibrium. We assume τ and the lump-sum transfers to entrepreneurs are the only policy instruments; the planner has to respect all other equilibrium constraints.

Because rent-seekers do not create any value in the economy, in the main text we consider the case with $\alpha = 0$, whereby the planner chooses the rental tax τ to maximize the value function of entrepreneurs while simultaneously disregard the welfare of the rent-seekers. The derivation with $\alpha \in [0, 1]$ is in Appendix A.7.

We now formalize the planner's problem. Fix the state variable $\chi \equiv X/K$, i.e., the relative stock of lemon capital. Given any rental rate p, the marginal entrepreneurs who rent out productive capital has productivity $\underline{z} = p$. The market clearing condition (4) pins down the marginal entrepreneurs who borrow, as the cut-off productivity $\bar{z}$ must satisfy

$$\frac{\phi}{1 - \phi}[1 - \bar{z}] = p + \chi. \tag{10}$$

The expected quality of traded assets, following (5), is

$$\rho = \frac{p}{p + \chi}.$$

The corresponding rental tax τ must therefore solve

$$\bar{z} = \tau p / \rho$$

$$\implies \tau = 1 + \frac{1}{p+\chi} - \frac{1}{\phi}.$$

The preceding equations imply that for any rental rate p such that the market clearing condition (10) can be satisfied (i.e., $\exists \bar{z} \geq p$ such that $\frac{\phi}{1-\phi}[1-\bar{z}] = p + \chi \iff p \in [0, \phi - \chi(1-\phi)]$), the planner can always choose $\tau = 1 + \frac{1}{p+\chi} - \frac{1}{\phi}$ to implement the rental rate p. Hence, in what follows we assume the planner can directly choose the rental rate p, with $\underline{z} = p$ and $\bar{z}$ solving (10).

Given p, the flow output is

$$Y(\chi, K, p) = Z(\chi, p) K = \frac{1}{2}\left(1 - p^2 + 2p - \frac{1-\phi}{\phi}p^2 - \frac{1-\phi}{\phi}p\chi\right) K$$

where the effective aggregate productivity $Z(\chi, p)$ follows from (8). Noting that both entrepreneurs and rent-seekers always consume $(1-\beta)$ fraction of their current wealth, we can formulate the planner's problem as

$$V^p(\chi, K) = \max_p \{\ln((1-\beta)[Z(\chi, p) - p\chi + (1-\delta)] K) + \beta V(\chi', K')\}$$

$$\text{s.t.} \qquad K' = \beta [Z(\chi, p) - p\chi + (1-\delta)] K \tag{11}$$

$$\chi' = \frac{\mu p + 1 - \delta}{Z(\chi, p) - p\chi + (1-\delta)}\chi \tag{12}$$

We make several observations. First, we specify that the planner chooses the aggregate consumption of all entrepreneurs; this is without loss of generality since the planner has access to entrepreneur-specific transfers and can address distributional concerns using those transfers. Second, the flow income of entrepreneurs is $(Z(\chi, p) - p\chi) K$; the second term inside the parenthesis reflects the rental payments made to the rent-seekers, reflecting the fact that the planner is subject to the same information asymmetry as entrepreneurs and cannot detect lemon assets ex-ante. Third, the law of motion of lemon assets (12) follows from (9) and is derived from the optimal consumption-saving decisions of the rent-seekers, who save a constant fraction β of the current wealth as lemon assets for the next period.

It is easy to see that the planner's value function must be log-linear in K; in fact, we can simplify the planner's value function as $V^p(\chi, K) = v^p(\chi) + \frac{1}{1-\beta}\ln K$, with $v^p(\chi)$ satisfying

$$v^p(\chi) = \max_p \left\{\frac{1}{1-\beta}\ln[Z(\chi, p) - p\chi + (1-\delta)] + \beta v^p(\chi')\right\} \qquad \text{s.t. (12).}$$

The next proposition solves the planner's problem as a first-order approximation around a steady-state.

Proposition 2. *Let $\{\bar{p}, \bar{\chi}\}$ denote the solution to the following two equations:*

$$p^{sp} = \phi - \frac{1+\phi}{2}\left(1 + \frac{\beta\mu p^{sp}}{Z\left(\chi^{sp}, p^{sp}\right) - p^{sp}\chi^{sp} + (1-\beta)(1-\delta) - \beta\mu p^{sp}}\right)\chi. \tag{13}$$

$$Z\left(\chi^{sp}, p^{sp}\right) - p^{sp}\chi^{sp} = \mu p^{sp} \tag{14}$$

The planner's solution features a steady-state characterized by $\{p^{sp}, \chi^{sp}\}$ that satisfy $\chi^{sp} = \max\{\bar{\chi}, 0\}$, $p^{sp} = \begin{cases} \bar{p} & \text{if } \bar{\chi} > 0 \\ \phi & \text{otherwise} \end{cases}.$

Correspondingly, the rental tax is

$$\tau = 1 + \frac{1}{p^{sp} + \chi} - \frac{1}{\phi}.$$

When the planner's steady-state is interior ($\chi^{sp} > 0$), then to first-order around the steady-state, the planner chooses p as

$$p = p^{sp} + \eta \cdot \left(\chi - \chi^{sp}\right), \tag{15}$$

where η is a scalar function of model parameters and is derived in appendix A.7.

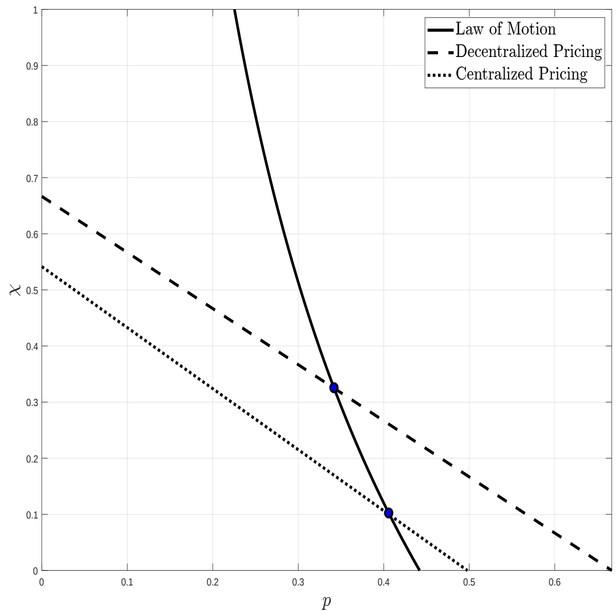
The planner's solution does not feature cycles.

We can compare the planner's rental rate with that in the decentralized economy (DE):

$$p = \phi - \chi. \tag{16}$$

The dotted line in Figure 8 plots the planner's pricing function (15) as a first-order approximation around the planner's steady-state and the dashed line plots the decentralized pricing function (16). The steady-state relationship (14) implied by the law of motion is shown in the solid line. The centralized and decentralized steady-state are indicated by the solid dots. We make several observations. First, for any given relative stock of lemon assets (χ), the planner would like to implement a rental rate p that is below the decentralized level through a rental tax. This is because from the planner's perspective, decentralized capital reallocation is excessive as it gives rise to lemon assets, and the planner uses the rental tax to reduce the creation of lemons. Second, the rental rate under the planner's solution declines faster in χ than in the decentralized equilibrium. This implies that the planner implements a higher tax rate when there is a higher fraction of lemon assets. Third, compared to the decentralized steady-state, the planner's steady-state features a lower relative stock of lemon assets (χ) and higher asset prices. Finally, Proposition 2 also implies that the planner's solution features a unique interior steady-state, and there does not exist cycles. This is reminiscent of the Turnpike theorem (McKenzie (1976)), suggesting that the endogenous cycles arising from the predator-prey dynamics between rent-seekers and entrepreneurs generate excessive volatility in asset prices and the volume of trades.

Figure 8. Centralized Allocation and Decentralized Allocation

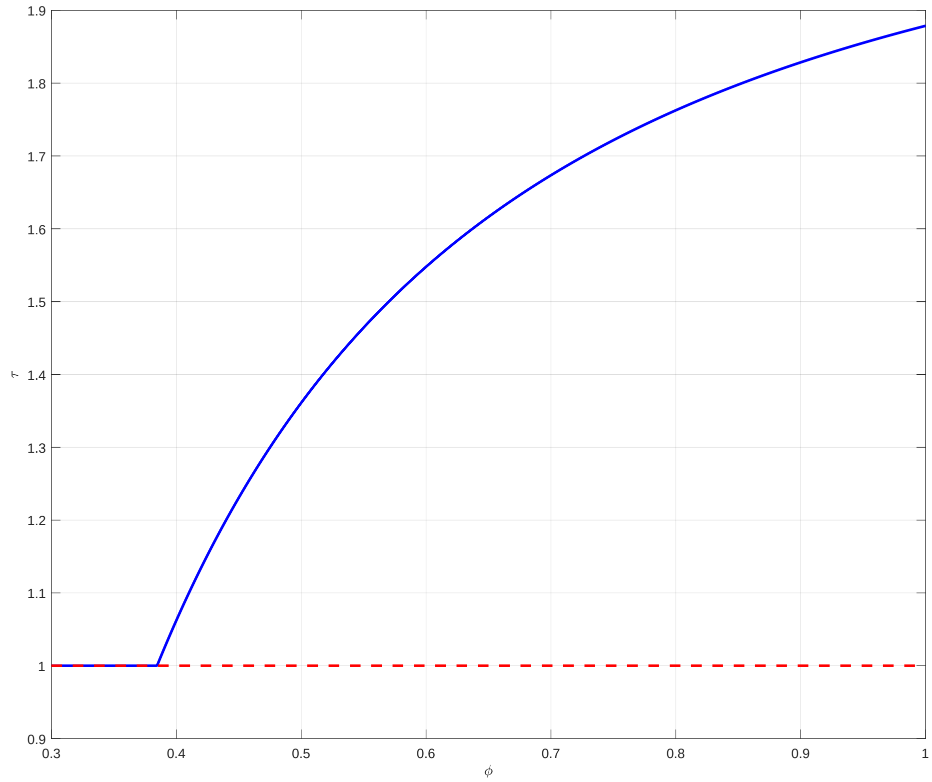


Notes: The black solid line depicts the steady state relationship $\mu p = Y(\chi, p)$, the dashed line is the decentralized pricing function, and the dotted line is the local centralized pricing function. Parameters used are $\beta = 0.95$, $\mu = 1.8$, $\delta = 0.05$ and $\phi = 0.67$.

Figure 9 shows the steady-state level rental tax rate τ as a function of the leverage parametrization ϕ. The figure shows that as ϕ increases—entrepreneurs feature higher leverage, thereby facilitating the creation of lemons—the planner implements a higher tax rate in the steady-state. When ϕ is sufficiently low—$\phi(2\mu - 1) \leq 1$, c.f. Lemma 5—the planner's steady-state coincides with the decentralized one, which features no lemon assets, and $\tau = 1$.

The society's discount rate β does not affect the decentralized steady-state but is an important parameter governing the planner's steady-state. From equation (13), we derive that for any given χ, a higher β translate into a lower planner's rental rate p in steady-state and thus lower degrees of asset reallocation. This is because a planner trades off between the current productive efficiency with future adverse selection and thus the efficiency in the future. A more patient social planner (higher β) has an incentive to even further distort the current-period rental market to depress the future stock of lemon assets, thereby improving productive efficiency in the future. As shown in Figure 10, with a higher β, the centralized steady-state features a lower level of χ but a higher level of p.

Figure 9. Steady-State Optimal Tax Rate

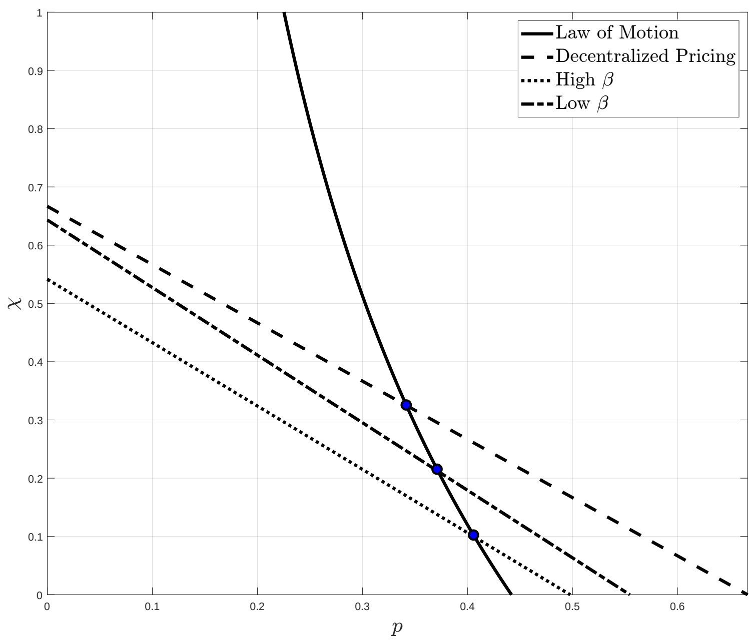


Notes: Social-optimal tax rate τ in the steady states as a function of ϕ. Parameters used are $\beta = 0.95$, $\mu = 1.8$ and $\delta = 0.05$.

5 Conclusion

This paper builds a simple model of endogenous credit cycles arising from the dynamics of adverse selection. Heterogeneous entrepreneurs trade productive assets in an anonymous market subject to financial frictions. Cream-skimming rent-seekers create durable lemon assets that are indistinguishable ex-ante from the productive assets but have no productive value ex-post. High asset prices today attract the creation of more lemons, thereby exacerbating adverse selection and depressing the future reallocation of productive assets and asset prices. The quality of assets evolves endogenously over time, and the equilibrium may feature endogenous cycles and chaos, with the credit market freezing and thawing recurrently and with deterministic ups and downs in asset prices and the volume of trades. We show that a social planner should adopt policies to lean against asset price movements and to eliminate endogenous cycles.

Figure 10. Higher $\beta \implies$ planner's steady-state features higher p and lower χ



Notes: Here we compare the social optimal allocations with $\beta = 0.95$ and $\beta = 0.85$. Other parameters used are $\mu = 1.8$, $\delta = 0.05$ and $\phi = 0.67$.

References

Aghion, Philippe, Abhijit Banerjee, and Thomas Piketty, "Dualism and macroeconomic volatility," *The Quarterly Journal of Economics*, 1999, *114* (4), 1359–1397.

Asriyan, Vladimir, William Fuchs, and Brett Green, "Liquidity sentiments," *American Economic Review*, 2019, *109* (11), 3813–48.

Azariadis, Costas and Bruce Smith, "Financial intermediation and regime switching in business cycles," *American economic review*, 1998, pp. 516–536.

_ , Leo Kaas, and Yi Wen, "Self-fulfilling credit cycles," *The Review of Economic Studies*, 2016, *83* (4), 1364–1405.

Bigio, Saki, "Endogenous liquidity and the business cycle," *American Economic Review*, 2015, *105* (6), 1883–1927.

Boissay, Frédéric, Fabrice Collard, and Frank Smets, "Booms and banking crises," *Journal of Political Economy*, 2016, *124* (2), 489–538.

Boissay, Frederic, Fabrice Collard, Jordi Galí, and Cristina Manea, "Monetary policy and endogenous financial crises," Technical Report 2021.

Boldrin, Michele and Michael Woodford, "Equilibrium models displaying endogenous fluctuations and chaos: a survey," *Journal of Monetary Economics*, 1990, *25* (2), 189–222.

Caramp, Nicolas, "Sowing the seeds of financial crises: Endogenous asset creation and adverse selection," *Available at SSRN 3009977*, 2017.

Chang, Briana, "Adverse selection and liquidity distortion," *The Review of Economic Studies*, 2018, *85* (1), 275–306.

Chiu, Jonathan and Thorsten V Koeppl, "Trading dynamics with adverse selection and search: Market freeze, intervention and recovery," *The Review of Economic Studies*, 2016, *83* (3), 969–1000.

Cui, Wei and Leo Kaas, "Default cycles," *Journal of Monetary Economics*, 2020.

Eisfeldt, Andrea L, "Endogenous liquidity in asset markets," *The Journal of Finance*, 2004, *59* (1), 1–30.

Farboodi, Maryam and Péter Kondor, "Cleansing by Tight Credit: Rational Cycles and Endogenous Lending Standards," 2021.

Fishman, Michael J, Jonathan A Parker, and Ludwig Straub, "A dynamic theory of lending standards," Technical Report 2020.

Fuchs, William and Andrzej Skrzypacz, "Costs and benefits of dynamic trading in a lemons market," *Review of Economic Dynamics*, 2019, *33*, 105–127.

Fukui, Masao, "Asset quality cycles," *Journal of Monetary Economics*, 2018, *95*, 97–108.

Gorton, Gary and Guillermo Ordonez, "Collateral crises," *American Economic Review*, 2014, *104* (2), 343–78.

_ and _ , "Good booms, bad booms," *Journal of the European Economic Association*, 2016.

Gorton, Gary B and Ping He, "Bank credit cycles," *The Review of Economic Studies*, 2008, *75* (4), 1181–1214.

Greenwood, Robin, Samuel G Hanson, Andrei Shleifer, and Jakob Ahm Sørensen, "Predictable financial crises," *The Journal of Finance*, 2022, *77* (2), 863–921.

Gu, Chao, Fabrizio Mattesini, Cyril Monnet, and Randall Wright, "Endogenous credit cycles," *Journal of Political Economy*, 2013, *121* (5), 940–965.

Guerrieri, Veronica and Robert Shimer, "Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality," *American Economic Review*, 2014, *104* (7), 1875–1908.

_ and _ , "Markets with multidimensional private information," *American Economic Journal: Microeconomics*, 2018, *10* (2), 250–74.

House, Christopher L, "Adverse selection and the financial accelerator," *Journal of Monetary Economics*, 2006, *53* (6), 1117–1134.

_ and Yusufcan Masatlioglu, "Managing markets for toxic assets," *Journal of Monetary Economics*, 2015, *70*, 84–99.

Hu, Yunzhi, "A dynamic theory of bank lending, firm entry, and investment fluctuations," *Journal of Economic Theory*, 2022, *204*, 105515.

Ikeda, Daisuke, "Adverse selection, lemons shocks and business cycles," *Journal of Monetary Economics*, 2019.

— **and Toan Phan**, "Toxic asset bubbles," *Economic Theory*, 2016, *61* (2), 241–271.

Kiyotaki, Nobuhiro and John Moore, "Credit cycles," *Journal of Political economy*, 1997, *105* (2), 211–248.

Krishnamurthy, Arvind and Tyler Muir, "How credit cycles across a financial crisis," Technical Report, National Bureau of Economic Research 2017.

Kurlat, Pablo, "Lemons markets and the transmission of aggregate shocks," *American Economic Review*, 2013, *103* (4), 1463–89.

Lee, Michael Junho and Daniel Neuhann, "Collateral quality and intervention traps," *Journal of Financial Economics*, 2023, *147* (1), 159–171.

Li, Tien-Yien and James A Yorke, "Period three implies chaos," *The theory of chaotic attractors*, 2004, pp. 77–84.

Lorenzoni, Guido, "Inefficient credit booms," *The Review of Economic Studies*, 2008, *75* (3), 809–833.

Martin, Alberto, "Endogenous credit cycles," *Available at SSRN 1001861*, 2005.

Matsuyama, Kiminori, "Growing through cycles," *Econometrica*, 1999, *67* (2), 335–347.

— , "Credit traps and credit cycles," *American Economic Review*, 2007, *97* (1), 503–516.

Maurin, Vincent, "Liquidity Fluctuations in Over-the-Counter Markets," *The Journal of Finance*, 2022, *77* (2), 1325–1369.

McKenzie, Lionel W, "Turnpike theory," *Econometrica: Journal of the Econometric Society*, 1976, pp. 841–865.

Mendoza, Enrique G, "Sudden stops, financial crises, and leverage," *American Economic Review*, 2010, *100* (5), 1941–66.

— **and Marco E Terrones**, "An anatomy of credit booms: evidence from macro aggregates and micro data," Technical Report, National Bureau of Economic Research 2008.

— **and** — , "An anatomy of credit booms and their demise," Technical Report, National Bureau of Economic Research 2012.

Mian, Atif, Amir Sufi, and Emil Verner, "Household debt and business cycles worldwide," *The Quarterly Journal of Economics*, 2017, *132* (4), 1755–1817.

__ , __ , and __ , “How does credit supply expansion affect the real economy? the productive capacity and household demand channels,” *The Journal of Finance*, 2020, *75* (2), 949–994.

Moll, Benjamin, “Productivity losses from financial frictions: Can self-financing undo capital misallocation?,” *American Economic Review*, 2014, *104* (10), 3186–3221.

Myerson, Paul, *Lean supply chain and logistics management*, McGraw-Hill Education, 2012.

Neuhann, Daniel, “Inefficient Asset Price Booms,” *Available at SSRN 3095730*, 2019.

Philippon, Thomas and Vasiliki Skreta, “Optimal interventions in markets with adverse selection,” *American Economic Review*, 2012, *102* (1), 1–28.

Reinhart, Carmen M and Kenneth S Rogoff, “The aftermath of financial crises,” *American Economic Review*, 2009, *99* (2), 466–472.

Schularick, Moritz and Alan M Taylor, “Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008,” *American Economic Review*, 2012, *102* (2), 1029–61.

Suarez, Javier and Oren Sussman, “Endogenous Cycles in a Stiglitz-Weiss Economy,” *Journal of Economic Theory*, 1997, *76* (1), 47–71.

Tirole, Jean, “Overcoming adverse selection: How public intervention can restore market functioning,” *American Economic Review*, 2012, *102* (1), 29–59.

Williams, Basil, “Search, liquidity, and retention: Screening multidimensional private information,” *Journal of Political Economy*, 2021, *129* (5), 1487–1507.

Zryumov, Pavel, “Dynamic adverse selection: Time-varying market conditions and endogenous entry,” *Available at SSRN 2653129*, 2015.

A Proof of Propositions and Lemmas

A.1 Proof of Lemma 1

Note that the entrepreneur’s optimization problem is:

$$y_i \equiv \max_{\ell_i, b_i} \quad z_i\left(k_i - \ell_i + \rho b_i\right) - p\left(b_i - \ell_i\right) = z_i k_i + (\rho z_i - p) b_i + (p - z_i)\ell_i$$

which is a linear function of (ℓ_i, b_i), then we immediately get the policy functions:

$$(b_i, \ell_i) = \begin{cases} (0, k_i), & \text{if } z_i < \underline{z} \\ (0, 0), & \text{if } z_i \in (\underline{z}, \overline{z}) \\ \left(\frac{\phi}{1-\phi} k_i, 0\right) & \text{if } z_i \geq \overline{z} \end{cases}.$$

And the two cutoffs are $(\underline{z}, \bar{z}) \equiv (p, p/\rho)$.

A.2 Proof of Lemma 2

As $z \sim U[0, 1]$, equation (7) can be rewritten as:

$$\frac{\phi}{1-\phi} \frac{1-\bar{z}}{\bar{z}} = 1,$$

and the asset quality is given by

$$\rho = \frac{\underline{z}}{\underline{z} + \chi}$$

so we immediately get $\bar{z} = \phi$, then from $\bar{z} = \underline{z}/\rho$ we get $\underline{z} = \phi - \chi = p$ and $\rho = \frac{\phi - \chi}{\phi}$. If $\phi > \chi$, the capital market exists, then the previous results holds. If $\phi < \chi$, the market collapses, and $p = 0$, ρ =0.

A.3 Proof of Lemma 3

When the capital market exists, capital of firms with $z \leq \underline{z}$ are sold to those with $z \geq \bar{z}$. Capital of firms with $\underline{z} < z < \bar{z}$ will be used on their own, so the aggregate productivity is:

$$Z = (F(\underline{z}) + 1 - F(\bar{z}))\, \mathbb{E}[z|z \geq \bar{z}] + (F(\bar{z}) - F(\underline{z}))\, \mathbb{E}[z|\underline{z} \leq z < \bar{z}]$$

Plug in the results of $\underline{z}$ and $\bar{z}$, and we immediately get:

$$Z = \frac{1 + (1 + \chi) \max\{\phi - \chi, 0\}}{2},$$

and the total capital reallocated is related to p or $\underline{z}$.

A.4 Proof of Lemma 4 and Proposition 1

Note that both the productive firms and lemon firms have log utility, so their saving rates are the same and are given by β. For the productive firms, the income in each period is given by:

$$\left[p\left(\chi\right)+\max\left\{z_i-p\left(\chi\right),0\right\}+\frac{\phi}{1-\phi}\max\left\{z_i\rho\left(\chi\right)-p\left(\chi\right),0\right\}+\left(1-\delta\right)\right]k,$$

and with a saving rate β, the capital stock in the next period is:

$$k'=\beta\left[p\left(\chi\right)+\max\left\{z_i-p\left(\chi\right),0\right\}+\frac{\phi}{1-\phi}\max\left\{z_i\rho\left(\chi\right)-p\left(\chi\right),0\right\}+\left(1-\delta\right)\right]k.$$

The analysis for the lemon firms are the same, and the income for lemon firms is $[1-\delta+\mu p(\chi)]x$, so the lemon stock in the next period is given by:

$$x'=\beta[1-\delta+\mu p(\chi)]x.$$

By integrating the individual firms' decisions we get the aggregate law of motion:

$$K'\left(\chi\right)=\beta\left[Z\left(\chi\right)-p\left(\chi\right)\chi+1-\delta\right]K,$$

$$X'(\chi)=\beta[1-\delta+\mu p(\chi)]X.$$

Then by dividing the two we get the one-equation law of motion that captures the dynamic of the economy:

$$\chi_{t+1}\equiv\Gamma\left(\chi_t\right)=\begin{cases}\frac{\mu(\phi-\chi_t)+1-\delta}{(1+(\phi-\chi_t)(1-\chi_t))/2+1-\delta}\chi_t, & \text{if } \chi_t<\phi\\ \frac{1-\delta}{1/2+(1-\delta)}\chi_t, & \text{otherwise.}\end{cases}.$$

A.5 Proof of Lemma 5

From the law of motion we know that when $\chi_t\geq\phi$ there cannot be any steady state with $\chi^*>0$, so we only consider the case $\chi^*<\phi$. Then the steady state χ^* is determined by $\Gamma(\chi^*)=\chi^*$, that is:

$$\frac{\mu\left(\phi-\chi^*\right)+1-\delta}{\left(1+\left(\phi-\chi^*\right)\left(1-\chi^*\right)\right)/2+1-\delta}=1,$$

which can be rearranged as:

$$\mu\left(\phi-\chi^*\right)+1-\delta=\left(1+\left(\phi-\chi^*\right)\left(1-\chi^*\right)\right)/2+1-\delta\Rightarrow g(\chi^*)=0.$$

This is a quadratic equation of χ^*. Note that when $\chi^*=\phi$ we have $g(\chi)=1>0$, so as to ensure a solution of $\chi^*<\phi$ we just need

$$g(0) = 1 - (2\mu - 1)\phi < 0,$$

so that $\phi(2\mu - 1) > 1$.

What's more we can prove that the first order derivation of Γ at χ^* is:

$$\frac{\partial \Gamma'(\chi^*)}{\partial \phi} = \frac{(\mu(\phi - \chi^*) + 1 - \delta)\left(\frac{1}{2} - \frac{1-\chi-2\mu}{1+\phi-2\chi-2\mu}\right) - \mu\left(1 - \frac{1-\chi-2\mu}{1+\phi-2\chi-2\mu}\right)\left(\frac{1}{2}(1+\phi) - \chi - \mu\right)}{(\mu(\phi - \chi^*) + 1 - \delta)^2},$$

numerator is

$$\begin{aligned} & (\mu(\phi - \chi^*) + 1 - \delta)\left(\frac{\frac{1}{2}(1+\phi) - 1 + \mu}{1+\phi-2\chi-2\mu}\right) - \mu\left(\frac{\phi - \chi}{1+\phi-2\chi-2\mu}\right)\left(\frac{1}{2}(1+\phi) - \chi - \mu\right) \\ = \ & \frac{1}{1+\phi-2\chi-2\mu}\left[\mu(\phi-\chi)(2\mu+\chi-1) + (1-\delta)\left(\frac{1}{2}(1+\phi) - 1 + \mu\right)\right], \end{aligned}$$

$$\phi(2\mu - 1) > 1$$

implies that the term in bracket is positive. Hence we just need to show $(1 + \phi - 2\chi - 2\mu) < 0$ to establish that $\frac{\partial \Gamma'(\chi^*)}{\partial \phi} < 0$. We note that since $\phi < 1$, we have $2\mu - 1 > 1$. Hence

$$1 + \phi - 2\chi - 2\mu < \phi - 2\chi - 1 < 0.$$

So as ϕ increases $\Gamma'(\chi^*)$ decreases, and when ϕ is so large that $\Gamma'(\chi^*) < -1$, the interior steady state becomes unstable and there may be endogenous cycles.

A.6 Proof of Proposition 2

Let $Y(\chi, p) \equiv Z(\chi, p) - p\chi = 1/2 - p^2/2 + p - p^2(1-\phi)/2\phi - p\chi(1 + (1-\phi)/2\phi)$, $a = 1 - \delta$. Then

$$V(\chi) = \max_p \frac{1}{1-\beta} \ln(Y(\chi, p) + a) + \beta V(\chi'),$$

subject to:

$$\chi' = \frac{\mu p + a}{Y(\chi, p) + a}\chi.$$

Then we get

$$\frac{\partial \chi'}{\partial p} = \chi \frac{\mu(Y(\chi, p) + a) - \frac{\partial Y}{\partial p}(\mu p + a)}{(Y(\chi, p) + a)^2},$$

and

$$\frac{\partial \chi'}{\partial \chi} = \frac{\mu p + a}{Y(\chi,p)+a} - \chi \frac{\partial Y}{\partial \chi}\frac{(\mu p + a)}{(Y(\chi,p)+a)^2}.$$

So FOC is:

$$\frac{1}{1-\beta}\frac{\partial Y/\partial p}{Y+a} + \beta V'(\chi')\frac{\partial \chi'}{\partial p} = 0.$$

Envelope theorem gives:

$$V'(\chi) = \frac{1}{1-\beta}\frac{\partial Y/\partial \chi}{Y+a} + \beta V'(\chi')\frac{\partial \chi'}{\partial \chi}.$$

In a steady state, $V'(\chi) = V'(\chi')$ hence

$$V'(\chi) = \frac{1}{1-\beta}\frac{\partial Y/\partial \chi}{Y+a} \Bigg/ \left(1 - \beta \frac{\partial \chi'}{\partial \chi}\right).$$

Substitute into FOC, we get:

$$\frac{\partial Y}{\partial p}\left(1-\beta\frac{\partial \chi'}{\partial \chi}\right) + \beta \frac{\partial Y}{\partial \chi}\frac{\partial \chi'}{\partial p} = 0,$$

Hence

$$\begin{aligned}
0 &= \frac{\partial Y}{\partial p}\left(1 - \beta\frac{\mu p + a}{Y(\chi,p)+a} + \beta\chi\frac{\partial Y}{\partial \chi}\frac{\mu p + a}{(Y(\chi,p)+a)^2}\right) + \beta\frac{\partial Y}{\partial \chi}\chi\frac{\mu(Y(\chi,p)+a) - \frac{\partial Y}{\partial p}(\mu p + a)}{(Y(\chi,p)+a)^2} \\
&= \frac{\partial Y}{\partial p}\left(1-\beta\frac{\mu p + a}{Y(\chi,p)+a}\right) + \frac{\partial Y}{\partial \chi}\frac{\beta\mu\chi}{Y(\chi,p)+a}
\end{aligned}$$

Now use the fact that

$$\frac{\partial Y}{\partial p} = 1 - p\phi^{-1} - \chi\frac{1+\phi}{2\phi}$$

$$\frac{\partial Y}{\partial \chi} = -p\frac{1+\phi}{2\phi}$$

We have

$$\left(1 - p\phi^{-1} - \chi\frac{1+\phi}{2\phi}\right)(Y(\chi,p) + a - \beta(\mu p + a)) = p\frac{1+\phi}{2\phi}\beta\mu\chi$$

$$\left(\phi - p - \chi\frac{1+\phi}{2}\right)(Y(\chi,p) + a - \beta(\mu p + a)) = \beta p \mu\frac{1+\phi}{2}\chi$$

$$(\phi - p)(Y(\chi,p) + a - \beta(\mu p + a)) = \chi\frac{1+\phi}{2}(Y(\chi,p) + (1-\beta)a)$$

$$\begin{aligned}
p &= \phi - \chi\frac{1+\phi}{2}\frac{Y(\chi,p) + (1-\beta)a}{Y(\chi,p)+(1-\beta)a - \beta\mu p} \\
&= \phi - \chi\frac{1+\phi}{2}\left(1 + \frac{\beta\mu p}{Y(\chi,p) + (1-\beta)(1-\delta) - \beta\mu p}\right)
\end{aligned}$$

Taking a first-order expansion around the steady-state gives rise to the linearized policy function in the proposition. We derive the linearized policy function in Appendix A.7 below. Using implicit function theorem, we can show that the law of motion of the state variable $\chi' = \Gamma(\chi)$ features a positive slope at the steady-state, thereby confirming that the planner's solution does not feature cycles.

A.7 Other setting 1: with Pareto Weight

In the above analysis, we calculated the social planner's solution when concerning only the welfare of the entrepreneurs. Now consider a more generally setting where the social planner gives a Pareto weight α to the rent seekers, and $1-\alpha$ to the entrepreneurs. The social planner's maximization problem is:

$$V^p(\chi, K) = \max_p \left\{(1-\alpha)\ln\left[(1-\beta)(Y+a)K\right] + \alpha \ln\left[(1-\beta)\frac{\mu p + a}{\mu}\chi K\right] + \beta V^p(\chi', K')\right\}$$

subject to the two constraints. The first term $\ln[(1-\beta)(Y+a)K]$ corresponds to welfare of the producers, where $Y = Y(\chi, p) \equiv Z(\chi, p) - p\chi$ and $a = 1-\delta$. The term $\ln\left[(1-\beta)\frac{\mu p+a}{\mu}\chi K\right]$ denotes the welfare of the rent seekers. Note that the budget constraint of the rent seekers is:

$$\frac{X' - (1-\delta)X}{\mu} = p(\chi)X - C^L$$

where $X' = \beta\left[\mu p + 1 - \delta\right]X$ so the aggregate consumption of the rent-seekers is $C^L = \frac{\mu p X - \beta[\mu p + 1 - \delta]X + (1-\delta)X}{\mu} = (1-\beta)\frac{\mu p + a}{\mu}\chi K$. Note that here we still find that the value function is log-linear in K, so that $V^p(\chi, K) = v(\chi) + b\ln K$, where b is a constant to be determined. Using this guess-and-verify we can rewite the social planner's value function as (note that here we drop the maximization symbol):

$$\begin{aligned} v(\chi) + b\ln K = \max_p (1-\alpha)\ln(Y+a) + (1-\alpha)\ln K + \alpha\ln\left(\frac{\mu p + a}{\mu}\chi\right) + \alpha \ln K \\ + \beta v(\chi') + \beta b \ln\left(\beta(Y+a)\right) + b\beta\ln K + \ln(1-\beta) \end{aligned}$$

Note that we can drop all the constant terms. By comparing coefficients we get that $b = \frac{1}{1-\beta}$, and that $v(\chi)$ satisfies:

$$\begin{aligned} v(\chi) &= \max_p (1-\alpha)\ln(Y+a) + \alpha\ln\left[(\mu p + a)\chi\right] + \frac{\beta}{1-\beta}\ln(Y+a) + \beta v(\chi') \\ &= \max_p \frac{1}{1-\beta}\ln(Y+a) + \alpha\ln\left(\frac{\mu p + a}{Y+a}\chi\right) + \beta v(\chi') \\ &= \max_p \frac{1}{1-\beta}\ln(Y+a) + \alpha\ln\chi' + \beta v(\chi') \end{aligned}$$

The first order condition is:

$$\frac{1}{1-\beta}\frac{1}{Y+a}\frac{\partial Y}{\partial p}+\frac{\alpha}{\chi'}\frac{\partial \chi'}{\partial p}+\beta v'\left(\chi'\right)\frac{\partial \chi'}{\partial p}$$

Envelope theorem gives:

$$v'(\chi)=\frac{1}{1-\beta}\frac{1}{Y+a}\frac{\partial Y}{\partial \chi}+\frac{\alpha}{\chi'}\frac{\partial \chi'}{\partial \chi}+\beta v'(\chi')\frac{\partial \chi'}{\partial \chi}\Rightarrow v'(\chi)=\left[\frac{1}{1-\beta}\frac{1}{Y+a}\frac{\partial Y}{\partial \chi}+\frac{\alpha}{\chi'}\frac{\partial \chi'}{\partial \chi}\right]\Big/\left(1-\beta\frac{\partial \chi'}{\partial \chi}\right)$$

So that the first order condition around the steady-state $\chi=\chi'$ is

$$\frac{1}{1-\beta}\frac{1}{Y+a}\frac{\partial Y}{\partial p}+\frac{\alpha}{\chi'}\frac{\partial \chi'}{\partial p}+\beta\frac{\partial \chi'}{\partial p}\left[\frac{1}{1-\beta}\frac{1}{Y+a}\frac{\partial Y}{\partial \chi}+\frac{\alpha}{\chi'}\frac{\partial \chi'}{\partial \chi}\right]\Big/\left(1-\beta\frac{\partial \chi'}{\partial \chi}\right)=0$$

$$\left(\frac{1}{1-\beta}\frac{1}{Y+a}\frac{\partial Y}{\partial p}+\frac{\alpha}{\chi'}\frac{\partial \chi'}{\partial p}\right)\left(1-\beta\frac{\partial \chi'}{\partial \chi}\right)+\beta\frac{\partial \chi'}{\partial p}\left[\frac{1}{1-\beta}\frac{1}{Y+a}\frac{\partial Y}{\partial \chi}+\frac{\alpha}{\chi'}\frac{\partial \chi'}{\partial \chi}\right]=0$$

Note that $\frac{\partial \chi'}{\partial \chi}=\frac{\mu p+a}{Y+a}-\chi\frac{\partial Y}{\partial \chi}\frac{\mu p+a}{(Y+a)^2}$ and $\frac{\partial \chi'}{\partial p}=\chi\frac{\mu(Y+a)-\frac{\partial Y}{\partial p}(\mu p+a)}{(Y+a)^2}$, so we get:

$$\frac{1}{1-\beta}\frac{1}{Y+a}\left[\frac{\partial Y}{\partial p}\left(1-\beta\frac{\mu p+a}{Y+a}+\beta\chi\frac{\partial Y}{\partial \chi}\frac{\mu p+a}{(Y+a)^2}\right)+\beta\frac{\partial Y}{\partial \chi}\chi\frac{\mu(Y+a)-\frac{\partial Y}{\partial p}(\mu p+a)}{(Y+a)^2}\right]$$
$$+\frac{\alpha}{\chi'}\left[\chi\frac{\mu(Y+a)-\frac{\partial Y}{\partial p}(\mu p+a)}{(Y+a)^2}\right]=0$$

which can be simplified to

$$\frac{1}{1-\beta}\left[\frac{\partial Y}{\partial p}\left(1-\beta\frac{\mu p+a}{Y+a}\right)+\beta\frac{\partial Y}{\partial \chi}\chi\frac{\mu}{Y+a}\right]+\alpha\frac{\mu(Y+a)-\frac{\partial Y}{\partial p}(\mu p+a)}{\mu p+a}=0$$

$$\frac{\partial Y}{\partial p}\left[1-\alpha(1-\beta)-\beta\frac{\mu p+a}{Y+a}\right]+\frac{\partial Y}{\partial \chi}\frac{\beta\mu\chi}{Y+a}+\alpha\mu(1-\beta)\frac{Y+a}{\mu p+a}=0$$

where $\frac{\partial Y}{\partial p}=1-p\phi^{-1}-\chi\frac{1+\phi}{2\phi}$ and $\frac{\partial Y}{\partial \chi}=-p\frac{1+\phi}{2\phi}$.

$$\left(1-\frac{p}{\phi}-\chi\frac{1+\phi}{2\phi}\right)\left[1-\alpha(1-\beta)-\beta\frac{\mu p+a}{Y+a}\right]-p\frac{1+\phi}{2\phi}\frac{\beta\mu\chi}{Y+a}+\alpha\mu(1-\beta)\frac{Y+a}{\mu p+a}=0$$

$$\left(\phi-p-\frac{1+\phi}{2}\chi\right)\left[1-\alpha(1-\beta)-\beta\frac{\mu p+a}{Y+a}\right]-\frac{1+\phi}{2}\chi\frac{\beta\mu p}{Y+a}+\alpha\phi\mu(1-\beta)\frac{Y+a}{\mu p+a}=0$$

$$p=\phi-\frac{1+\phi}{2}\chi\left(1+\frac{\beta\mu p}{(1-\alpha+\alpha\beta)(Y+a)-\beta(\mu p+a)}\right)+\frac{\alpha\phi\mu(1-\beta)(Y+a)^2/(\mu p+a)}{(1-\alpha+\alpha\beta)(Y+a)-\beta(\mu p+a)} \tag{A1}$$

Note that if $\alpha=0$, then the problem comes back to our baseline case.

We first consider the steady-state:

$$\left(\phi-p-\frac{1+\phi}{2}\chi\right)(1-\beta)(1-\alpha)-\frac{1+\phi}{2}\chi\frac{\beta\mu p}{\mu p+a}+\alpha\phi\mu(1-\beta)=0$$

$$(\phi-p)(1-\beta)(1-\alpha)-\frac{1+\phi}{2}\chi\left[(1-\beta)(1-\alpha)+\beta\frac{\mu p}{\mu p+a}\right]+\alpha\phi\mu(1-\beta)=0$$

$$\begin{aligned}p&=\phi-\frac{1+\phi}{2}\chi\left[1+\frac{\beta}{(1-\beta)(1-\alpha)}\frac{\mu p}{\mu p+a}\right]+\frac{\alpha\mu}{1-\alpha}\phi\\&=\frac{1-\alpha(1-\mu)}{1-\alpha}\phi-\frac{1+\phi}{2}\chi\left[1+\frac{\beta}{(1-\beta)(1-\alpha)}\frac{\mu p}{\mu p+a}\right]\end{aligned}$$

Next we linearize (A1).

$$\begin{aligned}\frac{\partial p}{\partial\chi}=&-\frac{1+\phi}{2}\left(1+\frac{\beta\mu p^*}{(1-\alpha)(1-\beta)(\mu p^*+a)}\right)\\&-\frac{1+\phi}{2}\chi^*\frac{\beta\mu\frac{\partial p}{\partial\chi}(1-\alpha)(1-\beta)(\mu p^*+a)-\beta\mu p^*\left[(1-\alpha+\alpha\beta)\frac{\partial Y}{\partial\chi}-\beta\mu\frac{\partial p}{\partial\chi}\right]}{(1-\alpha)^2(1-\beta)^2(\mu p^*+a)^2}\\&+\frac{\alpha\phi\mu(1-\beta)\frac{2(\mu p^*+a)^2\frac{\partial Y}{\partial\chi}-\mu\frac{\partial p}{\partial\chi}(\mu p^*+a)^2}{(\mu p^*+a)^2}}{(1-\alpha)(1-\beta)(\mu p^*+a)}-\frac{\alpha\phi\mu(1-\beta)(\mu p^*+a)\left[(1-\alpha+\alpha\beta)\frac{\partial Y}{\partial\chi}-\beta\mu\frac{\partial p}{\partial\chi}\right]}{(1-\alpha)^2(1-\beta)^2(\mu p^*+a)^2}\end{aligned}$$

$$\begin{aligned}\frac{\partial p}{\partial\chi}=&-\frac{1+\phi}{2}\left(1+\frac{\beta\mu p^*}{(1-\alpha)(1-\beta)(\mu p^*+a)}\right)-\frac{1+\phi}{2}\chi^*\frac{\beta\mu}{(1-\alpha)(1-\beta)(\mu p^*+a)}\frac{\partial p}{\partial\chi}\\&+\frac{1+\phi}{2}\chi^*\frac{\beta\mu p^*}{(1-\alpha)^2(1-\beta)^2(\mu p^*+a)^2}\left[(1-\alpha+\alpha\beta)\frac{\partial Y}{\partial\chi}-\beta\mu\frac{\partial p}{\partial\chi}\right]\\&+\frac{\alpha\phi\mu}{(1-\alpha)(\mu p^*+a)}\left(2\frac{\partial Y}{\partial\chi}-\mu\frac{\partial p}{\partial\chi}\right)-\frac{\alpha\phi\mu\left[(1-\alpha+\alpha\beta)\frac{\partial Y}{\partial\chi}-\beta\mu\frac{\partial p}{\partial\chi}\right]}{(1-\alpha)^2(1-\beta)(\mu p^*+a)}\end{aligned}$$

where $\frac{\partial Y}{\partial\chi}=Y_2\frac{\partial p}{\partial\chi}+Y_1$ with $Y_2=-p+1-2p(1-\phi)/2\phi-\chi\left(1+(1-\phi)/2\phi\right)$, $Y_1=-p\left(1+(1-\phi)/2\phi\right)$, define

$$\Xi=\frac{1+\phi}{2}\chi^*\frac{\beta\mu}{(1-\alpha)(1-\beta)(\mu p^*+a)}+\frac{\alpha\phi\mu^2}{(1-\alpha)(\mu p^*+a)}$$

$$\Pi = \frac{1+\phi}{2}\chi^* \frac{\beta\mu p^*}{(1-\alpha)^2(1-\beta)^2(\mu p^*+a)^2} - \frac{\alpha\phi\mu}{(1-\alpha)^2(1-\beta)(\mu p^*+a)}$$

$$\Upsilon = \frac{2\alpha\phi\mu}{(1-\alpha)(\mu p^*+a)}$$

Then the above equation can be simplified to:

$$\begin{aligned}\frac{\partial p}{\partial \chi} =& -\frac{1+\phi}{2}\left(1+\frac{\beta\mu p^*}{(1-\alpha)(1-\beta)(\mu p^*+a)}\right) - \Xi\frac{\partial p}{\partial \chi} \\ &+ \Pi\left[(1-\alpha+\alpha\beta)\left(Y_2\frac{\partial p}{\partial \chi}+Y_1\right) - \beta\mu\frac{\partial p}{\partial \chi}\right] + \Upsilon\left(Y_2\frac{\partial p}{\partial \chi}+Y_1\right)\end{aligned}$$

which gives:

$$\frac{\partial p}{\partial \chi} = \frac{\mathcal{A}}{\mathcal{B}}$$

where

$$\mathcal{A} = -\frac{1+\phi}{2}\left(1+\frac{\beta\mu p^*}{(1-\alpha)(1-\beta)(\mu p^*+a)}\right) - \left[\Pi(1-\alpha+\alpha\beta)+\Upsilon\right]Y_1$$

$$\mathcal{B} = 1 + \Xi - \left[\Pi(1-\alpha+\alpha\beta)+\Upsilon\right]Y_2 + \Pi\beta\mu$$

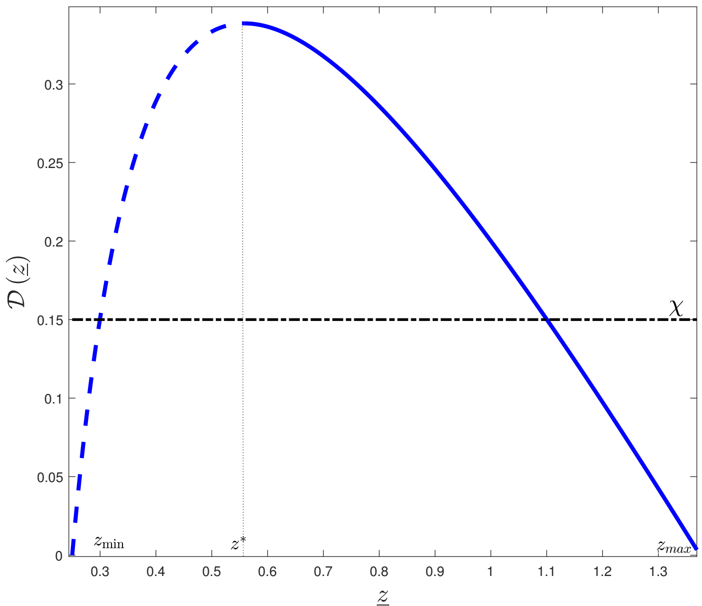
B General Characterization

We have so far used the uniform distribution $[0,1]$ and production function $y_i = z_i k_i$ to make sense of the key mechanism of the paper. Now we generalize the analysis by assuming $z_{\min} > 0$ and $y_i = A z_i k_i$, where A is an aggregate productivity shock. In this scenario, the market-clearing condition for the lemon asset (6) may have two solutions. To distinguish the two solutions, we assume that the net demand of lemon assets (which is characterized in Lemma 6) is mazimized at z^*,and $z^{**} = z^{**}(z^*;\phi)$ is an implicit function of z^* which is definded in equation (7).Then we have $\lim_{z^*\to z_{\min}} z^{**}(z^*;\phi) = z_{\max}$. Equation (6) can be rewritten as net demand and net supply of lemon assets, which is summarized in the following lemma.

Lemma 6. *If the market exists, there always exist multiple equilibria to $\underline{z}$, which satisfies*

$$\mathcal{D}(\underline{z}) = \chi, \tag{A2}$$

Figure A.1. $\mathcal{D}(\underline{z}) = \chi$



where the net demand of lemon assets is given by

$$\mathcal{D}(\underline{z}) \equiv \frac{\phi}{1-\phi}[1 - F(\bar{z})] - F(\underline{z}), \tag{A3}$$

and $\bar{z}$ is an implicit function of $\underline{z}$ from equation (7).

Proof. Note that the capital market clear condition is given by:

$$\frac{\phi}{1-\phi}[1 - F(\bar{z})] = F(\underline{z}) + \chi$$

Then we immediately get the equation (A3). Then from the asset quality condition:

$$\rho = \frac{F(\underline{z})}{F(\underline{z}) + \chi},$$

and that $\bar{z} = \underline{z}/\rho$, we can write $\bar{z}$ as a function of $\underline{z}$. □

Comments: If there were no adverse selection, i.e., $\chi = 0$, and thus $\bar{z} = \underline{z}$. In that case, equation (A3) can be further written as

$$\mathcal{D}^\star(\underline{z}) = \frac{\phi}{1-\phi} - \left(1 + \frac{\phi}{1-\phi}\right) F(\underline{z}) > \mathcal{D}(\underline{z}),$$

and the cutoff in equilibrium is determined by $\mathcal{D}^\star(\underline{z}) = 0$. That is, $\underline{z} = z^{**}(\phi) = F^{-1}(\phi)$.

We consider *Pareto-dominant equilibrium*, and thus $\underline{z} \in (z^*, z^{**}) \subset (z_{\min}, z_{\max})$, where

$$z^* = \arg\max_{z \in (z_{\min}, z^{**})} \mathcal{D}(z) = \mathcal{D}^{-1}(\overline{\chi}).$$

We can prove that $\underline{z}$ strictly increases with λ.

Proposition 3. *The market exists iff $\chi < \overline{\chi}$, where $\bar{\chi}$ is determined by*

$$\overline{\chi} = \max_{z \in (z_{\min}, z^{**})} \mathcal{D}(z) = \mathcal{D}(z^*), \tag{A4}$$

with $\overline{\chi}$ increasing with ϕ. Then the asset quality and asset price are respectively given by

$$\begin{aligned} \rho &= \begin{cases} \frac{F(\underline{z})}{F(\underline{z})+\chi}, & \text{if } \chi \leq \bar{\chi} \\ 0, & \text{if } \chi > \bar{\chi} \end{cases}, \\ p &= \begin{cases} A\underline{z}, & \text{if } \chi \leq \bar{\chi} \\ 0, & \text{if } \chi > \bar{\chi} \end{cases}. \end{aligned}$$

Proposition 4. *The law of motion of χ_t is characterized as below.*

$$\chi_{t+1} = \Theta_t(\chi_t) = \begin{cases} \Theta_t^+(\chi_t), & \text{if } \chi_t \leq \overline{\chi}_t \\ \Theta_t^-(\chi_t), & \text{if } \chi_t > \overline{\chi}_t \end{cases},$$

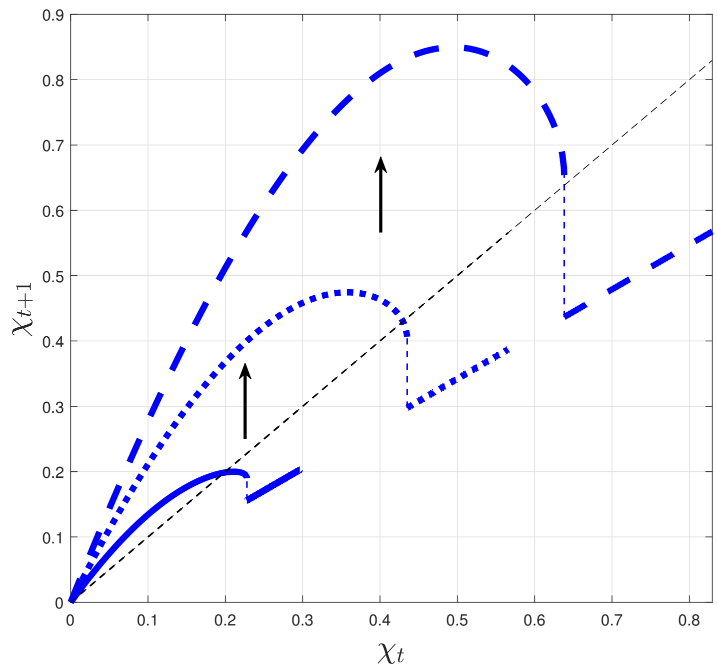
where $\overline{\chi}$ is defined in equation (A4), and $\left\{\Theta_t^+(\chi_t), \Theta_t^-(\chi_t)\right\}$ are defined as

$$\begin{aligned} \Theta^+(\chi) &= \frac{\mu p + (1-\delta)}{AZ - p\chi + (1-\delta)}\chi, \\ \Theta^-(\chi) &= \frac{1-\delta}{A\mathbb{E}(z) + (1-\delta)}\chi. \end{aligned}$$

Paradox of credit expansion: In the static environment, a higher ϕ always strengthen the risk capacity. The bright side of the market as insurance increases. The dark side is adverse selection and cream skimming. With a credit expansion, the high-productivity firms can borrow more to produce more, which in turn makes it more profitable for the lemon firms to transform capital to lemon, and the asset quality tends to decrease.

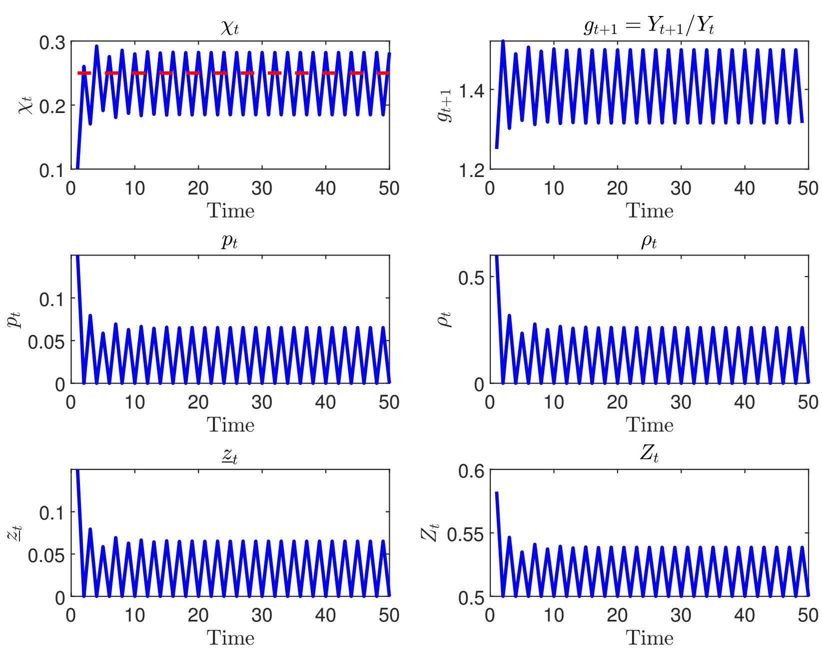
C Transition Paths with Cycles

Figure A.2. Law of motion on χ_t



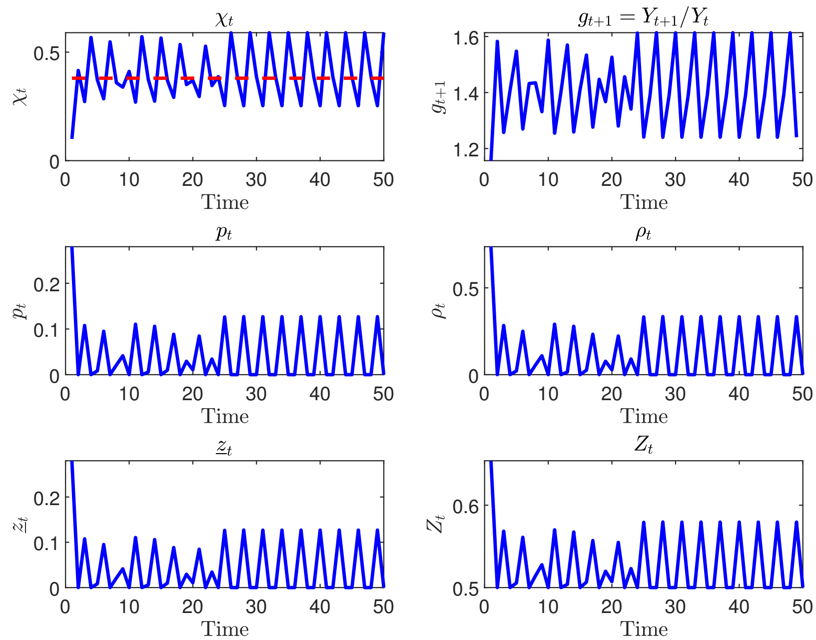
Notes: Law of motion on χ_t under different levels of ϕ.

Figure A.3. Transition dynamics of a 2-cycles



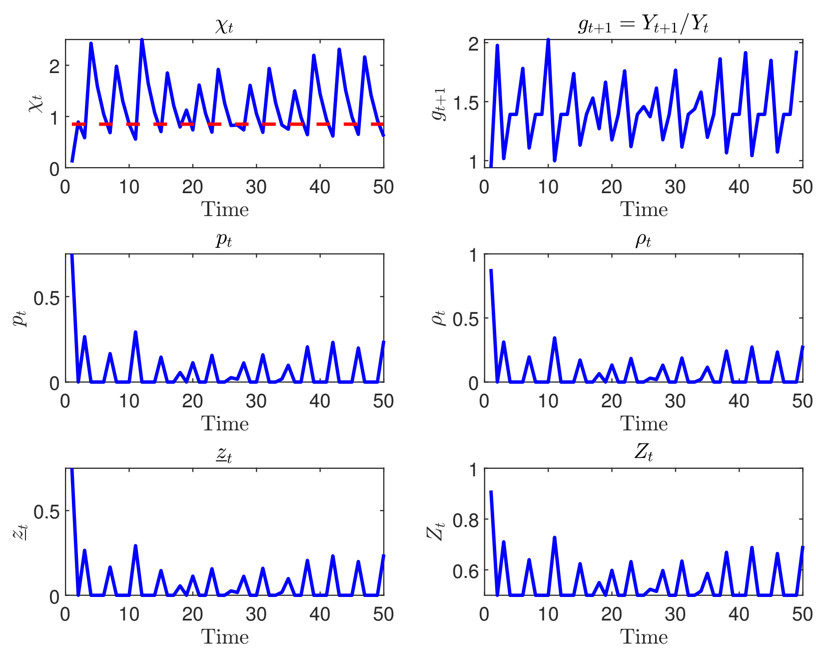
Notes: Parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$ and $\phi = 0.25$.

Figure A.4. Transition path of a 3-cycle



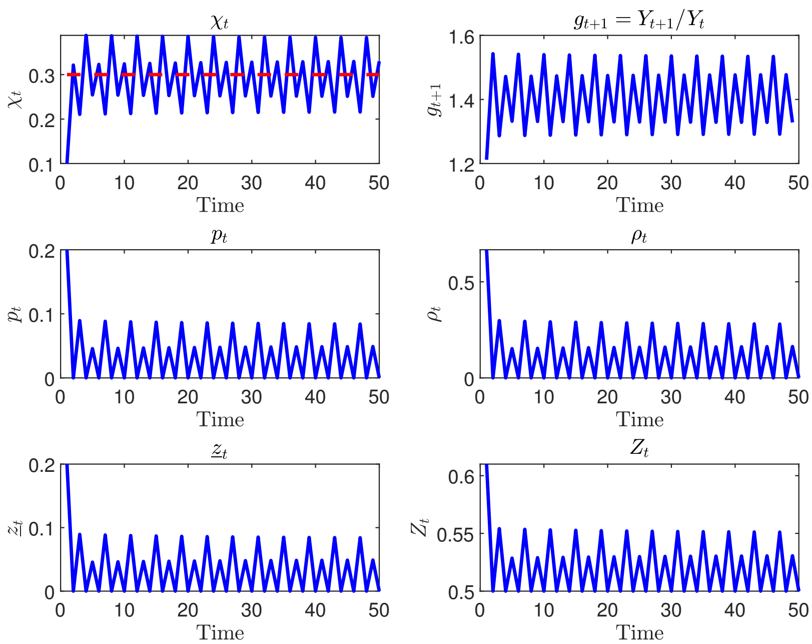
Notes: Parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$ and $\phi = 0.38$.

Figure A.5. Transition dynamics under chaos



Notes: Parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$ and $\phi = 0.85$.

Figure A.6. Transition dynamics in a 4-cycle



Notes: Parameters used are $\beta = 0.96$, $\mu = 20$, $\delta = 0.05$ and $\phi = 0.3$.